The goal of this project is to acquaint you with Matlab and \LaTeX. In class we discussed the linear spring model

\[ m \frac{d^2 y}{dt^2}(t) + c \frac{dy}{dt}(t) + ky(t) = f(t) \]

\[ y(0) = y_0, \quad \dot{y}(0) = v_0 \]

where \( m, c \) and \( k \) respectively denote the mass, damping and stiffness coefficients. We know that the analytic solution to (1) with \( f(t) = 0 \) is

\[ y(t) = e^{-c t / 2m} [A \cos(\nu t) + B \sin(\nu t)] \]

where

\[ \nu = \frac{\sqrt{4km - c^2}}{2m}, \quad A = x_0, \quad B = \left(x_1 + \frac{c}{2m} x_0\right) / \nu. \]

To numerically approximate the solution to (1), we discussed the implicit Euler algorithm

\[ \tilde{z}_{j+1} = [I - \Delta t A]^{-1} \tilde{z}_j + [I - \Delta t A]^{-1} \Delta t \tilde{F}(t_{j+1}) \]

where

\[ A = \begin{bmatrix} 0 & 1 \\ -k/m & -c/m \end{bmatrix}, \quad \tilde{F}(t) = \begin{bmatrix} 0 \\ f(t)/m \end{bmatrix} \]

and \( \tilde{z} = [y, \dot{y}]^T \). The true and approximate solutions, obtained with \( m = 4, c = 2, k = 16 \) and \( y_0 = 2, y_1 = 30 \) are plotted in Figure 1.

**Assignment:** Implement the trapezoid method and \texttt{ode23} for the model (1) and write up your results in \LaTeX. Compare the convergence rate of the trapezoid method with that of the implicit Euler algorithm and include a plot of your results. Include a table of results illustrating that you get the correct convergence rate when you double the number of gridpoints. You can work together on this but submit individual reports.

![Figure 1: True and approximate solutions to the spring model (1).](image-url)