THE IMPLICATIONS OF TAX FORESIGHT FOR OPTIMAL MONETARY POLICY

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ABSTRACT. Legislative and implementation lags inherent in the political process often allow private agents to receive news about their future tax rates, giving agents “tax foresight.” This paper investigates the effects of tax foresight on monetary policy under various assumptions about the information set of the monetary authority. We use a simple dynamic sticky price model extended to include possible foresight about changes in distortionary consumption and labor taxes. The optimal discretionary policy calls for the central bank to respond to tax news by moving the interest rate in the opposite direction from the optimal response to a tax realization. Standard Taylor-type rules cannot replicate these interest rate movements. Tax news increases the variances of endogenous variables which can worsen the welfare of private agents. When the monetary authority has partial information about the state of the economy, the presence of tax foresight makes the monetary authority unable to recover the true exogenous disturbances from its observable variables. This causes the monetary authority to induce history dependence in endogenous variables and makes the interest rate serially correlated.

Keywords: Optimal monetary policy, Anticipated taxes, Fiscal and monetary policy interactions

JEL Codes: E52, E63, H3

1. Introduction

Information about when and how taxes will change is often released in advance of an actual tax alteration, giving agents “tax foresight” or foreknowledge about future tax adjustments. Such foresight can result from two types of lags that mainly are attributed to the political process: a “legislative lag” between when a tax law is proposed and when it is passed and an “implementation lag” between when the legislation is signed into law and when it actually takes effect. The length of these lags can vary depending on the particular legislation considered (see Yang (2007) and Romer and Romer (2008)). Yang (2007) documents that since World War II, the average lag between an income tax policy announcement and enactment is seven months in the United States.

Previous theoretical studies of tax foresight demonstrate that the timing of tax changes is crucial for its effects on output, employment, and investment (examples include Mertens and Ravn (2009b), House and Shapiro (2006), and Yang (2005)). In particular, anticipated
tax cuts can be contractionary over the lag period between when a policy is announced and when it takes effect. This suggests that tax foresight’s presence could be important for monetary policy, since foresight has nontrivial consequences on the movements of aggregate variables. However, the theoretical implications of tax foresight for monetary policy remain largely unexplored.¹

In this paper, we characterize the behavior of a central bank when news about future changes in taxes is present in the economy. We focus on tax foresight’s effect(s) on optimal discretionary monetary policy, since discretionary optimization gives a reasonable approximation of the actual behavior of an optimizing central bank. We use a simple Calvo pricing model extended to include possible foresight about distortionary labor and consumption taxes.

We find that the timing of taxes is important for monetary policy and interest rate movements. Unlike unanticipated tax changes, the qualitative response of the nominal interest rate to anticipated tax changes is highly dependent on the manner in which the interest rate is set. When the monetary authority has full information about the state of the economy, the optimal discretionary rule calls for the central bank to respond to tax news by moving the interest rate in the opposite direction from the optimal interest rate response to a tax realization. By doing so, the central bank helps smooth the path of output and inflation. For example, following the realization of a labor tax increase, the monetary authority raises the nominal interest rate to offset increases in inflation. In contrast, the monetary authority decreases the nominal interest rate in response to news of a labor tax increase. News of a labor tax hike leads firms to increase their prices, in anticipation of future declines in output following the tax realization. The expansionary optimal monetary policy, however, increases output and encourages firms to reduce their prices, offsetting the effects of the tax news. Standard Taylor-type rules do not easily replicate these results, as the interest rate response to both tax news and a tax realization under such rules is often qualitatively the same.

We find that news of a tax rate change increases the variances of endogenous variables by introducing moving average components in their solution paths. The variances are increasing functions of the implementation lag.² This, in turn, causes anticipated tax changes to have higher unconditional welfare losses than unanticipated tax changes.

In addition to the importance of the timing of tax changes, we also find that the information set of the monetary authority is crucial for the dynamics. We consider the effects of discretionary monetary policy when there is a particular type of asymmetry between the information available to the central bank and that available to the private sector. Although tax news affects judgmental adjustments to central banks’ forecasts,³ formal central bank models do not explicitly incorporate fiscal news.⁴ We interpret the absence of this explicit modeling as the central bank not observing the tax news. We assume that the central bank

¹An exception is Leeper (1989), which shows that the presence of anticipated lump-sum taxes can reverse the Granger-causal ordering between deficits and seignorage.
²Feve, Matheron, and Sahuc (2009) obtain a similar result with a stylized model.
³Minutes from FOMC meetings document instances where news of future tax changes are discussed. Examples of such discussions are available in additional appendix from the author.
⁴Examples of central banks that have developed DSGE models are the Bank of Canada (ToTEM), Bank of England (BEQM), European Central Bank (NAWM), Sveriges Riksbank (RAMSES), and the US Federal Reserve (SIGMA and EDO). QPM of the Bank of Canada and FRBUS of the US Federal Reserve both allow
has limited information and does not directly observe the realizations of the structural and fiscal policy disturbances. Instead, it observes without noise the current values and history of a subset of variables. To focus on the implications of the monetary authority’s limited information set, we abstract from the private agents having any information uncertainty. That is, private agents have full information about the current state of the economy, observing the current and past realizations of all exogenous disturbances.

We assume that the central bank uses the Kalman filter to optimally extract information about the structural and policy disturbances from its observable variables. We solve for the monetary authority’s optimal discretionary policy using the method of Svensson and Woodford (2004). Even though the monetary authority and the private agents have asymmetric information sets, certainty equivalence holds in the sense that the optimal policy response to estimates of the state of the economy is independent of the degree of uncertainty present, as demonstrated in Svensson and Woodford (2004). However, the central bank’s estimation is not independent of its policy choice, which causes the monetary authority’s information set to affect the structure of the equilibrium. With limited information, the timing of taxes is essential. In the absence of tax foresight, the solution paths of endogenous variables are the same as those with full information. However, in the presence of fiscal foresight, the monetary authority is not able to correctly estimate the exogenous disturbances. Instead of recovering the current disturbances, the central bank recovers a linear combination of the current and past disturbances.

We quantify how the resulting equilibrium differs from the full information case and how the equilibrium depends on the set of observables available to the monetary authority. The response of the nominal interest rate can qualitatively vary from the response under full information, and the solution paths of endogenous variables can exhibit history dependence (even in a completely forward-looking model with i.i.d disturbances and no history dependence given full information). Previous research investigating the effects of optimal policy have found similar results when the central bank observes a limited set of variables with measurement error (see Aoki (2003) and Aoki (2006)). Our result differs in that it holds even when the central bank observes a subset of variables without noise. The result demonstrates that foresight alone affects the monetary authority’s optimal policy in an economy where agents have asymmetric information sets.

Recently, there has been increased interest in characterizing the theoretical and empirical effects of various types of anticipated disturbances: news about future tax changes (Blanchard and Perotti (2002), Mountford and Uhlig (2009), Romer and Romer (2010), and Mertens and Ravn (2009a)); anticipated changes in government spending (Ramey (2009) and Mertens and Ravn (2010)); news about future technology (Beaudry and Portier (2006) and Jaimovich and Rebelo (2009)); and anticipated wage mark-up disturbances (Winkler and Wohltmann (2009)). Any model with anticipated disturbances is susceptible to the issues outlined in this paper. Any anticipated cost push disturbance will have the same effects as an anticipated labor tax disturbance in our model, and any anticipated demand disturbance will have the same effects as an anticipated consumption tax disturbance in the model. Adding real and nominal frictions to our model will diversify the effects of different disturbances, but it will not overturn the importance of the timing of disturbances and information sets for anticipated and unanticipated policy changes. However, Sims (2002) suggests that the FRBUS model is rarely used for anticipated policy scenarios in practice.
of agents for the evolution of the nominal interest rate and aggregate variables. This suggests that the timing of structural and policy disturbances, as well as the information sets of economic agents, is central for understanding the effects of fiscal and monetary policy. Several recent empirical studies emphasize the role of anticipated disturbances as sources of macroeconomic fluctuations,\(^5\) which suggests that careful modeling and comprehension of these issues is crucial for policy analysis.

This paper is organized as follows. Section 2 outlines the model used in the analysis. Section 3 shows how tax foresight affects the solution paths of endogenous variables when the monetary authority has complete information and discusses the differences between the monetary authority’s response under discretionary policy and various Taylor-type rules. Section 4 sets up and solves the model when the monetary authority has limited information about the state of the economy and uses the Kalman filter to estimate the disturbances. It then examines the consequences of such limited information. Finally, section 5 concludes.

2. The Model

The model is a canonical New Keynesian model,\(^6\) extended to allow for advance news of tax policy changes. The economy consists of households, a continuum of monopolistic goods producing firms, a monetary authority, and a fiscal authority.

2.1. Households. There is a continuum of households of measure one. A representative household seeks to maximize the expected utility of consumption and leisure:

\[
E_t \sum_{t=0}^{\infty} \beta^t \left\{ \left[ \frac{C_t^{1-\sigma^{-1}}}{1-\sigma^{-1}} - \frac{\lambda}{1+\nu}H_{t+1}^{1+\nu} \right] \right\}
\]

where \(\beta \in (0, 1), \sigma > 0, \nu > 0, H_t \equiv \int_0^1 h_t(i)di\) is the total amount of labor supplied, and \(C_t \equiv \left[ \int_0^1 c_t(i)^{\sigma^{-1}}di \right]^{\frac{\sigma}{\sigma-1}}\) is the Dixit-Stiglitz aggregate of consumption of each of the differentiated goods. \(\theta > 1\) is the elasticity of substitution between differentiated goods.

The household’s nominal flow budget constraint is

\[
(1 + \tau_c^t)P_tC_t + B_t = R_{t-1}B_{t-1} + (1 - \tau_{w}^t)W_tH_t + \int_0^1 d_t(i)di - T_t
\]

where \(P_t\) is the price index, \(W_t\) is the nominal wage rate, \(d_t(i)\) are profits that are distributed lump sum to the households, \(T_t\) is a lump-sum tax, \(\tau_c^t\) is a tax on consumption goods, and \(\tau_{w}^t\) is a tax on labor income. For simplicity, we assume that the only assets traded are one-period riskless bonds, \(B_t\). Households take prices and wages as given and choose at each time period \(H_t, B_t,\) and \(c_t(i),\) for all \(i,\) to maximize their utility.

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2.2. Firms. There is a continuum of firms indexed by \( i \in [0, 1] \) producing differentiated goods. Firm \( i \) must produce enough to meet the demand for its good at the chosen price \( p_t(i) \) for its product. Household preferences imply that the demand for good \( i \) is \( y_t(i) = Y_t \left( \frac{p_t(i)}{P_t} \right)^{-\theta} \). Firms produce with a common technology given by \( y_t(i) = A_t h_t(i) \), where \( A_t \) is exogenous technological productivity that evolves according to

\[
\dot{a}_t \equiv \log A_t - \log \bar{A} = u^a_t, \quad u^a \sim N(0, \sigma^2_a).
\]  

Following Calvo (1983), producers fix the prices of their goods for a random interval of time. Each period, a fraction \( \alpha \in [0, 1) \) of producers are unable to change their price. The \( 1 - \alpha \) fraction of suppliers who do change their price in period \( t \) choose an optimal price, \( p^*_t(i) \), that maximizes the expected sum of future profits

\[
E_t \left\{ \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} Q_{t,T} \left[ p_t(i) y_T T \left( \frac{p_t(i)}{P_T} \right)^{-\theta} - \frac{W_T Y_T}{A_T} \left( \frac{p_t(i)}{P_T} \right)^{-\theta} \right] \right\}
\]

where \( \beta^{T-t} Q_{t,T} \) is the household’s stochastic discount factor. The price the firm sets is exclusive of the sales tax, \( \tau^c_t \). Assuming a symmetric equilibrium where \( p^*_t(i) = p^*_t \), the price index evolves according to

\[
P_t = \left[ (1 - \alpha) p_t^{1-\theta} + \alpha P_{t-1}^{1-\theta} \right]^{1/(1-\theta)}
\]

2.3. Monetary Policy. The central bank controls the riskless short-term gross nominal interest rate, \( R_t \). The monetary authority sets policy by minimizing the welfare losses from the welfare-theoretic loss function

\[
L \equiv E_t \left[ \sum_{i=0}^{\infty} \beta^i \left\{ \frac{q_t^\pi^2}{2} \pi^2_{t+i} + \frac{q_t^y}{2} \dot{y}^2_{t+i} \right\} | I_t \right]
\]

where \( \pi_t \equiv \frac{P_t}{P_{t-1}} \) is the gross inflation rate and hats denote variables measured in percentage deviations from their steady-state values in a steady state with zero inflation. \( \dot{y}_t \) is the welfare relevant output gap, expressing the difference between output and its efficient level. Since technology and both types of taxes cause exogenous disturbances to production, the output gap can be written as \( \dot{y}_t = \dot{Y}_t + g_c \dot{x}^c + g_w \dot{x}^w - \psi(1+\nu) \dot{a}_t \) where \( g_w, g_c, \psi > 0 \). \( E[|I_t] \) represents the expectation with respect to the central bank’s information set. Appendix A derives the loss function (6) from a quadratic approximation to the sum of the expected utility of a representative agent, following the method of Benigno and Woodford (2005). Thus, ranking policies in terms of \( L \) is equivalent to ranking policies based on utility maximization. We consider the effects of discretionary policy, where the monetary authority re-optimizes the loss function each period, taking as given the private agents’ behaviors.

