Parameter Selection Techniques

**Reading:** Chapter 6

**Motivation:** Consider spring model

\[ m \frac{d^2z}{dt^2} + kz = 0 \]

\[ z(0) = z_0, \quad \frac{dz}{dt}(0) = 0 \]

with solution \( z(t) = z_0 \cos(\sqrt{k/m} \cdot t). \)

**Observation:** Parameters \( q = [k,m] \) not uniquely determined by displacement data.

**Admissible Parameter Space:** \( \mathbb{Q} = (0, \infty) \times (0, \infty) \)

**Note:** Determination of slope equivalent to specifying \( \theta \)

\[ I(q) = \{ \theta = \arctan(k/m) \mid 0 < \theta < \pi/2 \} \]

\[ NI(q) = \left\{ r = \sqrt{k^2 + m^2} \mid r > 0 \right\} \]

**Note:** \( \mathbb{Q} = I(q) \oplus NI(q) \)

- See Definitions 6.1 and 6.2 for definitions of identifiable and influential subspaces.
Linearly Parameterized Problems

Problem:

\[ y = Aq , \quad y \in \mathbb{R}^n , \quad q \in \mathbb{R}^p \]
Nonlinearly Parameterized Problems

Problem:
\[ y = f(q) \quad \text{or} \quad y = f(t, q) \quad \text{or} \quad y = f(x, q) \]

Note: Two basic strategies (global or pseudo-global sensitivity analysis)
- Variance-based (Sobol) methods
- Methods based on model linearization
  - Local sensitivity-based methods; e.g.,
    \[ \chi_{ij}(q^*) = \frac{\partial f}{\partial q_j}(t_i, q^*) \]
  - Morris screening