Abstract

This paper uses Bayesian methods to estimate the ‘fiscal limit’ distribution for Greece implied by a rational expectations framework. We build a real business cycle model that allows for interactions among fiscal policy instruments, the stochastic ‘fiscal limit,’ and sovereign default. A fiscal limit measures the debt level beyond which the government is no longer willing to finance, causing a partial default to occur. Using the particle filter to perform likelihood-based inference, we estimate the full nonlinear model with post-EMU data until 2010Q4. We find that the probability of default on Greek debt remained close to zero until 2009, and then rose sharply to the range of 5% to 10% by 2010Q4. The model predicts a probability of default between 60-80% by 2011Q4, while the 2011 surge in the Greek real interest rate is within model forecast bands. The results suggest that a nonlinear rational expectations environment can account for the Greek interest rate path.

JEL Classification: C11; C15; E62; H30; H60
Keywords: Sovereign default risk, sequential Monte Carlo, Bayesian estimation

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† Bank of Canada, 234 Wellington Street, Ottawa, ON K1A0G9, Canada; hbi@bank-banque-canada.ca
‡ North Carolina State University, Nelson Hall 4108, Box 8110 Raleigh, NC 27695-8110; nora.traum@ncsu.edu
1. Introduction

During the past five years, there has been a concern over the fiscal positions of several Eurozone nations, as evidenced by the rapid increase in long-term interest rate spreads of several countries’ bonds against German bonds. For instance, the spread between Greek bonds and German bonds rose from 2.35 percentage points in 2009 to nearly 30 points by the end of 2011.

Such marked increases in spreads have stimulated a debate over whether financial markets have mispriced default risks in the Eurozone periphery countries before and/or after the start of the crisis. Aizenman, Hutchison, and Jinjarak (2012) find a dynamic panel model cannot predict the magnitude of sovereign risk premia in the Eurozone periphery countries during the crisis, either in or out of sample. They conjecture that the market has either mispriced the risk, or has priced based on future, not current, fundamentals. Using a similar approach, De Grauwe and Ji (2012) take a stronger stand, arguing that the financial markets underpriced the sovereign risk before the crisis and overpriced afterwards.

One issue with these approaches is that the reduced-form panel regressions lack the structure to extensively explore how macroeconomic fundamentals explain interest rate dynamics. This paper introduces a different approach to contribute to the pricing debate by estimating a rational expectations model that allows for sovereign default. Using Bayesian method, the structural model is estimated for Greece during the post-EMU period until the end of 2010.

Two key findings emerge. First, the rapid deterioration of confidence in Greek debt over 2011 is consistent with behavior from a simple rational expectations model. Using the estimated structural model, we find that the model-implied default probability remained close to zero from 2001 until 2009, and then rose sharply to the range of 5% to 10% by the fourth quarter of 2010. The model also predicts a probability of default between 60% to 80% by the end of 2011, consistent with the debt restructuring arrangements that took place at the beginning of 2012. In addition, the surge in the real interest rate observed in Greece over 2011 is well within the forecast bands of our rational expectations model. These results suggest that Greek debt was not mispriced in 2011, as the interest rate path can be accounted for by macroeconomic fundamentals. Second, model comparisons based on Bayes factors strongly favor the nonlinear model specification with an endogenous probability of default over a log-linearized model specification that does not allow for default.

We consider a closed economy in which the government finances transfers and expenditures by collecting distortionary income taxes and issuing bonds.\footnote{Ceteris paribus, the assumption that the economy is open and all debt is held by foreigners raises the observed risk premium relative to the closed economy environment, as foreigners do not experience the}
not enforceable and depends on the maximum debt-to-GDP ratio that the government is politically able to service, a so-called ‘fiscal limit.’ Given the stochastic environment, we do not assume that the fiscal limit is a deterministic point, but rather follows a distribution. Previous studies have shown that economic models can give rise endogenously to such a distribution through dynamic Laffer curves and the political inability to cut public expenditures (see Bi (2012), and Juessen, Linnemann, and Schabert (2011)). In the case of Greece, the protests against austerity measures in 2010 and 2011 suggest the relevance of political considerations. Instead of estimating a structural political economy model that captures such underlying political factors, we follow the approach of Davig, Leeper, and Walker (2010) and take the distribution of the fiscal limit as exogenously given. The parameters associated with the distribution are estimated, so that the fiscal limit is data-driven in our framework. In addition, we show that the estimates for the fiscal limit our robust to several robustness checks on the estimation.