2.4. Fiscal Policy. In order to illustrate the effects of tax foresight and keep the model as straightforward as possible, we assume a simple tax information process: agents at time

\[\text{\textsuperscript{7}}g_w, g_c, \text{ and } \psi’s dependence on the underlying structural parameters of the model are derived in Appendix A.\]
receive a signal that tells them exactly what tax rate they will have in period \( t + j \). Specifically, tax rates evolve according to

\[
\hat{\tau}^c_t = u^c_{t-j}, \quad u^c \sim N(0, \sigma^2_c) \tag{7}
\]

\[
\hat{\tau}^w_t = u^w_{t-j}, \quad u^w \sim N(0, \sigma^2_w) \tag{8}
\]

The subscripts on \( u^c \) and \( u^w \) indicate the period in which information about the values of \( u^c \) and \( u^w \) are received. Thus, \( j \) denotes the degree of fiscal foresight present in the economy. Note that these tax rules imply that at time \( t \) agents have perfect knowledge of \( \{\hat{\tau}^c_t + j, \hat{\tau}^w_t + j, \hat{\tau}^c_{t+1}, \hat{\tau}^w_{t+1}, \ldots\} \). \( u^c \), \( u^w \), and \( u^a \) are assumed to be independent and serially uncorrelated at all leads and lags. We assume that lump-sum transfers adjust each period so that the government budget constraint is met.

There is no government consumption, and the aggregate resource constraint for the economy is given by

\[ Y_t = C_t \tag{9} \]

### 2.5. Approximated Model.

The structural equations of the economy can be summarized by a log-linearized Phillips curve and a consumption Euler equation given by

\[
\hat{\pi}_t = \kappa \hat{y}_t + \beta E_t \hat{\pi}_{t+1} + \nu^c \hat{\tau}^c_t + \nu^w \hat{\tau}^w_t \tag{10}
\]

\[
\hat{y}_t = E_t \hat{y}_{t+1} - \sigma (\hat{\tau}_t - E_t \hat{\tau}_{t+1}) + r^n_t \tag{11}
\]

where \( \nu^c \equiv \kappa(\psi - \rho_c) > 0 \), \( \nu^w \equiv \kappa(\psi - \rho_w) > 0 \), and, following Woodford (2003), \( r^n_t \) is the natural rate of interest given by \( r^n_t \equiv \rho_w \hat{\tau}^w_t + (\rho_c - \sigma) \hat{\tau}^c_t - \psi(1 + \nu) \hat{a}_t - [\rho_w E_t \hat{\tau}^w_{t+1} + (\rho_c - \sigma) E_t \hat{\tau}^c_{t+1} - \psi(1 + \nu) E_t \hat{a}_{t+1}] \). Derivations for these equations are given in appendix A.

### 3. Foresight Implications with Full Information

When the monetary authority has full information about the economy, minimizing the loss function (6) subject to the equilibrium conditions (10) and (11) leads to the inflation targeting rule:

\[
\hat{\pi}_t = -\frac{q_y}{q_y \kappa} \hat{y}_t \tag{12}
\]

Equation (12) specifies a targeting rule for optimal monetary policy that is robust in the sense of Giannoni and Woodford (2003). In a model where the central bank has no model uncertainty and perfectly observes \( \hat{y}_t \) and \( \hat{\pi}_t \), fiscal foresight is irrelevant for monetary policy; the monetary authority commits to a rule that is independent of the tax processes. This result is not specific to this particular model; Woodford (2003) shows that many model specifications can be written in terms of a targeting or interest rate rule that is independent of the exogenous processes in the economy. As long as the monetary authority observes the output gap, he will have no problem implementing the optimal policy. However, if the monetary authority does not observe the output gap, fiscal foresight presents some challenges to implementing the optimal policy. These complications are discussed in section 4.
Substituting the inflation targeting equation into the Phillips curve, equation (10), and solving the difference equation that follows leads to the solution path for the output gap:

\[
\hat{y}_t = -E_t \sum_{j=0}^{\infty} \left[ \frac{\beta q_y}{q_y + \kappa^2 q_{\pi}} \right]^{-(j+1)} \frac{\kappa q_{\pi}}{q_y} (\nu_w \hat{\tau}_t^w + \nu_c \hat{\tau}_t^c).
\] (13)

Given our assumptions about the stochastic processes \( \hat{\tau}_t^w \) and \( \hat{\tau}_t^c \), this solution reduces to

\[
\hat{y}_t = -\nu_w q_y \sum_{s=0}^{j} (q_y + \kappa^2 q_{\pi})^{-s} \hat{\tau}_t^w - \nu_c q_y \sum_{s=0}^{j} (q_y + \kappa^2 q_{\pi})^{-s} \hat{\tau}_t^c.
\] (14)

Substituting the solution for the output gap into the targeting rule, equation (12), gives the solution for the inflation path:

\[
\hat{\pi}_t = \frac{\nu_w q_y}{q_y + \kappa^2 q_{\pi}} \sum_{s=0}^{j} (q_y + \kappa^2 q_{\pi})^{-s} \hat{\tau}_t^w + \frac{\nu_c q_y}{q_y + \kappa^2 q_{\pi}} \sum_{s=0}^{j} (q_y + \kappa^2 q_{\pi})^{-s} \hat{\tau}_t^c.
\] (15)

The output gap decreases with either news of a consumption or labor tax increase or the realization of an increase in either tax rate. In contrast, the inflation rate increases following either news or a realization of an increase in either tax rate. The realization of a labor tax increase makes people less willing to work, as they earn less disposable income for each hour worked. This causes wages to rise and output to fall, leading firms to sell their goods at a higher price. Following news of a labor tax increase, future income and output are expected to be lower when the tax change is realized. In anticipation of these changes, firms that can adjust their price will increase it, causing inflation to increase and current output to decline. The realization of a consumption tax increase usually has both demand and supply-side effects: households demand less goods as the price increases and cut back on labor supply. However, with optimal policy the monetary authority is able to perfectly offset changes in demand by adjusting the interest rate. Thus, only the supply-side effects are felt, causing a consumption tax increase to look similar to a labor tax increase. In anticipation of the future supply-side effects, firms that can adjust their price following news of a consumption tax increase will increase their price, leading inflation to rise and output to fall. Notice as well that more recent news is discounted relative to distant news. Although tax rates are discounted in the usual way (this can be seen from equation (13)), tax news is discounted in the opposite manner, as more recent news affects tax rates farther in the future.

Examining equations (14) and (15) reveals that tax foresight increases the variances of endogenous variables (by introducing moving average terms into their solution paths). Feve, Matheron, and Sahuc (2009) obtain a similar result for a simple stylized model economy. This result suggests that if tax foresight is present, the unconditional welfare of agents will be reduced. A common measure of welfare is to calculate the unconditional expectation of the loss function over all possible histories of disturbances (see Woodford (1999) and Giannoni (2001) for examples):

\[
E[L] = E \left[ \sum_{i=0}^{\infty} \beta^i \left( \frac{q_{\pi}}{2} E_t[\hat{\pi}_{t+i}] + \frac{q_y}{2} E_t[\hat{y}_{t+i}] + \frac{q_{\pi}}{2} \text{Var}_t[\hat{\pi}_{t+i}] + \frac{q_y}{2} \text{Var}_t[\hat{y}_{t+i}] \right) \right].
\]

This value indicates the loss (expressed as a percentage of steady state consumption) due to temporary disturbances in excess of the steady-state loss. As can be seen from equations (14) and (15), foresight does not change the unconditional means of the endogenous variables; the
means are zero with or without fiscal news. However, foresight does increase the variability of endogenous variables, which causes reductions in the unconditional welfare of private agents.

The solutions for the output gap and gross inflation rate can be substituted into the consumption Euler equation, equation (11), to derive the solution for the interest rate. When \( j = 1 \), i.e. when there is one period of fiscal foresight, the solution path for the interest rate is given by

\[
\hat{R}_t = \left[ \frac{\kappa q_y \nu_w}{\sigma(q_y + \kappa^2 q_\pi)} + \frac{\nu_w}{\sigma} \right] a_t^w + \left[ \frac{\kappa q_y \nu_c}{\sigma(q_y + \kappa^2 q_\pi)} + \frac{\nu_c}{\sigma} - 1 \right] a_{t-1}^c - \frac{\psi(1 + \nu)}{\sigma} a_t^c \]

\[
+ \left[ \frac{\nu_w q_y (\sigma - \theta)}{\sigma(q_y + \kappa^2 q_\pi)} + \frac{\beta \kappa q_y \nu_c \nu_e}{\sigma(q_y + \kappa^2 q_\pi)^2} - \frac{\nu_c}{\sigma} \right] a_t^w + \left[ \frac{\nu_e q_y (\sigma - \theta)}{\sigma(q_y + \kappa^2 q_\pi)} + \frac{\beta \kappa q_y \nu_e \nu_e}{\sigma(q_y + \kappa^2 q_\pi)^2} - \frac{\nu_e}{\sigma} + 1 \right] a_t^e \]  

(16)

Table 2 indicates the percentage of times the interest rate increases in response to a one percent increase in the tax disturbances for a range of parameter values considered in the literature. The results are based on 250,000 draws from uniform distributions for the parameters \( \sigma, \nu, \alpha, \) and \( \theta \) (see table 1). We assume that \( \theta \in [5, 15] \), implying the steady state price markup is between 7 and 25 percent and within the estimates of the average price markup of U.S. firms (see Basu and Fernald (1995) and Basu and Fernald (1997)). We allow \( \sigma \) to range from zero to three, given the large range of values for the intertemporal elasticity of substitution (see Guvenen (2006) for a survey of the literature). We allow the Calvo pricing parameter, \( \alpha \), to range between zero and one and the inverse of the Frisch labor elasticity, \( \nu \), to range between zero and five. The discount factor, \( \beta \), is fixed at 0.99. The steady state labor and consumption tax rates are set at the mean values of U.S. federal tax rates over the period 1960Q1 to 2008Q1 (constructed following Leeper, Plante, and Traum (2010)).

