Math 524 – Learning Objectives

This handout describes specific learning objectives for Math 524. What is a learning objective? This is something that you should know and also something you should be able to do by the end of the class. In particular, these are the things that you must be able to do for the exams.

1. Material up to the midterm

By the time of the midterm, a student in Math 524 will be able to:

- **State** the definition of formal power series, generating functions, exponential generating function.
- **Manipulate** generating functions via basic operations: addition and multiplication of generating functions, substituting one generating function into another, differentiating and integrating generating functions.
- **State** and **apply** the fundamental theorem of rational generating functions. **Solve** linear recurrences with constant coefficients. **Find** the ordinary generating function for polynomial sequences.
- **Use** standard generating functions in calculus to prove combinatorial identities: Newton’s binomial theorem, Taylor series
- **State** and **prove** key results about binomial and multinomial coefficients and things that they count: subsets of $[n]$, $k$-compositions, weak $k$-compositions, lattice paths, words, multisets.
- **Use** generating functions and combinatorial arguments to prove results about binomial and multinomial coefficients.
- **Define** various statistics on permutations in the symmetric group: cycle type, inv$(w)$, $D(w)$, des$(w)$, signless Stirling numbers of the first kind. **Explain** how to derive main theorems for generating functions involving these statistics.
- **Use** the generating function for the cycle indicator of the symmetric group to find the exponential generating function for various sequences of numbers associated to the symmetric group.
- **State** the definition of the $q$-analogue of $n$, the $q$-analogue of $n!$ and the $q$-binomial coefficients and give combinatorial interpretations for the coefficients of $q^l$ in these polynomials. **Use** combinatorial interpretation to prove results about these $q$-analogues. **Relate** $q$-analogues to counting problems about vector spaces over finite fields, when appropriate.
- **Derive** generating functions for partition numbers and for counting problems on partitions with restrictions on number and type of parts. **Use** generating functions and combinatorial arguments to prove equalities between various partitions numbers.
- **Derive** all entries in the 12-fold way table.
- **Prove** basic facts about set partitions, Stirling numbers of the second kind, and Bell numbers. **Derive** formulas for these numbers using combinatorial arguments and generating functions.
2. Material after the midterm

- *State* the principle of inclusion/exclusion in both the set version and the function version. *Use* the principle of inclusion/exclusion to solve counting problems. *Apply* to familiar counting problems including counting derangements.
- *State* main definitions involving posets: definition of poset, chain, antichain, order ideal, rank function, graded poset, lattice, distributive lattice, rank generating function $F(P, x)$, linear extension, atoms, semi-modularity, geometric lattice.
- *Work with* familiar examples of posets: $B_n$, $D_n$, $\Pi_n$, $B_n(q)$.
- *Construct* new posets from old posets using various operations: disjoint union, ordinal sum, direct product, dual poset.
- *Prove* that given posets have or lack various properties from the previous list. *Construct* examples of posets having or lacking various combinations of the properties above.
- *Use* the “shortcut” to prove a finite poset is a lattice by only proving the existence of meets or joins (a finite meet semilattice with $\hat{1}$ is a lattice).
- *State* the fundamental theorem of finite distributive lattices. Given a finite distributive lattice $L$ find a poset $P$ such $L = J(P)$.
- *Compute* with the incidence algebra of a poset. *Use* elementary properties of matrices to deduce combinatorial formula for various functions in the incidence algebra: zeta function, mobius function.
- *Compute* the number of maximal chains or linear extensions of a poset using various tools.
- *State* the mobius inversion formula and relate to the principle of inclusion/exclusion. *Compute* the mobius function for our favorite example posets. *Use* mobius inversion to solve various combinatorial problems (e.g. counting aperiodic things).
- *Relate* the mobius function to constructions in geometry and topology: the Euler characteristic of a simplicial complex (via the order complex of a poset), and the theory of hyperplane arrangements.
- *State* the definition of a matroid in terms of independent sets. Give definitions for other key objects associated with a matroid: bases, rank function, circuits, closure, flats. *Prove* basic properties of these objects.
- *Use* equivalent formulations of definition of a matroid (cryptomorphism). *Construct* the lattice of flats of a matroid, and construct the unique simple matroid associated to a geometric lattice.
- *Compute* with key examples of matroids: representable matroids, cycle matroids of graphs, the uniform matroid $U_{k,n}$, Fano plane $F_7$.
- *Use* the pictorial representations of rank 2 and 3 matroids to construct matroids with interesting properties.
- *Compute* the restriction and contraction of a matroid with respect a set.
- *Compute* the characteristic polynomial of a matroid.