We do not model default as a strategic decision made by a benevolent government. Instead, we appeal to political frictions that make the default decision intrinsically uncertain. Default is possible at any point on the distribution of the fiscal limit. Randomness inherent in the politically-determined default decision is modeled as a random draw of the ‘effective fiscal limit’ from the distribution. Each period, the government reneges on a fraction of its debt if the debt-to-GDP ratio surpasses the effective fiscal limit; otherwise, it fully honors the debt obligations.

Since the economy switches between the default and the no-default regimes endogenously depending upon the level of government debt, the model cannot be solved using a first-order approximation. Instead, it is solved using the monotone mapping method (see Davig (2004) and Coleman (1991)) and estimated using Bayesian inference methods and a sequential Monte Carlo approximation of the likelihood (see Fernandez-Villaverde and Rubio-Ramirez (2007), Doh (2011), and Amisano and Tristani (2010)).

2. Contacts with the Literature

An important branch of the sovereign default literature builds on Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), and Arellano (2008). In those models, sovereign default is the optimal decision made by a benevolent social planner in response to unlucky external shocks. Our approach abstracts from the government’s strategic incentives to default; instead, it lets negative wealth effects of debt and, in turn, have less incentive to hold debt. Therefore, the estimates from our closed economy framework can be thought as the lower bound to estimates of the fiscal limit. See section 4.2.3 for more details.
the data determine the distribution of the fiscal limit, which can be driven by both bad policy and bad luck.

A related branch of the empirical literature focuses on estimating first order approximations of small, open economy DSGE models to study real business cycle properties. Examples include Garcia-Cicco, Pancrazi, and Uribe (2010) and Guerron-Quintana (forthcoming). In contrast, we estimate a full, nonlinear model that incorporates a richer fiscal policy specification than often is encountered in this literature to study properties related to default.\footnote{Our modeling of private behavior is simpler than the frameworks often found in this literature, which is a trade-off made to ensure the full nonlinear estimation is computationally feasible.}

This paper is also related to a large empirical literature that studies the determinants of sovereign default risk premia through reduced-form panel regressions. Examples include Alesina, De Broeck, Prati, and Tabellini (1992), Bernoth, von Hagen, and Schuknecht (2006), Aizenman, Hutchison, and Jinjarak (2012), and De Grauwe and Ji (2012). This literature has not fully agreed upon the relative importance of different factors in determining risk premia, suggesting there is an importance of country-specific macroeconomic fundamentals in explaining sovereign risk premia. In addition, linear regressions that are often used in this line of work cannot capture the strong nonlinearity in sovereign risk premia that rational expectations models predict, see Arellano (2008) and Bi (2012) among others.\footnote{Some papers in this literature allows weak nonlinearity by including quadratic terms.}

Nonlinearity may be particularly relevant in a crisis, such as the one the Eurozone is facing now. Using a VAR model based on rolling-window estimation, Polito and Wickens (2012) compute the time-varying forecasts for the debt-to-GDP ratio and their volatility for the United States, and then calculate the time-varying default probabilities for different ad-hoc debt limits. Related work by Ostry, Ghosh, Kim, and Qureshi (2010) estimate historical fiscal responses to construct debt limits which are backward-looking by construction.

This paper builds upon our earlier work of Bi and Traum (2012), which provides a coherent framework for estimating forward-looking fiscal limits, but does not examine the extent to which a rational expectations model can explain the interest rate surges on sovereign debt. Unlike Bi and Traum (2012), we use unfiltered fiscal data measured in terms of GDP for the estimation and specify a fiscal limit distribution in terms of the debt-to-GDP ratio as opposed to the debt level.