With the discretionary optimal policy, the central bank always responds to tax news by moving the interest rate in the opposite direction from the response to a tax realization. Following the realization of a labor tax increase, the monetary authority increases the nominal interest rate to offset increases in inflation. In contrast, the monetary authority decreases the nominal interest rate in response to news of a labor tax increase to help smooth the path of inflation and output. The expansionary policy encourages agents to work more and increases output, leading firms to want to decrease their price level. This, in turn, partially offsets the firms’ desires to increase prices due to the expectations of future declines in output following the realization of the labor tax increase. These actions help smooth the path of inflation. Analogously, in order to neutralize the demand effects of a consumption tax increase, the central bank decreases the nominal interest rate with the consumption tax realization and increases the nominal interest rate following news of the consumption tax change. The results suggest that the timing of tax changes is crucial for understanding how tax changes affect interest rates.

3.1. Optimal Policy Comparisons to Taylor Rules. As suggested above, for reasonable parameter calibrations, the central bank’s optimal response to tax news differs qualitatively from its response to a tax realization. In contrast, if the monetary authority follows a simple Taylor rule where the interest rate responds to inflation and some measure of output, then the central bank’s response to tax news and tax realizations are often qualitatively the same.

Table 2 indicates the percentage of times the interest rate increases in response to a one percent increase in the tax disturbances for 250,000 draws from the parameter distributions given in table 1. The results are for three Taylor rules: one that responds to output \( \left( \hat{R}_t = \right) \)
1.5\(\hat{\pi}_t\) + 0.1\(\hat{Y}_t\)), one that responds to the welfare theoretic output gap (\(\hat{R}_t = 1.5\hat{\pi}_t + 0.1\hat{y}_t\)), and one that responds to output growth (\(\hat{R}_t = 1.5\hat{\pi}_t + 0.1[\hat{Y}_t - \hat{Y}_{t-1}]\)). We fix the inflation and the output measure coefficients in the Taylor rules to common estimates in the literature (e.g. Taylor (1993)).

Contrary to the response under discretionary policy, in most cases the interest rate increases following news of a labor tax increase. With the Taylor rule, the response of the nominal interest rate is driven in large part by the increase in inflation, resulting from expectations of future declines in output. The only cases where this does not occur are when \(\kappa\), the measure of the speed of price adjustment, is very high or very low. When \(\kappa\) is close to zero, there is a very high probability that the intermediate firms’ prices will remain fixed (i.e. \(\alpha \approx 1\)). In this case, firms are rarely allowed to adjust prices in anticipation of future declines in output, and inflation is almost constant. Thus, the dominant response of the Taylor rule in this case is to the declines in output, causing the interest rate to decrease. When \(\kappa\) is very high, prices adjust very quickly and approximate a flexible price equilibrium. In this case, households decrease their labor supply following news of a labor tax increase, resulting in reduced demand for goods as income declines. This in turn leads to declines in inflation which cause the nominal interest rate to decrease.

Since increases in consumption taxes have both demand and supply-side effects, the interest rate’s qualitative response to consumption tax news is more variable under the Taylor rules. The response depends on the expectations of future output and inflation following the tax realization. These expectations, in turn, depend on how likely demand versus supply-side effects dominate, which depend on the particular parameter combinations of the model.

Table 3 indicates the percentage of times the interest rate increases in response to a one percent unanticipated increase in the tax and technology disturbances. Contrary to anticipated tax changes, the qualitative responses of the nominal interest rate to unanticipated changes are the same for all monetary policy rules. This further illustrates how central the timing of tax changes is for understanding tax effects and interest rate movements.

### 4. Foresight Implications with Imperfect Information

It is not obvious how the monetary authority can implement the optimal discretionary policy described in the previous section. In a real economy, it is unlikely that the central bank directly observes the exogenous disturbances \(u^a, u^c, u^w\) or knows how private agents will respond to them. Without such knowledge, the monetary authority cannot determine the welfare theoretic output gap necessary to implement its optimal policy. Instead, to implement its policy the central bank must form an estimate of the output gap from the variables it does observe.

To explore the repercussions of this issue in the presence of tax foresight, we now assume that the monetary authority does not observe the path of exogenous disturbances but does perfectly observe a subset of all current and past variables. In order to illustrate the effects

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\(^8\)The responses from the simple Taylor rules approximate the optimal policy response as the Taylor rule inflation coefficient tends to infinity. The larger the interest rate rule’s response to inflation, the less variable inflation is since the central bank stabilizes inflation. In this case, the dominant response of the Taylor rule is always to the declines in output.
of tax foresight and keep the model as tractable as possible, we focus on one period of labor tax news: \( \hat{\tau}_t^w = u_{t-1}^w \), as Yang (2007) documents news of U.S. income tax changes lasting a little over a quarter. Although tax news affects judgmental adjustments to central banks’ forecasts, formal central bank models do not explicitly incorporate fiscal news.\(^9\) We interpret the absence of this explicit modeling as the central bank not observing the tax news.\(^10\) To focus the analysis on the implications of the monetary authority’s limited information set, we abstract from any information uncertainty facing private agents. That is, we assume that private agents have full information about the current state of the economy.

We assume the following time sequence. At the beginning of time \( t \), the exogenous disturbances hit the economy. Then, all endogenous variables are determined simultaneously. Based on the observation of a subset of variables, the central bank forms its estimates of the exogenous disturbances using the Kalman filter and sets the nominal interest rate in order to minimize the loss function (6). With imperfect information, the timing of taxes is essential. In the absence of tax foresight, the solution paths of endogenous variables are the same as those with full information.\(^11\) However, in the presence of fiscal foresight, the monetary authority is not able to correctly estimate the exogenous disturbances and can induce history dependence in the solution paths of endogenous variables.

Our model falls within the general framework of Svensson and Woodford (2004) to determine the optimal discretionary policy when private agents and the monetary authority have asymmetric information sets. As demonstrated in Svensson and Woodford (2004), certainty equivalence holds, given that the loss function is quadratic and the structural equations are linear and given our assumptions about information sets. Thus, the optimal discretionary policy is the same as the policy with full information, except that the central bank responds to an efficient estimate of the state of the economy rather than to its actual value.

The central bank’s estimation and policy is dependent on its set of observables. When the monetary authority’s observables include endogenous variables, the central bank must sort through a simultaneity problem when utilizing the Kalman filter to estimate the exogenous disturbances. Observed inflation and output are forward-looking variables that depend on both the private agents’ current expectations of future inflation and output and the current monetary policy. The central bank’s current expectations and policy, in turn, depend on its estimates of the exogenous disturbances. These estimates depend on the monetary authority’s observations of inflation and/or output.

\(^9\)QPM of the Bank of Canada and FRBUS of the US Federal Reserve both allow for anticipated and unanticipated policy changes. However, Sims (2002) suggests that the FRBUS model is rarely used for anticipated policy scenarios in practice.

\(^10\)Alternatively, one could interpret the central bank’s limited information as reflecting aggregate uncertainty. Private agents, as workers in firms, have more information about variations in production in their firm (due to the technological productivity shocks) than the monetary authority does. In addition, private agents, when given tax news, are able to calculate how much their own personal, future tax rates will change and plan their personal consumption/savings response to such change. In contrast, the monetary authority must determine whether or not agents respond to news and how aggregate average tax rates are affected, which in turn depend on endogenous variables such as output.

\(^11\)This follows from our assumption that the central bank perfectly observes a subset of variables and knows the underlying structural parameters of the economy.
Svensson and Woodford (2004) employ a guess and verify method to solve for the equilibrium in this type of model. We use this method to solve our model and qualify how the resulting equilibrium depends on the central bank’s observables and tax foresight. In what follows, we define $E_t z_s$ to be the rational expectation of the variable $z_s$ given the private agent’s information in period $t$. We define $z_{s|t}$ to be the rational expectation of variable $z_s$ given the monetary authority’s information in period $t$.

4.1. **Technology and Labor Tax Disturbances.** To keep the model as simple and tractable as possible, we assume for now that consumption taxes are not subject to exogenous fluctuations and remain fixed at their steady state level $\tau^*$. Thus, the only disturbances in the economy are unanticipated changes to technological productivity and one-period news of future labor tax changes. With certainty equivalence, the monetary authority ensures the interest rate satisfies its estimate of equation (16):

$$
\hat{R}_t = \psi(1 + \nu) - \frac{\sigma}{f_1} u^a_{t|t} + \frac{\nu q_y}{\sigma(q_y + \kappa^2 q_\pi)} + \frac{\beta \kappa \tau q_\pi}{\sigma(q_y + \kappa^2 q_\pi)^2} u^w_{t|t} + \frac{\kappa \tau q_\pi}{\sigma(q_y + \kappa^2 q_\pi)} u^w_{t-1|t}
$$

(17)

In equilibrium, inflation and output will depend on both the true values of the exogenous disturbances and the monetary authority’s estimates of these variables. We characterize how the central bank’s estimates of $u^a$ and $u^w$ and the resulting equilibrium depend on alternative sets of observables.

4.1.1. **Tax and Inflation Observables.** When the monetary authority observes the sequence of current and past inflation and labor tax rates, $\{\hat{\pi}_{t-j}, \hat{\tau}_{t-j}\}_{j=0}^{\infty}$, inflation and output evolve according to

$$
\begin{bmatrix}
\hat{\pi}_t \\
\hat{Y}_t
\end{bmatrix} = \begin{bmatrix}
-\kappa \psi(1 + \nu) & \kappa \psi f_3(1 + \beta + \kappa) \\
0 & \sigma \psi - f_3(1 + \kappa)
\end{bmatrix} \begin{bmatrix}
u q_y \\
\frac{\beta \kappa \tau q_\pi}{\sigma(q_y + \kappa^2 q_\pi)^2}
\end{bmatrix}
\begin{bmatrix}
u q_y + \kappa \tau q_\pi \\
\frac{\beta \kappa \tau q_\pi}{\sigma(q_y + \kappa^2 q_\pi)}
\end{bmatrix} \begin{bmatrix}
\hat{u}^w_{t|t} \\
\hat{u}^w_{t-1|t}
\end{bmatrix}
$$

(18)

Derivations are given in Appendix B. If the monetary authority correctly estimates the exogenous disturbances, so that $u^a_{t|t} = u^a_{t}$, $u^w_{t|t} = u^w_{t}$ and $u^w_{t-1|t} = u^w_{t-1}$, equation (18) reduces to the solution paths of inflation and output with full information, given by equations (14) and (15). However, given the monetary authority’s observables, its estimates at time $t$ of the exogenous disturbances are

$$
\begin{bmatrix}
\hat{u}^a_{t|t} \\
\hat{u}^w_{t|t} \\
\hat{u}^w_{t-1|t}
\end{bmatrix} = \begin{bmatrix}
\sigma_a^2 \psi(1 + \nu)(\psi - \sigma f_3) \\
\sigma_w^2 (\psi - \sigma f_3)(1 + \beta + \kappa) - \psi(1 + \kappa) \\
\phi \\
\sigma_a^2 \psi(1 + \nu)
\end{bmatrix} \begin{bmatrix}
\hat{\pi}_t \\
\hat{\tau}_t
\end{bmatrix}
$$

(19)

where $\sigma_a^2$ and $\sigma_w^2$ are the variances of the technology and labor tax disturbances and $\phi > 0$ is defined in Appendix B. Substituting this expression into equation (18) gives the solution paths of inflation and output in terms of the exogenous disturbances:

$$
\hat{\pi}_t = \phi_1 u^a_t + \phi_2 u^w_t + \frac{\nu q_y}{q_y + \kappa^2 q_\pi} u^w_{t-1}
$$

(20)

$$
\hat{Y}_t = \phi_3 u^a_t + \phi_4 u^w_t - \frac{\theta_2 q_y + \psi \kappa^2 q_\pi}{q_y + \kappa^2 q_\pi} u^w_{t-1}
$$

(21)
Derivations and expressions for the $\phi$’s are given in Appendix B. Comparing these expressions to equations (14) and (15) reveals that the responses of endogenous variables to the realization of a labor tax change ($u_{t-1}^w$) are the same as the responses with full information. This stems from the fact that the monetary authority directly observes the realization of labor tax changes at time $t$ by observing the labor tax rate $\hat{\tau}_t^w$. However, because the observables do not span the same space as the exogenous disturbances, the central bank does not recover the true values of $u_t^a$ and $u_t^w$ from the Kalman filter. To see this, we express the monetary authority’s estimates of $u_t^a$ and $u_t^w$ in terms of their true underlying values by substituting equation (20) into equation (19) and simplifying, which leads to

$$u_{t+1}^a = \frac{\sigma_a^2 q_y (1 + \theta \kappa)^2 (1 + \nu)^2 u_t^a + \sigma_a^2 q_y (1 + \theta \kappa)(1 + \nu) (1 + \beta + \kappa \sigma) - q_y [\beta + \kappa (\sigma - \theta)]}{\phi} u_t^w$$

(22)

$$u_{t+1}^w = \frac{\sigma_a^2 q_y (1 + \theta \kappa)(1 + \nu) (1 + \beta + \kappa \sigma) - q_y [\beta + \kappa (\sigma - \theta)]}{\phi} u_t^a + \frac{\sigma_a^2 q_y [\beta + \kappa (\sigma - \theta)] - \Phi (1 + \nu) (1 + \beta + \kappa \sigma)}{\phi} u_t^w$$

(23)

where $\phi$ is defined in Appendix B. The responses of endogenous variables to changes in technological productivity or labor tax news differ from the responses with full information. Figure 1 plots the first coefficient of equation (22), which gives the weight the monetary authority’s estimate of the technology shock assigns to the shock’s true value, as a function of various parameters. The figure also plots the second coefficient of equation (23), which gives the weight the monetary authority’s estimate of the labor news assigns to the true value of the news, as a function of various parameters. Although the central bank does very well in estimating the technology disturbance for most parameter values, it does very poorly in estimating the labor tax news, since very little weight is placed on the true value of the labor tax news, $u_t^w$.

