Math 421 – Introduction to Probability – Learning Objectives

This handout describes specific learning objectives for Math 421, Introduction to Probability. What is a learning objective? This is something that you should know and also something you should be able to do by the end of the class. In particular, these are the things that you must be able to do for the homework assignments and exams.

By the end of the course, a student in Math 421 will be able to:

1. Material Up to the First Midterm

- **Use** basic counting principles to answer combinatorial counting problems. Principles to be used include: multiplication principle for independent experiments, breaking the set to be counted into easier to count disjoint sets, counting the complimentary set.
- **Define** permutations and use them in combinatorial problems.
- **Define** binomial coefficients and use them in combinatorial problems.
- **State** and **use** the binomial theorem.
- **Define** multinomial coefficients and use them to answer combinatorial problems.
- **State** and **use** basic properties of intersection, union, and complements of sets.
- **State** the three axioms of probability and **use** them to derive basic facts about a probability function.
- **Use** basic properties of probability to calculate probabilities in examples: probability of a complimentary set, breaking up an event into a disjoint union of simpler events.
- **State** and **use** the simple and full version of the principle of inclusion/exclusion to calculate probabilities.
- **Calculate** probabilities of events in common scenarios with equally likely outcomes, including but not limited to: hands of card games, dice rolling problems, balls in urns, etc.
- **Define** conditional probability and calculate it in examples. **Use** basic properties of conditional probability: multiplication rule.
- **Use** the idea of conditioning to simplify the calculation of complex probability problems.
- **State** and **apply** Bayes’ formula (Bayes’ rule) to calculate conditional probabilities.
- **State** definition of independence of events and **determine** whether or not two (or more) events are independent in concrete and abstract examples.

2. Material Between the First and Second Midterms

- **State** the definition of a random variable, and key definitions associated with random variables in the discrete and continuous case including: probability mass function, probability density function, cumulative distribution function, expectation, variance.
- **Define** discrete random variable and state main properties.
- **Define** and **compute with** key examples of discrete random variables and **state** their main properties (probability mass function, expectation, variance) including:
Bernoulli random variable, Binomial random variable, Poisson random variable, and geometric random variable.

- **Compute** with newly encountered discrete random variables including computing expectations and variances.
- **Use** theorem on expectation of sums of random variables to simplify calculations of expected values in complex examples.
- **State** main properties of the cumulative distribution function and use to compute probabilities of events related to random variables.
- **Define** continuous random variables and state main properties.
- **Define** and **compute with** key examples of continuous random variables and state their main properties (probability density function, expectation, variance) including: uniform random variable, normal random variable, exponential random variable.
- **Use** the Poisson approximation to the binomial distribution in appropriate situations.
- **Use** the normal approximation to the binomial distribution (de Moivre-Laplace theorem) in appropriate situations, including incorporating the continuity correction.
- **Compute** the cumulative distribution function and densities of functions of continuous random variables.
- **Identify** random variables in applied problems as being familiar examples of random variables and use these identifications to solve problems.

### 3. Material After the Second Midterm

- **State** the definition of jointly distributed random variables and key definitions associated with joint distribution: joint probability mass function, joint density function, marginal distributions.
- **Define** conditional densities and probability mass functions of jointly distributed random variables. **Compute** conditional densities and probability mass functions in examples.
- **Use** Bayes theorem in context of random variables.
- **State** and **use** convolution formula to compute the density of sums of random variables.
- **Compute** cumulative distribution function and density of functions of continuous random variables.
- **Define** and **compute** the covariance and correlation of jointly distributed random variables.
- **State** and **use** key results on computing expectation and variance of sums of random variables.
- **State** key limit theorems including the Strong Law of Large Numbers and the Central Limit Theorem.