## 3. Model

Following Bi (2012), our model is a closed economy with linear production technology, whereby output depends on the level of productivity \( A_t \) and the labor supply \( n_t \). Private
consumption \((c_t)\) and government purchases \((g_t)\) satisfy the aggregate resource constraint,
\[ c_t + g_t = A_t n_t. \]  
(1)

The deviation of technological productivity \((A_t)\) from its steady state value \((A)\) follows an \(AR(1)\) process,
\[ A_t - A = \rho^A (A_{t-1} - A) + \varepsilon_t^A, \quad \varepsilon_t^A \sim \mathcal{N}(0, \sigma_A^2). \]  
(2)

### 3.1 Government

The government finances lump-sum transfers to households \((z_t)\) and exogenous, unproductive purchases by levying a tax \((\tau_t)\) on labor income and issuing one-period bonds \((b_t)\). We denote \(q_t\) as the price of the bond in units of consumption at time \(t\). For each unit of the bond, the government promises to pay the household one unit of consumption in the next period. However, the bond contract is not enforceable. At time \(t\), the government may partially default on its liability \((b_{t-1})\) by a fraction of \(\Delta_t\), with the post-default liability being denoted as \(b_{d, t}\),
\[ \tau_t A_t n_t + b_t q_t = (1 - \Delta_t) b_{t-1} + g_t + z_t. \]  
(3)

The default scheme at each period depends on an effective fiscal limit \((s_t^*)\), specified in terms of the debt-to-GDP ratio. If current outstanding debt obligations, relative to GDP, are below the effective fiscal limit, then the government fully repays its liabilities; otherwise, the government partially defaults on its obligations by a fixed share of \(\delta\). The amount of unpaid bonds in any period \((\Delta_t)\), which we call the default rate, is summarized by
\[ \Delta_t = \begin{cases} 0 & \text{if } s_{t-1} < s_t^* \\ \delta & \text{if } s_{t-1} \geq s_t^* \end{cases} \]

where \(s_{t-1} = b_{t-1}/y_{t-1}\). The effective fiscal limit \((s_t^*)\) is stochastic and drawn from an exogenous distribution, \(s_t^* \sim \mathcal{S}^*\). Bi (2012) shows that a similar rational expectations model can give rise to an endogenous distribution of the fiscal limit that is strongly nonlinear — once the default probability begins to rise, it does so rapidly. To capture this strong nonlinearity, we follow Davig, Leeper, and Walker (2010) and model the cumulative density function of the fiscal limit distribution as a logistic function with parameters \(\eta_1\) and \(\eta_2\) dictating its
\[ p_{t-1} \equiv P(s_{t-1} \geq s^*_t) = \frac{\exp(\eta_1 + \eta_2 s_{t-1})}{1 + \exp(\eta_1 + \eta_2 s_{t-1})}, \tag{4} \]

where \( p_{t-1} \) is the default probability associated with the debt-to-GDP ratio of \( s_{t-1} \). The higher the government liability, the higher the default probability. The distribution is uniquely determined by the \( \eta_1 \) and \( \eta_2 \) parameters. As elaborated in section 4.2.1, these parameters are inferred from the estimation, so that the associated distribution reflects the economic and political fiscal limit implied by the data. With the estimated distribution, we can compute the default probability for the observed Greek debt-to-GDP ratios.