Figure 2 gives the impulse response functions for news in period one of a 1% labor tax increase when the central bank has limited information (dashed lines). The results are compared to the result with full information (solid lines), the policy the monetary authority would like to achieve. Impulse responses were generated from parameter calibrations common in the literature: $\alpha = 0.75$, $\theta = 8$, $\nu = 1.5$, $\sigma = 0.5$, and $\sigma_a = \sigma_w = 1$. Because the central bank can identify past tax disturbances, the impulse responses differ only in the initial period, when the central bank’s estimate of the disturbance is inaccurate. Because of the small weight the monetary authority’s estimate of labor tax news gives to the true labor tax disturbance, the nominal interest rate and inflation barely move in period one. The responses make it appear as if agents are unresponsive to news, as inflation and the nominal interest rate do not change significantly until period two when the labor tax increase is enacted. In actuality, agents are responding to news and the muted response is driven by the central bank’s actions.

4.1.2. Inflation and Technological Productivity Observables. When the monetary authority observes the sequence of current and past inflation and technological productivity, $\{\hat{a}_{t-j}, \hat{\pi}_{t-j}\}_{j=0}^\infty$, its estimates of the exogenous variables are of the form

$$\begin{bmatrix} u_t^a \\ u_t^w \\ u_{t-1}^w \\ u_{t-1}^a \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & c_1 & 0 \\ 0 & c_2 & 0 \end{bmatrix} \begin{bmatrix} u_{t-1}^a \\ u_{t-1}^w \\ u_{t-2}^w \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ c_3 & c_4 \\ c_5 & c_6 \end{bmatrix} \begin{bmatrix} \hat{a}_t \\ \hat{\pi}_t \end{bmatrix}$$

(24)
for some constant $c_i$'s. Notice that this implies that the central bank’s estimate of the labor tax news, $\hat{u}_{t|t-1}^w$, depends on the entire history of past observables. Thus, instead of recovering the correct estimate of the tax news, the monetary authority recovers a weighted average of all current and past exogenous disturbances. This, in turn, causes the solution paths of endogenous variables to exhibit history dependence. To see this, note that substituting equation (24) into equation (28) implies that the interest rate is set according to the rule
\begin{equation}
\hat{R}_t = c_1 \hat{R}_{t-1} + (f_1 + f_2 c_3)(1 - c_1 L)u_t^a + (f_2 + f_3 c_2 L)c_3 u_t^a \\
+ f_2 c_6(1 - c_1 L)\hat{\pi}_t + (f_2 + f_3 c_2 L)c_4 \hat{\pi}_t
\end{equation}

Equation (25) gives a feedback rule for optimal policy that is dependent only on observable variables. The rule implies that the interest rate responds to the lagged interest rate. The degree of interest-rate smoothing depends on the value of $c_1$, the parameter governing how much weight the central bank places on its previous estimate of tax news when updating its estimate. A well-documented result in the literature on monetary policy is that the U.S. federal funds rate is serially correlated. Looking at the solution path for the interest rate, it is clear that the interest rate will be serially correlated in this set-up. The reason why the optimal policy feedback rule involves the lagged interest rate is that the central bank needs to use the past interest rate, as well as past observables, to help identify the tax disturbances. This is due entirely to the presence of tax foresight, which causes previous variables to contain information about current tax rates.

The interest rate is not the only variable that exhibits history dependence. The solution paths for inflation and output are of the form:
\begin{align}
\hat{\pi}_t &= \zeta_1^\pi \hat{\pi}_{t-1} + (\zeta_2^\pi - \zeta_3^\pi \zeta_4^\pi L)u_t^a + (\zeta_4^\pi - \zeta_5^\pi \zeta_6^\pi L - \zeta_7^\pi \zeta_8^\pi L^2)u_t^w \\
\hat{Y}_t &= \zeta_1^\pi \hat{Y}_{t-1} + (\zeta_2^\pi - \zeta_3^\pi \zeta_4^\pi L)u_t^a + (\zeta_4^\pi - \zeta_5^\pi \zeta_6^\pi L - \zeta_7^\pi \zeta_8^\pi L^2)u_t^w
\end{align}

for some constant $\zeta_j$’s. The history dependence is a direct consequence of the presence of fiscal foresight and the limited information available to the central bank. Interestingly, even in this simple set-up where the structural disturbances are uncorrelated, the solution paths of endogenous variables involve lagged endogenous variables. This is due to the fact that inflation and technological productivity are not sufficient statistics for the identification of the underlying exogenous disturbances. Previous research investigating the consequences of the monetary authority having limited information [see Aoki (2003) and Aoki (2006)] has focused on circumstances where the monetary authority observes a noisy measurement of current variables. In such cases, the monetary authority’s policy can induce history dependence in the solution path of variables only if the structural shocks follow AR($p$) processes. Our analysis shows that the central bank’s policy can induce history dependence if some disturbances have MA($q$) components, even if the monetary authority’s observables do not have noise.

Returning to figure 2, we compare how different information sets affect the equilibrium dynamics. When taxes are not included in the observables (the dotted-dashed lines), the

---

12 Analytical solutions are not available in this case. These results are based on numerical results for various parameter calibrations. For a nonexplosive, unique equilibrium to exist, $|c_1| < 1$ and $|c_2| < 1$.

13 Examples include Goodfriend (1991) and Rudebusch (1995).

14 In models without foresight, it is necessary to introduce such noise to make the partial information case nontrivial.
effects of labor tax disturbances are more persistent. This is because the central bank's policy creates history dependence in the solution paths of variables (this can be seen by comparing equations (15) and (26)). Although the monetary authority decreases the nominal interest rate when he has full information (see the dashed line), the central bank increases the interest rate following labor tax news in this case. The response is driven by the history dependence in the interest rate rule, given by equation (25), which causes the interest rate’s response to be smoothed over time.

4.2. Consumption and Labor Tax Disturbances. We now assume that technological productivity is not subject to exogenous fluctuations and remains fixed at its steady state level $\bar{A}$. Thus, the only disturbances in the economy are unanticipated changes to consumption taxes and one-period news of future labor tax changes. With certainty equivalence, the monetary authority ensures the interest rate satisfies its estimate of equation (16):

$$\tilde{R}_t = \frac{\kappa q \nu_c}{\sigma(q_y + \kappa^2 q_y)} - \frac{\varphi_c}{\sigma} - 1 u^i_{it} + \left[ \frac{\nu q \nu_c (\sigma - \theta)}{\sigma(q_y + \kappa^2 q_y)} + \frac{\beta \kappa q \nu_c q_y}{\sigma(q_y + \kappa^2 q_y)^2} - \varphi_w \right] u^w_{it} + \left[ \frac{\kappa q \nu_w}{\sigma(q_y + \kappa^2 q_y)} + \frac{\varphi_w}{\sigma} \right] u^w_{t-1,i} \quad (28)$$

Figure 2 gives the impulse response functions for news of a 1% unanticipated increase in the consumption tax rate and news of a 1% increase in the labor tax rate when the central bank has limited information and various sets of observables. The results are compared to the result under discretionary policy with full information (solid line), the policy the monetary authority would like to achieve. Impulse responses were generated from parameter calibrations common in the literature: $\alpha = 0.75$, $\theta = 8$, $\nu = 1.5$, $\sigma = 0.5$, and $\sigma_c = \sigma_w = 1$.

When the labor tax rate is observable (dotted-dashed line), the monetary authority correctly estimates the history of labor tax disturbances and only has trouble identifying the current tax news. Thus, the impulse responses following a 1% labor tax increase differ from the response under full information only in the initial period. At the same time, the monetary authority attributes part of the consumption tax disturbance to the labor tax news, causing the responses following a consumption tax increase to differ quantitatively from the full information case.

When the consumption tax rate is observable (dashed line), the responses of all endogenous variables to a consumption tax shock match the responses under full information. In contrast, the responses to labor tax news are more persistent than the full information case. Similarly to the case in section 4.1.2, when the labor tax rate is no longer observable, the monetary authority incorrectly estimates the entire history of labor tax disturbances, causing the solution path of endogenous variables to exhibit history dependence. When inflation and output are observables (dotted line), the responses following both the consumption and labor tax disturbances are more persistent than the full information case. This is because the monetary authority incorrectly estimates both disturbances in this case, which causes the interest rate’s operational policy rule to include the lagged interest rate as well as lagged inflation and output. This, in turn, induces history dependence in the solution paths of all endogenous variables.

4.3. General Implications. Although the above results were for specific, simple examples, several results generalize. As long as the tax process is exogenous and the tax rate is observable, the central bank will be able to correctly estimate all past exogenous tax disturbances. For instance, this result holds for more general tax specifications, such as $AR(p)$ processes.
As long as taxes are observable, the monetary authority can recover the realization of tax changes at time $t$ (that is, $u_{t-j}$ with $j$ periods of foresight) and will always respond to the realization in the same manner as in the full information case. When the central bank does not include taxes in its observables, the solution paths of endogenous variables will exhibit history dependence, no matter which observables the monetary authority uses. Without taxes as an observable, the monetary authority cannot correctly recover the history of past tax disturbances. Instead, the estimate of the tax news will be a weighted average of all current and past exogenous disturbances.

In general, the central bank’s policy induces history dependence in the model and thus increases the variances and serial correlations of endogenous variables. Unless the monetary authority directly observes a particular exogenous disturbance, tax foresight’s presence will make it impossible to correctly estimate that disturbance. Thus, even inferences from the central bank’s estimates of unanticipated disturbances will be misleading. Qualitatively, these results hold for longer periods of tax foresight as well.

It is important to note that our examples are for the simplest and best-case scenarios possible. With more complicated stochastic processes for the exogenous disturbances and a more complicated model, the innovations that the monetary authority recovers could differ more substantially from the true exogenous disturbances. In reality, tax rates are likely to respond to endogenous variables. In such cases, including tax rates as an observable would not guarantee that the monetary authority would correctly estimate all past exogenous tax disturbances. Our results suggest that more realistic modeling of both the tax process and the monetary authority’s information set are important for understanding the effects of tax policy.

