1. Solve the system of equations:

\[
\begin{align*}
    x - 3y + z &= -6 \\
    2x + 3y + 3z &= 5 \\
    3x - y + z &= 0
\end{align*}
\]

2. Holiday Airlines wants to fly 1000 members of a travel club to Rome. The Airline owns two types of planes. Each DC-10 can carry 100 passengers and each Lockheed 1011 can carry 200 passengers. Each DC-10 will cost $10,000 for the trip and each Lockheed 1011 will cost $12,000. Each plane requires eight stewardesses and there are only 48 stewardesses available. How many planes of each type should be used to minimize the cost? (You can solve this geometrically or by the simplex method, but show your work either way.)

3. A motorcycle company conducted a survey of 100 people shopping at one of their stores. Of these people, 65 were married, 78 owned autos, and 31 owned motorcycles. Moreover, 60 were married and owned autos, 11 were married and owned motorcycles, and 13 owned both an auto and a motorcycle. Finally, 8 people were married and owned both an auto and a motorcycle.

   (a) How many single people in this survey owned neither an auto nor a motorcycle?
   (b) If a person in this survey is married, what is the probability that he/she owns a motorcycle?

4. (a) A social studies teacher has a list of five different history books and seven different geography books from which each student must choose three history books and five geography books to read. How many ways can a student choose his/her reading assignment?
   (b) If 8 books are placed on a shelf in a row, what is the probability that two particular books are placed next to each other?

5. When "the Force" is with him, young Luke Warmwater has a probability 1/2 of beating his tutor Ben Adobe at chess. When "the Force" is not with Luke, he only has a 1/4 probability of winning. At any time, there is a 3/4 probability that "the Force" is with Luke.

   (a) What is the probability that Luke will win a game of chess with Ben?
   (b) If Luke wins a game of chess with Ben, what is the probability that "the Force" is with him?

6. Tom is playing a game in which he rolls a pair of dice, and he wins $5 if the sum is seven, and loses $1 otherwise. If the dice are fair and Tom plays the game 10 (independent) times, what is the probability that he comes out ahead (wins more money than he loses)?

7. Indicate whether the following transition matrices for Markov chains are regular or absorbing. Justify your answers carefully from the definitions of regular and absorbing Markov chains.

   (a) \[ T = \begin{pmatrix} 1/4 & 1/4 & 1/2 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{pmatrix} \]
   (b) \[ T = \begin{pmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ 1/4 & 1/4 & 1/2 \end{pmatrix} \]
8. In Glenwood, California, there is a shift in population from the city to the suburbs. Each year, 7% of those in the city move to the suburbs, while only 1% of those in the suburbs move to the city.

(a) If the Johnson family lives in the city in August 1999, what is the probability that they live in the city in August 2001?
(b) In 1999, 85% of the metropolitan population lives in the city. If the present trend continues, approximately what percentage of the population will live in the city 50 years from now?
(c) If the present trend continues indefinitely, find the exact theoretical fraction of the population living in the city in the long run.

9. The victims of a new deadly disease being treated at Stanford Medical Center are classified annually as follows: cured, in temporary remission, sick, or dead from the disease. Once a victim is cured, he/she is permanently immune. Each year, those in remission get sick again or are cured with probabilities 1/2 each, while those who are sick get cured, go into remission, or die of the disease with probabilities of 1/3 each. If Jack gets sick with this disease,

(a) What is the probability he is eventually cured?
(b) How many years are expected to pass until Jack is cured or dies from the disease?