We assume that the government follows simple fiscal rules.\(^5\) It may raise the income tax rate or reduce government spending when the level of debt increases:

\[ \tau_t = (1 - \rho^\tau) \tau + \rho^\tau \tau_{t-1} + \varepsilon^\tau_t + \gamma^\tau (b^\tau_t - b) \quad \gamma^\tau > 0, \quad \varepsilon^\tau_t \sim \mathcal{N}(0, \sigma^2_\tau) \tag{5} \]

\[ g_t = (1 - \rho^g) g + \rho^g g_{t-1} + \varepsilon^g_t - \gamma^g (b^g_t - b) \quad \gamma^g > 0, \quad \varepsilon^g_t \sim \mathcal{N}(0, \sigma^2_g) \tag{6} \]

\( AR(1) \) components are denoted as \( u^\tau_t \) and \( u^g_t \). \( g, \tau, \) and \( b \) represent the deterministic steady-state levels of government spending, the tax rate, and government debt respectively. \( \gamma^\tau \) and \( \gamma^g \) are the fiscal adjustment parameters. A larger \( \gamma^\tau \) or \( \gamma^g \) implies that the government is more willing to retire debt by raising the tax rate or cutting government spending.

The non-distortionary transfers are modeled as a residual in the government budget constraint, exogenously determined by an \( AR(1) \) process,

\[ z_t - z = \rho^z (z_{t-1} - z) + \varepsilon^z_t \quad \varepsilon^z_t \sim \mathcal{N}(0, \sigma^2_z). \tag{7} \]

Since transfers are not included as an observable in our estimation, they can be thought of as a residual capturing all movements in government debt that are not explained by the model.

\(^4\) Davig, Leeper, and Walker (2010) specify the fiscal limit in terms of a tax rate, while we specify the fiscal limit in terms of the debt-to-GDP ratio. Using a similar model, Bi and Traum (2012) shows how the estimated logistic function can be related to an underlying endogenous distribution from the model reflecting a country’s economic and political willingness and ability to sustain various debt levels. In addition, the logistic function can approximate the endogenous distribution implied by models of strategic default (see for example the distribution arising in figure 6 of Gordon and Guerrero-Quintana (2013)).

\(^5\) Our fiscal rules are comparable to those specified in log-linearized models, see for example Leeper, Plante, and Traum (2010). In addition, Schmitt-Grohe and Uribe (2007) consider simple tax rules similar to equation (5) and show they can approximate the Ramsey optimal policy quite well.
3.2 Household

With access to the sovereign bond market, a representative household chooses consumption \( (c_t) \), hours worked \( (n_t) \), and bond purchases \( (b_t) \) by solving,

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t (\log (c_t - h\bar{c}_{t-1}) + \phi \log(1 - n_t)) \tag{8}
\]

s.t. \( A_t n_t (1 - \tau_t) + z_t - c_t = b_t q_t - (1 - \Delta_t) b_{t-1} \tag{9} \)

where \( \beta \) is the discount factor. The household’s utility for consumption is relative to a habit stock that is given by a fraction of aggregate consumption from the previous period, \( h\bar{c}_{t-1} \) with \( h \in [0, 1] \).

The household’s first-order condition requires the marginal rate of substitution between consumption and leisure equates to the after-tax wage. The bond price reflects the household’s expectation about the probability and magnitude of sovereign default in the next period.

\[
\phi \frac{c_t - h\bar{c}_{t-1}}{1 - n_t} = A_t (1 - \tau_t) \tag{10}
\]

\[ q_t = \beta E_t \left( (1 - \Delta_{t+1}) \frac{c_t - h\bar{c}_{t-1}}{c_{t+1} - h\bar{c}_t} \right). \tag{11} \]

The optimal solution to the household’s maximization problem must also satisfy the transversality condition,

\[
\lim_{j \to \infty} E_t \beta^{j+1} \frac{u_c(t+j+1)}{u_c(t)} (1 - \Delta_{t+j+1}) b_{t+j} = 0. \tag{12}
\]

3.3 Model Solution

The complete model consists of a system of nonlinear equations, including the household’s optimization conditions, household and government budget constraints, specifications of fiscal policy, the default scheme, shock processes, and the transversality condition. The solution, based on Coleman (1991) and Davig (2004), conjectures candidate decision rules that reduce the equilibrium conditions to a set of expectation first-order difference equations.