5. Conclusion

We have shown that the timing of taxes is crucial to monetary policy and interest rate movements. When tax foresight is present, the dynamics of a simple New Keynesian model—one with three equilibrium conditions describing output, inflation, and interest rate movements—can vary substantially depending on the monetary authority’s policy and information set.

When the central bank has full information about the economy, foresight does not alter the optimal discretionary rule. The monetary authority commits to an inflation targeting rule regardless of the tax process. The discretionary rule calls for the central bank to respond to tax news by moving the interest rate in the opposite direction from the discretionary interest rate response to a tax realization. This action offsets private agents’ responses to anticipated movements in future output and inflation, and allows the central bank to smooth the path of output and inflation. In contrast, standard Taylor-type rules cannot replicate these interest rate movements, since the interest rate response to both tax news and a tax realization is qualitatively the same for standard calibrations.

\footnote{For instance, recent empirical work has found that income taxes respond to contemporaneous output and to government debt (examples include Blanchard and Perotti (2002) and Leeper, Plante, and Traum (2010)).}
When the central bank does not observe the structural and policy disturbances, the central bank’s policy can change the structure of the equilibrium. To implement the discretionary policy, the monetary authority estimates the exogenous disturbances using the Kalman filter and its set of observables. Based on these estimates, the central bank sets the interest rate to minimize its loss function. In the presence of tax foresight, the monetary authority cannot correctly estimate the exogenous disturbances. This is due to the fact that the central bank’s observables are not sufficient statistics to separately identify responses to news versus actual exogenous changes in the economy. If the tax rate is not included as an observable, the central bank responds to past observables, as they contain information about future tax rates. This causes the monetary authority’s optimal feedback rule to respond to the lagged interest rate and induces history dependence in the solution paths of all endogenous variables.

In general, the central bank’s policy induces history dependence in the model and thus increases the variances and serial correlations of endogenous variables. Unless the monetary authority directly observes a particular exogenous disturbance, tax foresight’s presence will make it impossible to correctly estimate that disturbance. Qualitatively, these results will hold for more general ARMA($p,q$) specifications for the exogenous processes as well.

Taken together, these results highlight the importance of considering both the timing of taxes and monetary and fiscal policy interactions when setting policy. As has been demonstrated, the monetary authority’s actions can influence substantially the transmission of tax changes when tax foresight is present. More generally, any model with anticipated disturbances is susceptible to these issues. Our results demonstrate that anticipated disturbances can have important implications for monetary policy. Thus, understanding the timing of structural and policy disturbances, as well as the information sets of economic agents, is crucial for policy analysis.
THE IMPLICATIONS OF TAX FORESIGHT FOR OPTIMAL MONETARY POLICY

<table>
<thead>
<tr>
<th>Fixed Parameters</th>
<th>Varying Parameters’ Distributions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta ), discount factor 0.99</td>
<td>( \alpha ), price stickiness Uniform(0,1)</td>
</tr>
<tr>
<td>( \tau^c ), cons. tax rate 0.223</td>
<td>( \theta ), price markup Uniform(5,15)</td>
</tr>
<tr>
<td>( \tau^w ), labor tax rate 0.028</td>
<td>( \sigma ), elast. of substitution Uniform(0,3)</td>
</tr>
<tr>
<td>( \nu ), labor preference Uniform(0,5)</td>
<td></td>
</tr>
</tbody>
</table>

**Table 1. Parameters.** Fixed parameter values and uniform distributions for parameters allowed to vary.

<table>
<thead>
<tr>
<th>Monetary Policy</th>
<th>( u^w_t ) ↑ ⇒ ( \hat{R}_t ) ↑</th>
<th>( u^w_{t-1} ) ↑ ⇒ ( \hat{R}_t ) ↑</th>
<th>( u^c_t ) ↑ ⇒ ( \hat{R}_t ) ↑</th>
<th>( u^c_{t-1} ) ↑ ⇒ ( \hat{R}_t ) ↑</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discretionary Rule</td>
<td>0%</td>
<td>100%</td>
<td>100%</td>
<td>0%</td>
</tr>
<tr>
<td>Taylor Rules:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \hat{R}_t = 1.5 \hat{\pi}_t + 0.1 \hat{Y}_t )</td>
<td>95%</td>
<td>100%</td>
<td>48%</td>
<td>0%</td>
</tr>
<tr>
<td>( \hat{R}_t = 1.5 \hat{\pi}_t + 0.1 \hat{y}_t )</td>
<td>79%</td>
<td>100%</td>
<td>48%</td>
<td>0%</td>
</tr>
<tr>
<td>( \hat{R}_t = 1.5 \hat{\pi}_t + 0.1 (\hat{Y}<em>t - \hat{Y}</em>{t-1}) )</td>
<td>90%</td>
<td>100%</td>
<td>13%</td>
<td>0%</td>
</tr>
</tbody>
</table>

**Table 2. One Period Tax Foresight Economy.** Percentage of times the interest rate increases in response to a one percent increase in the tax news \( (u^w_t, u^c_t) \) and the tax realizations \( (u^w_{t-1}, u^c_{t-1}) \). The results are based on 250,000 draws from the parameter distributions given in table 1.

<table>
<thead>
<tr>
<th>Monetary Policy</th>
<th>( u^w_t ) ↑ ⇒ ( \hat{R}_t ) ↑</th>
<th>( u^w_{t-1} ) ↑ ⇒ ( \hat{R}_t ) ↑</th>
<th>( u^c_t ) ↑ ⇒ ( \hat{R}_t ) ↑</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discretionary Rule</td>
<td>0%</td>
<td>100%</td>
<td>0%</td>
</tr>
<tr>
<td>Taylor Rules:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \hat{R}_t = 1.5 \hat{\pi}_t + 0.1 \hat{Y}_t )</td>
<td>0%</td>
<td>100%</td>
<td>0%</td>
</tr>
<tr>
<td>( \hat{R}_t = 1.5 \hat{\pi}_t + 0.1 \hat{y}_t )</td>
<td>0%</td>
<td>100%</td>
<td>0%</td>
</tr>
<tr>
<td>( \hat{R}_t = 1.5 \hat{\pi}_t + 0.1 (\hat{Y}<em>t - \hat{Y}</em>{t-1}) )</td>
<td>0%</td>
<td>100%</td>
<td>0%</td>
</tr>
</tbody>
</table>

**Table 3. No Tax Foresight Economy.** Percentage of times the interest rate increases in response to a one percent increase in the tax \( u^w_t, u^c_t \) and the technology \( u^c_t \) disturbances. The results are based on 250,000 draws from the parameter distributions given in table 1.
Figure 1. Column one plots the first coefficient of equation (22), which gives the weight the monetary authority’s estimate of the technology shock assigns to the shock’s true value, as a function of various parameters. Column two plots the second coefficient of equation (23), which gives the weight the monetary authority’s estimate of the labor news assigns to the true value of the news, as a function of various parameters.

Figure 2. Impulse responses to news in period 1 of a 1% increase in the labor tax rate effective in period 2. Solid line: Discretionary policy when the central bank has full information. Dashed line: Discretionary policy when the central bank observes inflation and the labor tax rate. Dotted-dashed line: Discretionary policy when the central bank observes inflation and technological productivity.
Figure 3. Impulse responses to a 1% increase in the consumption tax rate (first row) and news in period 1 of a 1% increase in the labor tax rate effective in period 2 (second row). Solid line: Discretionary policy when the central bank has full information. Dashed line: Discretionary policy when the central bank observes inflation and the consumption tax rate. Dotted-dashed line: Discretionary policy when the central bank observes inflation and the labor tax rate. Dotted line: Discretionary policy when the central bank observes inflation and output.
REFERENCES


Appendix A. Derivations of Structural Equations and the Loss Function

This appendix gives derivations of the log-linearized structural equations and the loss function in the text. These derivations are in large part a reproduction of the results in Benigno and Woodford (2005), and the interested reader should refer to Benigno and Woodford (2005) for further derivations.

A.1. Derivation of the Consumption Euler Equation, (11). From the first order conditions of the household’s maximization problem and using the aggregate resource constraint, (9), we get an Euler equation:

\[ R_t = E_t \left( \frac{Y_t^{\Gamma - \sigma^{-1}} \pi_{t+1}}{\beta Y_{t+1}^{\Gamma - \sigma^{-1}} (1 + \pi_t^c)} \right) \]

Log-linearizing this expression and re-writing it in terms of the welfare relevant output gap, \( \hat{y}_t = \hat{Y}_t + \frac{g_c}{\omega} \hat{\pi}_t^c + \frac{g_w}{\omega} \hat{\pi}_t^w - \psi(1 + \nu) \hat{a}_t \), gives equation (11) in the text.

A.2. Derivation of the Phillips Curve, (10). We derive a second order approximation of the Phillips curve since this will be needed to derive the quadratic welfare measure. Reducing this result to a first order approximation will give equation (10) in the text. Assuming a symmetric equilibrium where \( p_t^e(i) = p_t^e \), the solution to equation (4) is

\[ E_t \left\{ \prod_{T=t}^{\infty} (\alpha \beta)^{T-t} \frac{Y_T^{1-\sigma^{-1}}}{1 + \pi_T^c} \frac{(P_T^c)}{(P_t^c)^{\theta-1}} \left( \frac{p_t^e}{P_t^c} \right)^{-\theta} \right\} = E_t \left\{ \prod_{T=t}^{\infty} (\alpha \beta)^{T-t} \frac{\theta}{\theta - 1} \frac{Y_T^{1+\nu} A_T^{-(1+\nu)}}{(1 - \tau_T^{\sigma})} \left( \frac{p_t^e}{P_t^c} \right)^{-\theta(1+\nu)-1} \left( \frac{P_T}{P_t^c} \right)^{\theta(1+\nu)} \right\} \]

where we have substituted for \( Q_{t,T} \) and utilized the first order conditions of the household’s maximization problem. This equation can be written more compactly as

\[ \left( \frac{p_t^e}{P_t^c} \right)^{1+\nu \theta} = K_t \]

where

\[ K_t \equiv E_t \left\{ \prod_{T=t}^{\infty} (\alpha \beta)^{T-t} \frac{\theta}{\theta - 1} \frac{Y_T^{1+\nu} A_T^{-(1+\nu)}}{(1 - \tau_T^{\sigma})} \left( \frac{P_T}{P_t^c} \right)^{\theta(1+\nu)} \right\} \]

\[ F_t \equiv E_t \left\{ \prod_{T=t}^{\infty} (\alpha \beta)^{T-t} \frac{1-\sigma^{-1}}{1 + \pi_T^c} \left( \frac{P_T}{P_t^c} \right)^{\theta-1} \right\} \]

Combining equation (A.2) with the price index gives the aggregate supply equation

\[ \frac{1 - \alpha \Pi_t^{\theta-1}}{1 - \alpha} = \left( \frac{K_t}{F_t} \right)^{\frac{1}{1+\nu \theta}} \]  \( \text{(A.5)} \)

Notice that this implies \( \hat{F} = \hat{K} \) since \( \Pi = 1 \). We rewrite this equation as

\[ \log \left( \frac{1 - \alpha \Pi_t^{\theta-1}}{1 - \alpha} \right) = \frac{\theta - 1}{1 + \omega \theta} [\log F_t - \log K_t] \]

A second order Taylor series expansion of the left-hand side with respect to \( \Pi_t \) around \( \Pi = 1 \) yields

\[ \hat{\pi}_t + \frac{\theta - 1}{2} \pi_t^2 = \frac{1 - \alpha}{\alpha (1 + \nu \theta)} (\hat{K}_t - \hat{F}_t) + O(||\xi||^2) \]  \( \text{(A.7)} \)
where, following Benigno and Woodford (2005), $O(||\xi||^2)$ will be used throughout as a shorthand for $O(||\xi, \hat{X}_{t_0-1}^1, \hat{X}_{t_0}||^3)$ where $\hat{X}_{t_0}$ are state-contingent commitments for period $t_0$. Note that

$$K_t + \frac{1}{2}K_t^2 + O(||\xi||^3) = (1 - \alpha\beta)E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}[\hat{k}_{t,T} + \frac{1}{2}K_t^2] + O(||\xi||^3)$$

