1. Let \( \mathbf{u} = \begin{pmatrix} 1 \\ 2 \\ -4 \\ -2 \end{pmatrix} \) and \( \mathbf{v} = \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix} \).

a. Determine the Euclidean distance between \( \mathbf{u} \) and \( \mathbf{v} \).

b. Find a vector of unit length in the direction of \( \mathbf{u} \).

c. Find the 1- and \( \infty \)-norms of \( \mathbf{u} \) and \( \mathbf{v} \).

d. Find the Manhattan distance between \( \mathbf{u} \) and \( \mathbf{v} \).

2. What is the 2-norm of a unit vector?

3. Describe in words what happens when you take the inner product of a vector with itself, \( \mathbf{x}^T \mathbf{x} \). How does this computation relate to the 2-norm of \( \mathbf{x} \), \( \| \mathbf{x} \|_2 \)?

4. (True/False) When we have a system of equations for regression analysis, \( \mathbf{X}\beta = \mathbf{y} \) which has no exact solutions, the goal of the Least-Squares method is to find a solution \( \hat{\beta} \) such that \( \| \mathbf{X}\hat{\beta} - \mathbf{y} \|_2^2 \) is minimized.

(\text{In case the notation looks confusing, that is the two-norm squared, } \| \mathbf{x} \|_2^2 = (\| \mathbf{x} \|_2)^2.)