The state vector \( (\psi_t) \) includes the post-default debt level \( (b^d_t) \), the consumption from previous period \( (c_{t-1}) \), and exogenous state variables \( (A_t, u^g_t, z_t, u^\tau_t) \). The decision rule for the end-of-period debt, \( b_t = f^b(\psi_t) \), is solved in the following steps: \(^6\)

\(^6\) The online appendix provides more details.
1. Discretize the state space with grid points of \(n_b = 25, n_c = 5, n_A = 5, n_g = 7, n_z = 12, n_\tau = 11^7\). Make an initial guess for the decision rule, \(f^b_0\), over the state space.

2. At each grid point, solve the model and update the decision rule \(f^b_i\) using the given rule \(f^b_{i-1}\):
   (a) Make a guess for the end-of-period debt, \(b_t = f^b_{i-1}(\psi_t)\), for each grid point using the existing rule.
   (b) Compute the consumption \((c_t)\) using the household optimization condition for labor supply and the aggregate resource constraint, equations (10) and (3).
   (c) Compute the government spending \((g_t)\) and the tax rate \((\tau_t)\) using the specified fiscal rules, equations (5) and (6).
   (d) Compute the bond price \((q_t)\) using the household Euler equation,
   \[
   q_t = \beta E_t \frac{c_t - h\Delta_{t+1}}{c_{t+1} - h\Delta_{t+1}} (1 - \Delta_{t+1}).
   \]
   The integral on the right-hand side is evaluated using numerical quadrature and linear interpolation,
   \[
   E_t \frac{1 - \Delta_{t+1}}{c_{t+1} - h\Delta_{t}} = \left(1 - p_t\right) \int_{\epsilon_t}^{\epsilon_{t+1}} \int_{\epsilon_g}^{\epsilon_{g+1}} \int_{\epsilon_\tau}^{\epsilon_{\tau+1}} \frac{1}{c_{t+1} - h\Delta_{t}} |_{\text{no default}} \\
   + p_t \int_{\epsilon_t}^{\epsilon_{t+1}} \int_{\epsilon_g}^{\epsilon_{g+1}} \int_{\epsilon_\tau}^{\epsilon_{\tau+1}} \frac{1 - \delta}{c_{t+1} - h\Delta_{t}} |_{\text{default}}
   \]
   where \(p_t\) is the default probability at the beginning of period \(t + 1\) and can be obtained directly from the logistical function for the fiscal limit distribution, equation (4).
   (e) Update the decision rule for the end-of-period debt, \(b_t = f^b_i(\psi_t)\), using the government budget constraint,
   \[
   f^b_i(\psi_t) = \frac{b^d_t + z_t + g_t - \tau_t(c_t + g_t)}{q_t}
   \]

3. Check the convergence of the decision rule. If \(|f^b_i - f^b_{i-1}|\) is above the desired tolerance (set to \(1e-5\)), go back to step 2; otherwise, \(f^b_i\) is the decision rule.

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\(^7\)The grid boundaries are fixed throughout the estimation to ensure the same solution precision for different parameter draws.
After obtaining the decision rule for government bond, \( f^b(\psi_t) \), the pricing rule for the government bond, \( q_t = f^q(\psi_t) \), can be computed using the government budget constraint for each grid point.

\[
f^q(\psi_t) = \frac{b^d_t + z_t + g_t - \tau_t(c_t + g_t)}{f^b(\psi_t)}
\]

We evaluate the accuracy of numerical solutions using the dynamic Euler-equation test proposed by Den Haan (2010). The idea is to compare a time series for consumption \((c_t)\) that is constructed using the decision rule directly, with an alternative series \((\tilde{c}_t)\) that is implied by the Euler equation and the budget constraint. For a simulation with 500 periods, the errors, defined as the percentage differences between the two time series, are low. The mean error is 0.08%, implying that on average households make a 8 cent mistake for each $100 dollars spent.\(^8\) The online appendix provides more details on the test procedure.