(A.8)

$$\hat{F}_t + \frac{1}{2}F_t^2 + O(||\xi||^3) = (1 - \alpha\beta)E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}[\hat{f}_{t,T} + \frac{1}{2}F_t^2] + O(||\xi||^3)$$

(A.9)

where

$$\hat{k}_{t,T} = \frac{(1 + \nu)Y_T - (1 + \nu)\hat{\alpha}_t + \hat{\tau}_t}{\hat{\kappa}_T} + \theta(1 + \nu) \sum_{s=t+1}^{T} \pi_s,$$

(A.10)

$$\hat{f}_{t,T} = \frac{(1 - \sigma^{-1})Y_T - \hat{\tau}_f}{\hat{f}_T} + (\theta - 1) \sum_{s=t+1}^{T} \pi_s,$$

(A.11)

and $\hat{\alpha}_t = \log \frac{A_t}{1 + \nu}$, $\hat{\tau}_t = \frac{T}{1 + \nu} \log \frac{\hat{\tau}_w}{\hat{\tau}_c}$, and $\hat{\tau}_f = \frac{T}{\hat{\kappa}} \log \frac{\hat{\tau}_w}{\hat{\tau}_c}$. Combining equations (A.8) and (A.9) gives

$$\hat{K}_t - \hat{F}_t = (1 - \alpha\beta)E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}[\hat{k}_{t,T} - \hat{f}_{t,T}] + \frac{1}{2}(\hat{k}_T^2 - \hat{f}_T^2)$$

\[= \frac{1}{2}(1 - \alpha\beta) \frac{\alpha(1 + \nu\theta)}{1 - \alpha} \hat{\pi}_T E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \hat{\pi}_T + O(||\xi||^3) \]

(A.12)

Note that we can further expand the first term of equation (A.12):

$$E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}[\hat{k}_{t,T} - \hat{f}_{t,T}] = E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}[\hat{k}_T - \hat{f}_T] + \frac{(1 + \nu\theta)}{1 - \alpha\beta} \sum_{T=t+1}^{\infty} (\alpha\beta)^{T-t} \hat{\pi}_T$$

(A.13)

Also, note that

$$\frac{1}{2}E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}[\hat{k}_{t,T}^2 - \hat{f}_{t,T}^2] = \frac{1}{2}E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}[\hat{k}_T^2 - \hat{f}_T^2] + E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \hat{\pi}_T N_T$$

\[+ \frac{1}{2}(2\theta + \theta \nu - 1)(1 + \theta \nu) \frac{E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \hat{\pi}_T(\hat{\pi}_T + 2V_T)}{1 - \alpha\beta} \]

(A.14)

where

$$N_t \equiv E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}[\theta(1 + \nu)\hat{k}_T + (1 - \theta)\hat{f}_T]$$

$$V_T \equiv E_t \sum_{s=T+1}^{\infty} (\alpha\beta)^{s-T} \hat{\pi}_s$$
Then it follows that
\[
\begin{align*}
\hat{K}_t - \hat{F}_t &= (1 - \alpha \beta)E_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} [\hat{k}_T - \hat{f}_T] + (1 + \nu \theta)E_t \sum_{T=t+1}^{\infty} (\alpha \beta)^{T-t} \hat{\pi}_T - \frac{1}{2} (1 - \alpha \beta) \frac{\alpha(1 + \nu \theta)}{1 - \alpha} \hat{\pi}_t Z_t \\
&\quad + \frac{1}{2} (1 - \alpha \beta)E_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} [\hat{k}_T^2 - \hat{f}_T^2] + (1 - \alpha \beta)E_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} \hat{\pi}_T N_T \\
&\quad + \frac{1}{2} (2 \theta + \theta \nu - 1)(1 + \theta \nu)E_t \sum_{T=t+1}^{\infty} (\alpha \beta)^{T-t} \hat{\pi}_T (\hat{\pi}_T + 2V_T) + \mathcal{O}(||\xi||^3)
\end{align*}
\] (A.15)

This can be written recursively as
\[
\begin{align*}
\hat{K}_t - \hat{F}_t + \frac{1}{2} (1 - \alpha \beta) \frac{\alpha(1 + \nu \theta)}{1 - \alpha} \pi_t Z_t &= (1 - \alpha \beta) [\hat{k}_T - \hat{f}_T + \frac{1}{2} (\hat{k}_T^2 - \hat{f}_T^2)] + \alpha \beta (1 + \nu \theta) E_t \hat{\pi}_{t+1} \\
&\quad + (1 - \alpha \beta) \alpha \beta E_t \pi_{t+1} N_{t+1} \\
&\quad + \frac{1}{2} (2 \theta + \theta \nu - 1)(1 + \theta \nu) \alpha \beta E_t \hat{\pi}_{t+1} (\hat{\pi}_{t+1} + 2V_{t+1}) \\
&\quad + \alpha \beta E_t [\hat{K}_{t+1} - \hat{F}_{t+1} + \frac{1}{2} (1 - \alpha \beta) \alpha(1 + \nu \theta) \pi_{t+1} Z_{t+1}] + \mathcal{O}(||\xi||^3)
\end{align*}
\] (A.16)

Substitute out \( \hat{K}_t - \hat{F}_t \) using equation (A.7):
\[
\begin{align*}
\hat{\pi}_t + \frac{1}{2} \frac{\theta - 1}{1 - \alpha} \hat{\pi}_t^2 + \frac{1}{2} (1 - \alpha \beta) \hat{\pi}_t Z_t &= \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha(1 + \nu \theta)} [\hat{k}_T - \hat{f}_T + \frac{1}{2} (\hat{k}_T^2 - \hat{f}_T^2)] \\
&\quad + (1 - \alpha) \beta E_t \hat{\pi}_{t+1} + \frac{1}{2} (1 - \alpha \beta)(1 - \alpha) \beta E_t \pi_{t+1} N_{t+1} \\
&\quad + \frac{1}{2} (2 \theta + \theta \nu - 1)(1 - \alpha) \beta E_t \hat{\pi}_{t+1} (\hat{\pi}_{t+1} + 2V_{t+1}) \\
&\quad + \alpha \beta E_t [\hat{\pi}_{t+1} + \frac{1}{2} \frac{\theta - 1}{1 - \alpha} \hat{\pi}_{t+1}^2 + \frac{1}{2} (1 - \alpha \beta) \hat{\pi}_{t+1} Z_{t+1}] + \mathcal{O}(||\xi||^3)
\end{align*}
\] (A.17)

Now note that \( N_t \) can be rewritten as:
\[
N_t = \frac{1}{2} (1 + \theta \nu) Z_t + \frac{1}{2} (2 \theta + \theta \nu - 1) \frac{\alpha(1 + \theta \nu)}{(1 - \alpha)(1 - \alpha \beta)} \hat{\pi}_t - \frac{(2 \theta + \theta \nu - 1)(1 + \theta \nu)}{1 - \alpha \beta} V_t
\] (A.18)

Substituting equation (A.18) into equation (A.17) yields
\[
\begin{align*}
\hat{\pi}_t + \frac{1}{2} \frac{\theta - 1}{1 - \alpha} \hat{\pi}_t^2 + \frac{1}{2} (1 - \alpha \beta) \hat{\pi}_t Z_t &= \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha(1 + \nu \theta)} [\hat{k}_T - \hat{f}_T + \frac{1}{2} (\hat{k}_T^2 - \hat{f}_T^2)] \\
&\quad + (1 - \alpha) \beta E_t \hat{\pi}_{t+1} + \frac{1}{2} (1 - \alpha \beta)(1 - \alpha) \beta E_t Z_{t+1} \hat{\pi}_{t+1} \\
&\quad + \frac{1}{2} (2 \theta + \theta \nu - 1) \alpha \beta E_t \hat{\pi}_{t+1}^2 \\
&\quad - (2 \theta + \theta \nu - 1)(1 - \alpha) \beta E_t V_{t+1} \hat{\pi}_{t+1} \\
&\quad + \frac{1}{2} (2 \theta + \theta \nu - 1)(1 - \alpha) \beta E_t \hat{\pi}_{t+1} (\hat{\pi}_{t+1} + 2V_{t+1}) \\
&\quad + \alpha \beta E_t [\hat{\pi}_{t+1} + \frac{1}{2} \frac{\theta - 1}{1 - \alpha} \hat{\pi}_{t+1}^2 + \frac{1}{2} (1 - \alpha \beta) \hat{\pi}_{t+1} Z_{t+1}] + \mathcal{O}(||\xi||^3)
\end{align*}
\] (A.19)
After some algebra and collecting terms, this can be written as

\[ \hat{\pi}_t + \frac{1}{2} \theta - \frac{1}{2} \pi^2_t + \frac{1}{2} (1 - \alpha) \pi_t Z_t = \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha(1 + \nu \theta)} [\hat{k}_T - \hat{f}_T + \frac{1}{2} (\hat{k}^2_T - \hat{f}^2_T)] \]

\[ + \beta E_t \hat{\pi}_{t+1} + \frac{1}{2} \beta(1 - \alpha \beta) E_t \hat{\pi}^2_{t+1} Z_{t+1} \]

\[ + \beta E_t \left[ \frac{1}{2} \theta - \frac{1}{2} \pi^2_{t+1} \right] + \frac{1}{2} \theta(1 + \nu) \beta E_t \hat{\pi}^2_{t+1} + O(||\xi||^3) \]  \hspace{1cm} (A.20)

Define \( X_t \equiv \hat{\pi}_t + \frac{1}{2} \theta - \frac{1}{2} \pi^2_t + \frac{1}{2} (1 - \alpha \beta) \pi_t Z_t + \frac{1}{2} \theta(1 + \nu) \pi^2_t \). Then equation (A.20) can be written as

\[ X_t = \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha(1 + \nu \theta)} [\hat{k}_T - \hat{f}_T + \frac{1}{2} (\hat{k}^2_T - \hat{f}^2_T)] + \frac{1}{2} \theta(1 + \nu) \pi^2_t + \beta E_t X_{t+1} \]  \hspace{1cm} (A.21)

Substituting for \( k_T \) and \( f_T \), we arrive at the second order approximation of equation (A.5):

\[ X_t = \kappa \left\{ \hat{Y}_t + (\nu + \sigma^{-1})^{-1} \left[ \frac{\hat{\pi}_t}{\nu} + \frac{\hat{c}_t}{\nu} \right] - (\nu + \sigma^{-1})^{-1} (1 + \nu) \hat{a}_t + \frac{1}{2} d_{yy} \hat{Y}^2_t - \hat{Y}_t d_{yy} \xi_t + \frac{1}{2} d_{\pi} \hat{\pi}^2_t \right\} \]

\[ + \beta E_t X_{t+1} + \text{t.i.p.} + O(||\xi||^3) \]  \hspace{1cm} (A.22)

where

\[ \kappa \equiv \frac{(1 - \alpha \beta)(1 - \alpha)(\nu + \sigma^{-1})}{\alpha(1 + \theta \nu)} > 0 \]

\[ d_{yy} \equiv 2 + \nu - \sigma^{-1} \]

\[ d_{yy} \xi_t \equiv (\nu + \sigma^{-1})^{-1} \left[ (1 + \nu)^2 \hat{a}_t - (1 + \nu) \hat{\pi}_t^w - (1 - \sigma^{-1}) \hat{c}_t \right] \]

\[ d_{\pi} \equiv \frac{\theta(1 + \nu)}{\kappa} > 0 \]

and t.i.p. refers to terms that are not dependent on policy (i.e. terms that only involve the exogenous variables). To a first order approximation, equation (A.22) reduces to

\[ \hat{\pi}_t = \kappa \left[ \hat{Y}_t + \psi(\hat{\pi}_t^w + \hat{c}_t^w) - \psi(1 + \nu) \hat{a}_t \right] + \beta E_t \hat{\pi}_{t+1} \]

where

\[ \psi \equiv (\nu + \sigma^{-1})^{-1} > 0 \]

Re-writing this expression in terms of the welfare relevant output gap, \( \hat{y}_t = \hat{Y}_t + \rho_\pi \hat{c}_t^w + \rho_y \hat{\pi}_t^w - \psi(1 + \nu) \hat{a}_t \), gives equation (10) in the text.

### A.3. Derivation of the Quadratic Loss Function, equation (6).

The utility function (1) can be written as

\[ E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{Y_{t+i}^{1-\sigma^{-1}} - 1}{1 - \sigma^{-1}} - \frac{\lambda}{1 + \nu} \frac{Y_{t+i}^{1+\nu}}{A_{t+i}^{1+\nu}} \Delta_{t+i} \right] \]  \hspace{1cm} (A.23)

where \( \Delta_t \) is the measure of price dispersion:

\[ \Delta_t \equiv \int_0^1 \left( \frac{p_t(i)}{P_t} \right)^{-\theta(1+\nu)} di \]

A second order Taylor approximation of the first term of equation (A.23) gives

\[ u(Y_t) = \tilde{Y} \tilde{u}_t [\tilde{Y}_t + \frac{1}{2} (1 - \sigma^{-1}) \tilde{Y}^2_t] + \text{t.i.p.} + O(||\xi||^3) \]  \hspace{1cm} (A.24)
where t.i.p. stands for terms that are independent of policy, specifically $\bar{u}$. These terms can be ignored since they are not relevant for the welfare ranking of alternative policies. A second order expansion of the second term of equation (A.23) gives

$$v(Y_t)|\Delta_t = \bar{v}_y\bar{Y} \left[ \frac{\Delta_t - 1}{1 + \nu} + \bar{Y}_t + \frac{1}{2}(1 + \nu)\bar{Y}_t^2 + (\Delta_t - 1)\bar{Y}_t - (1 + \nu)\bar{a}_t - (\Delta_t - 1)\bar{a}_t \right] + \text{t.i.p.} + \mathcal{O}(||\xi||^3)$$

(A.25)

It is useful to express $\Delta_t - 1$ in terms of $\hat{\Delta}_t$. Towards this end, note that using the price index, the price dispersion measure can be written as

$$\Delta_t = \alpha \Delta_{t-1}\Pi_t^{\theta(1+\nu)} + (1 - \alpha) \left( \frac{1 - \alpha \Pi_t^{\theta(1+\nu)}}{1 - \alpha} \right)$$

Taking a Taylor series expansion of this equation around $\bar{\Pi} = 1$ and $\bar{\Delta} = 1$ gives

$$\hat{\Delta}_t = \alpha \hat{\Delta}_{t-1} + \frac{\alpha \theta(1+\nu)(1+\theta\nu)\hat{\pi}_t^2}{2} + \mathcal{O}(||\xi||^3)$$

(A.26)

This equation has no linear inflation terms, thus $\hat{\Delta}_t = \mathcal{O}(||\xi||^2)$ for all $t > t_0$ if $\hat{\Delta}_{t_0-1} = \mathcal{O}(||\xi||^2)$. It follows that $\Delta_t - 1 = \mathcal{O}(||\xi||^2)$ for all $t > t_0$. Substituting this into equation (A.25) gives

$$v(Y_t)|\Delta_t = \bar{v}_y\bar{Y} \left[ \frac{\hat{\Delta}_t}{1 + \nu} + \bar{Y}_t + \frac{1}{2}(1 + \nu)\bar{Y}_t^2 - (1 + \nu)\bar{a}_t \right] + \text{t.i.p.} + \mathcal{O}(||\xi||^3)$$

From the household’s first order condition for labor, $v_y$ can be expressed as a function of $u_y$:

$$v_y = \frac{(1 - \tau^w)(\theta - 1)}{\theta(1 + \tau^c)} u_y$$

Substituting this in the above equation gives

$$v(Y_t)|\Delta_t = (1 - \Phi)\bar{u}_c\bar{Y} \left[ \frac{\hat{\Delta}_t}{1 + \nu} + \bar{Y}_t + \frac{1}{2}(1 + \nu)\bar{Y}_t^2 - (1 + \nu)\bar{a}_t \right] + \text{t.i.p.} + \mathcal{O}(||\xi||^3)$$

(A.27)

where $\Phi \equiv 1 - \frac{(1 - \tau^w)(\theta - 1)}{\theta(1 + \tau^c)} < 1$. Combining equations (A.24) and (A.27) gives a second order expression for the utility function

$$U = \bar{Y}\bar{u}_cE_t \sum_{i=0}^{\infty} \beta^i \left[ \Phi\bar{Y}_t^{\hat{\pi}_t} - \frac{1}{2} \{ (\nu + \sigma^{-1}) - \Phi(1 + \nu) \} \bar{Y}_t^{\hat{\pi}_t^2} + (1 - \Phi)(1 + \nu)\bar{Y}_t^{\hat{\pi}_t}\bar{a}_t^{\hat{\pi}_t} - \frac{1}{1 + \nu} \Phi^{\hat{\pi}_t} \right] + \text{t.i.p.} + \mathcal{O}(||\xi||^3)$$

(A.28)

Since $\alpha < 1$, using equation (A.26), $\hat{\Delta}_t$ can be written in terms of $\hat{\pi}_t$.

$$\hat{\Delta}_t = \alpha^{t+1}\hat{\Delta}_{-1} + \frac{\alpha}{1 - \alpha} \theta(1 + \nu)(1 + \nu\theta) \sum_{s=0}^{t} \alpha^{t-s} \hat{\pi}_t^2 + \mathcal{O}(||\xi||^3)$$

Multiplying this equation by $\beta^t$ and summing over $t$ gives

$$\sum_{t=0}^{\infty} \beta^t \hat{\Delta}_t = \frac{\alpha}{1 - \alpha\beta} \hat{\Delta}_{-1} + \frac{\alpha}{(1 - \alpha)(1 - \alpha\beta)} \theta(1 + \nu)(1 + \nu\theta) \sum_{t=0}^{\infty} \beta^t \hat{\pi}_t^2 + \mathcal{O}(||\xi||^3)$$

Substituting this expression into equation (A.28) gives

$$U = \bar{Y}\bar{u}_cE_t \sum_{i=0}^{\infty} \beta^i \left[ \Phi\bar{Y}_t^{\hat{\pi}_t} - \frac{1}{2} \{ (\nu + \sigma^{-1}) - \Phi(1 + \nu) \} \bar{Y}_t^{\hat{\pi}_t^2} + (1 - \Phi)(1 + \nu)\bar{Y}_t^{\hat{\pi}_t}\bar{a}_t^{\hat{\pi}_t} \right]$$

$$- \bar{Y}\bar{u}_c \frac{(1 - \Phi)\theta}{(\nu + \sigma^{-1})^{-1} \kappa} E_t \sum_{i=0}^{\infty} \beta^i \hat{\pi}_t^{\hat{\pi}_t} + \text{t.i.p.} + \mathcal{O}(||\xi||^3)$$

(A.29)
Multiplying the second order approximation of the aggregate supply equation, (A.22), by $\frac{\Phi\hat{Y}_u}{\kappa}$ and subtracting the resulting equation from equation (A.29) gives:

$$U = -\hat{Y}_cE_t\sum_{i=0}^{\infty}\beta^i\left\{\frac{q_\pi}{2}\pi_{t+i}^2 + \frac{q_y}{2}(\hat{Y}_{t+i} - \hat{Y}_{t+i}^*)^2\right\} + \text{t.i.p.} + O(||\xi||^2) \quad (A.30)$$

where

$$q_\pi = \frac{\theta}{\kappa}q_y > 0, \quad q_y = \nu + \Phi + \sigma^{-1}(1 - \Phi) > 0$$

$$\hat{Y}_{t}^* = \psi(1 + \nu)\hat{a}_t - \omega^w\hat{r}_t^w - \omega_c\hat{r}_t^c$$

$$\omega^w = q_y^{-1}\Phi(1 + \nu)\psi > 0, \quad \omega_c = q_y^{-1}\Phi(1 - \sigma^{-1})\psi$$

Noting that $\hat{C} = \hat{Y}$ and rearranging terms in equation (A.30) gives

$$\frac{(U - \hat{U})}{C\hat{u}_c} \approx -E_t\sum_{i=0}^{\infty}\beta^i\left\{\frac{q_\pi}{2}\pi_{t+i}^2 + \frac{q_y}{2}(\hat{Y}_{t+i} - \hat{Y}_{t+i}^*)^2\right\}$$

which, expresses the welfare approximation as a fraction of steady state consumption. Maximizing this value is the same as minimizing the loss function given by equation (6) in the text.

**Appendix B. Partial Information Solutions**

This Appendix gives derivations of the equilibrium when the monetary authority has imperfect information. We assume there is one period of labor tax news and consumption taxes are kept at their steady state level. The derivations are in large part a reproduction of the general procedure in Svensson and Woodford (2004), and more details for the procedure can be found in Svensson and Woodford (2004). The structural equations (10) and (11) can be written as

$$\begin{bmatrix} X_{t+1} \\ E_t x_{t+1} \end{bmatrix} = \begin{bmatrix} a^{q+1}_{t+1} \\ a^{w+1}_{t+1} \\ u^w_t \\ E_t \hat{a}_{t+1} \\ E_t \hat{Y}_{t+1} \end{bmatrix} + \begin{bmatrix} \kappa\psi(1+\nu) \\ 0 \\ \kappa(\sigma\psi(1+\nu)) \\ \kappa(\psi(1+\nu)) \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \kappa \beta \\ \kappa \beta \\ \kappa \beta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \sigma \beta \\ \sigma \beta \end{bmatrix} + \begin{bmatrix} R_t \\ 0 \end{bmatrix} \quad (B.1)$$

where $X_t \equiv [a^{q+1}_t \ a^w_t \ u^w_{t-1}]$ is a vector of the exogenous variables, $x_t \equiv [\hat{a}_t \ \hat{Y}_t]$ is a vector of the forward-looking variables, $\hat{R}_t$ is the central bank’s policy instrument, and $u_t$ is a vector of i.i.d. shocks with mean zero and covariance matrix $\Sigma_u = \begin{bmatrix} \sigma^2_a & 0 \\ 0 & \sigma^2_w \end{bmatrix}$. The matrices $A$ and $B$ are decomposed according to $X_t$ and $x_t$,

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

**B.1. Inflation and Labor Tax Observables.** In this case, the vector of observable variables, $Z_t$, can be written as

$$Z_t = \begin{bmatrix} \hat{\pi}_t^w \\ \hat{\pi}_t \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_t \\ x_t \end{bmatrix} \quad (B.2)$$
where $D$ is decomposed according to $X_t$ and $x_t$, $D = [D_1 \ D_2]$. Svensson and Woodford (2004) show that in this set-up, certainty equivalence holds and the monetary authority’s policy instrument is a function of the current estimates of the exogenous variables. Thus, the monetary authority sets the interest rate so that equation (28) in the text holds. Notice that this equation can be written as

$$\hat{R}_t = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ F \end{bmatrix} X_{t|t}$$  \hspace{1cm} (B.3)

Furthermore, Svensson and Woodford (2004) show that the estimate of the forward-looking variables is given by

$$x_{t|t} = GX_{t|t}, \text{ where } G = (A_{22} - GA_{12})^{-1}[-A_{12} + GA_{11} + (GB_1 - B2)F] \hspace{1cm} (B.4)$$

The solution for the matrix $G$ is given by

$$G = \begin{bmatrix} -\kappa[\psi(1 + \nu) + \sigma f_1] & \kappa[\psi(\beta + \kappa \sigma) - \sigma f_2 - \sigma f_3(1 + \beta + \kappa \sigma)] & \kappa(\psi - \sigma f_3) \\ -\sigma f_1 & \kappa[\psi - f_2 - f_3(1 + \kappa \sigma)] & -\sigma f_3 \end{bmatrix}$$

We posit that the forward-looking variables are functions of the the monetary authority’s estimates of the exogenous variables and their true values in the form:

$$x_t = G^1 X_t + (G - G^1)X_{t|t}, \hspace{0.5cm} G^1 = \begin{bmatrix} g_{11}^1 & g_{12}^1 & g_{13}^1 \\ g_{21}^1 & g_{22}^1 & g_{23}^1 \end{bmatrix} \hspace{1cm} (B.5)$$

for some matrix $G^1$. Taking $G^1$ as given for the time, we proceed to solve for the monetary authority’s estimates of the exogenous variables, $X_{t|t}$. We then use these estimates to find the fixed point solution to $G^1$. Towards this end, note that it follows from combining equations (B.1)-(B.5) that

$$X_{t+1} = HX_t + u_{t+1} \hspace{1cm} (B.6)$$

$$Z_t = LX_t + MX_{t|t} \hspace{1cm} (B.7)$$

where

$$H \equiv A_{11} + A_{12}G^1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \hspace{1cm} (B.8)$$

$$L \equiv D_1 + D_2G^1 = \begin{bmatrix} 0 & 0 & 1 \\ g_{11}^1 & g_{12}^1 & g_{13}^1 \end{bmatrix} \hspace{1cm} (B.9)$$

$$M \equiv D_2(G - G^1) = \begin{bmatrix} 0 & 0 & 0 \\ -\kappa[\psi(1 + \nu) + \sigma f_1] - g_{11}^1 & g_{12}^1 - g_{12}^1 & \kappa(\psi - \sigma f_3) - g_{13}^1 \end{bmatrix} \hspace{1cm} (B.10)$$

Equations (B.6-B.7) are a transition and measurement equation for a filtering problem. However, as Svensson and Woodford (2004) note, these equations are not expressed as transition and measurement equations for a standard Kalman filter problem because of the appearance of $X_{t|t}$ on the right-hand side of the measurement equation. This is due to the simultaneity problem of having endogenous variables as the monetary authority’s observables. Ignoring this issue for the moment, the Kalman filter updating equation for $X_{t+1|t+1}$ is given by

$$X_{t+1|t+1} = X_{t+1|t} + K(Z_{t+1} - LX_{t+1|t} - MX_{t+1|t+1})$$

$$= X_{t+1|t} + K[L(X_{t+1} - X_{t+1|t})] \hspace{1cm} (B.11)$$

where $K = PL'(LPL')^{-1}$ and $P = \text{Cov}[X_{t+1} - X_{t+1|t}]$ and must satisfy

$$P = H[P - PL'(LPL')^{-1}LP]H' + \Sigma_u \hspace{1cm} (B.12)$$
where $\sigma_a$ and $\sigma_w$ are the standard deviations of the technology and labor tax disturbances. Solving for $X_{t+1|t+1}$ from equation (B.11) gives

$$X_{t+1|t+1} = (I + KM)^{-1}[(I - KL)X_{t+1|t} + KZ_{t+1}]$$

Equation (B.14) gives an updating equation for the central bank’s estimates of the exogenous disturbances. All that remains to be determined is the matrix $G^1$. Following the derivations in Svensson and Woodford (2004), $G^1$ is determined by the relation

$$G^1 = A_{22}^{-1} \{ -A_{21} + [G^1 + (G - G^1)KL]H \}$$

Solving for $G^1$ gives

$$G^1 = \begin{bmatrix} -\kappa \psi (1 + \nu) & \kappa \left[ \psi (\beta + \kappa a) - \sigma f_3 (1 + \beta + \kappa a) \right] & \kappa \psi \\ 0 & \sigma [\kappa \psi - f_3 (1 + \kappa a)] & 0 \end{bmatrix}$$

Given $G^1, K, H, L$ and $M$, we use equation (B.14) to find the updating equation for $X_{t|t}$, which is also equation (19) in the text:

$$\begin{bmatrix} u^a_{t|t} \\ u^w_{t|t} \\ u^w_{t-1|t} \end{bmatrix} = \begin{bmatrix} \sigma_a^2 \psi (1+\nu)(\psi - f_3) \\ \sigma_a^2 (\psi - f_3) [\kappa f_3 (1 + \beta + \kappa a) - \psi (\beta + \kappa a)] \\ 0 \end{bmatrix} \begin{bmatrix} \phi_t \\ \phi_t \\ 1 \end{bmatrix} + \begin{bmatrix} \sigma_a^2 \kappa^{-1} \psi (1+\nu) \\ \sigma_a^2 \kappa^{-1} [\psi (\beta + \kappa a) - \sigma f_3 (1 + \beta + \kappa a)] \\ 0 \end{bmatrix} \begin{bmatrix} \hat{=} \psi_t \\ \hat{=} \psi_t \\ 0 \end{bmatrix}$$

where

$$\phi = \sigma_a^2 \sigma f_3 (1 + \beta + \kappa a) [f_2 + f_3 (1 + \beta + \kappa a)] + \sigma \psi \{ \sigma_a^2 f_1 (1 + \nu) - \sigma_w^2 (\beta + \kappa a) [f_2 + 2f_3 (1 + \beta + \kappa a)] \} + \psi^2 [\sigma_a^2 (1 + \nu)^2 + \sigma_w^2 (\beta + \kappa a)^2]$$

We then find the equilibrium paths for inflation and output from equation (B.5):

$$\begin{bmatrix} \hat{=} \psi_t \\ \hat{=} \psi_t \end{bmatrix} = \begin{bmatrix} -\kappa \psi (1 + \nu) & \kappa \left[ \psi (\beta + \kappa a) - \sigma f_3 (1 + \beta + \kappa a) \right] & \kappa \psi \\ 0 & \sigma [\kappa \psi - f_3 (1 + \kappa a)] & 0 \end{bmatrix} \begin{bmatrix} u^a_{t|t} \\ u^w_{t|t} \\ u^w_{t-1|t} \end{bmatrix} + \begin{bmatrix} -f_1 \kappa a & -f_2 \kappa a & -f_3 \kappa a \\ -f_1 a & -f_2 a & -f_3 a \end{bmatrix} \begin{bmatrix} u^a_{t|t} \\ u^w_{t|t} \\ u^w_{t-1|t} \end{bmatrix}$$

Substituting equation (B.16) into equation (B.17) and simplifying gives the solution paths of output and inflation

$$\hat{=} \psi_t = \phi_1 u^a_{t|t} + \phi_2 u^w_{t|t} + \frac{q_y \psi u^w_{t-1}}{q_y + a^2 \psi}$$

$$\hat{=} \psi_t = \phi_3 u^a_{t|t} + \phi_4 u^w_{t|t} - \frac{q_y \psi u^w_{t-1}}{q_y + a^2 \psi}$$

where

$$\phi_1 = \frac{A}{B}, \quad \phi_2 = \frac{C}{q_y (1 + \theta \kappa) B}$$

$$\phi_3 = \frac{D}{B}, \quad \phi_4 = \frac{E}{q_y (1 + \theta \kappa) B}$$
\[ A = \sigma_y^2 \beta \kappa \psi (1 + \nu) \{ q_y - \Phi (1 + \nu) \} (q_y + \nu \{ q_y \} (1 + \beta + \kappa \sigma))^2 \]

\[ B = (1 + \theta \kappa) \{ q_y^2 + \sigma_y^2 (1 + \kappa \sigma) \} (1 + \nu)^2 + \sigma_y^2 (1 + \nu)^2 \{ q_y \} (1 + \beta + \kappa \sigma) (1 + \nu)^2 \]

\[ C = \sigma_y^2 \beta \kappa \psi \{ q_y \} (1 + \nu)^2 [q_y \beta + q_y \kappa \sigma]^2 \]

\[ D = \psi (1 + \nu) \{ \sigma_q^2 (1 + \nu)^2 (1 + \nu)^2 + \kappa \beta \kappa \sigma] (1 + \nu) - \sigma_q^2 \Phi q_y (1 + \nu)^2 \{ \beta + \kappa \sigma - \beta (1 + 2 \nu) \} (1 + \nu)^2 \]

\[ E = -\beta \psi q_y \{ q_y^2 (1 + \nu)^2 + \sigma_q^2 \kappa \sigma \} (1 + \nu)^2 + \sigma_q^2 \Phi q_y (1 + \nu)^2 \{ \beta + \kappa \sigma \} (1 + \nu)^2 \]

Finally, we can express the monetary authority’s estimates of \( u_t^a \) and \( u_t^w \) in terms of their true underlying values by substituting the solution for inflation into equation (B.16) and simplifying, which leads to

\[ u_t^w = \frac{\sigma_y^2 (1 + \theta \kappa)^2 (1 + \nu)^2}{\varphi} u_t^w + \frac{\sigma_y^2 q_y (1 + \theta \kappa) (1 + \nu) \{ \Phi (1 + \nu) (1 + \beta + \kappa \sigma) - q_y \beta + q_y \kappa \sigma \}}{\varphi} u_t^w \]

\[ u_t^w = \frac{\sigma_y^2 q_y (1 + \theta \kappa) (1 + \nu) \{ \Phi (1 + \nu) (1 + \beta + \kappa \sigma) - q_y \beta + q_y \kappa \sigma \}}{\varphi} u_t^w \]

where

\[ \varphi = \sigma_y^2 (1 + \nu)^2 \frac{(1 + \theta \kappa)^2 (1 + \nu)^2 - 2 \sigma_y^2 \Phi q_y (1 + \nu)^2 (1 + \beta + \kappa \sigma) \beta + \kappa \sigma (1 + \nu)^2}{\varphi} + \sigma_y^2 \Phi^2 (1 + \nu)^2 (1 + \beta + \kappa \sigma)^2 + q_y^2 (\beta - \theta \kappa + \kappa \sigma)^2. \]

(B.18)

B.2. Inflation and Technology Observables. In this section, we outline what changes inflation and technology observables cause to the derivations for the equilibrium given in the previous section. In this case, the vector of observable variables, \( Z_t \), can be written as

\[ Z_t = \begin{bmatrix} \tilde{a}_t \\ \tilde{x}_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_t \\ x_t \end{bmatrix} \]

The observables affect the measurement equation, \( Z_t = LX_t + MX_{t|t} \), where the matrices \( L \) and \( M \) are now given by

\[ L = \begin{bmatrix} 1 & 0 & 0 \\ g_{11} & g_{12} & g_{13} \end{bmatrix} \]

\[ M = \begin{bmatrix} -\kappa \psi (1 + \nu) + \sigma f_1 \\ -g_{12} - g_{13} \end{bmatrix} \]

The Kalman gain \( K \) and the covariance matrix \( P \) for the prediction errors \( X_{t+1} - X_{t+1|t} \) are given by

\[ P = \begin{bmatrix} \sigma_y^2 & 0 & 0 \\ 0 & \sigma_q^2 & 0 \\ 0 & 0 & \sigma_q^2 \end{bmatrix} \]

\[ K = \begin{bmatrix} 1 \\ -g_{11}^2 \\ g_{11} \end{bmatrix} \]

It is important to note that when solving for the matrix \( P \), two solutions are possible: the one given above and \( P = \begin{bmatrix} \sigma_y^2 & 0 & 0 \\ 0 & \sigma_q^2 & 0 \\ 0 & 0 & \sigma_q^2 \end{bmatrix} \). As explained in Rondina and Walker (2010), the two solutions
to the Ricatti matrix $P$ represent two unique information equilibriums.\footnote{An information equilibrium is an equilibrium for particular information sets of agents.} The latter solution is an information equilibrium when information is specified so that all agents can observe the exogenous variables, which is not the case studied here. In contrast, the former solution is an equilibrium when the central bank does not observe the exogenous variables, which corresponds to our information assumptions.

Given these matrices, the equilibrium can be solved for following the procedure in section B.1.