4. Estimation

The model is estimated for Greece over the post-EMU period, 2001Q1-2010Q4, as interest rates during the pre-Euro period are susceptible to exchange rate risk from which our model abstracts. Five observables are used for the estimation: real output, the government spending-to-GDP ratio, the tax revenue-to-GDP ratio, the government debt-to-GDP ratio, and the 10-year sovereign real interest rate. For the estimation, we use percentage deviations of each observable from its mean value over the sample. We linearly detrend the real output measure and leave all other series unfiltered. Figure 3 depicts the data used for estimation. Appendix A.1 provides a detailed description of the data.

4.1 Methodology

We estimate the model using Bayesian methods. The equilibrium system is written in the nonlinear state-space form, linking observables \(v_t\) to model variables \(x_t\):

\[
x_t = f(x_{t-1}, \epsilon_t, \theta) \tag{13}
\]
\[
v_t = Ax_t + \xi_t \tag{14}
\]

\(^8\)We perform this exercise with parameters set at the posterior median from the baseline case with \(\delta^a = 0.3\). A longer simulation is more likely to generate larger errors; therefore, the errors generated by a simulation of 500 periods sets an upper bound for our model that uses a short data sample of 40 periods.
where $\theta$ denotes model parameters and $\xi_t$ is a vector of measurement errors distributed $N(0, \Sigma)$. We assume that $\Sigma$ is a diagonal matrix and calibrate the standard deviation of each measurement error to be 20% of the standard deviation of the corresponding observable variable.\(^9\)

We use a particle filter to approximate the likelihood function. For a given sequence of observations up to time $t$, $v^t = [v_1, ..., v_t]$, the particle filter approximates the density $p(x_t | v^t, \theta)$ by applying a law of large numbers to a series of simulations using a swarm of particles $x_i^t$ ($i = 1, ..., N$), see appendix A.2 for more details. The particle filter is applicable for nonlinear and non-Gaussian distributions, and it is increasingly used to estimate nonlinear DSGE models, to which class our model belongs. Recent examples include An and Schorfheide (2007), Fernandez-Villaverde and Rubio-Ramirez (2007), Amisano and Tristani (2010), Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez, and Uribe (2011), and Doh (2011).

We combine the likelihood $L(\theta | v^T)$ with a prior density $p(\theta)$ to obtain the posterior kernel, which is proportional to the posterior density. We assume that parameters are independent a priori. However, we discard any prior draws that lead to an explosive path for variables. We construct the posterior distribution of the parameters using the random walk Metropolis-Hastings algorithm, see appendix A.3 for more details. In each estimation, we sample one million draws from the posterior distribution and discard the first 500,000 draws.\(^10\) The likelihood is computed using 60,000 particles, and posterior analysis is conducted using every 50th draw from the chains.

### 4.2 Prior Distributions

As shown in table B, we impose dogmatic priors over some parameters. The discount rate is 0.99, so that the deterministic net interest rate is 1%.\(^11\) We calibrate the household’s leisure preference parameter $\phi$ such that a household spends 25% of its time working at the steady state. We calibrate the deterministic debt-to-GDP ratio, government spending-to-GDP ratio, and tax rate to the mean values of the data sample.\(^12\)

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\(^9\) Estimating measurement errors provides complications with nonlinear estimation techniques. See Doh (2011) for more discussion of the role of measurement error in nonlinear DSGE model estimation.

\(^10\) We use Fortran MPI code compiled in Intel Visual Fortran for the estimation. We use the computer server system at the Bank of Canada, each CPU of which uses Xeon CPU X5680 at 3.33GHz and has 23 processors with 64G RAM. One evaluation using the particle filter takes 3 seconds. These computational constraints limit the number of draws from the Metropolis-Hastings algorithm. The acceptance rate for both model specifications is 15%.

\(^11\) The mean of our data is 0.8% for Greece.

\(^12\) We calibrate the steady state to match sample means since Greece’s inclusion in the European union. Prior to union inclusion, Greece’s monetary and fiscal policy actions could have been based on different long run targets, implying different steady state values for policy instruments. As our model specification
The prior for habit persistence $h$ is similar to those in the linear DSGE estimation literature, for instance Smets and Wouters (2007). For the remaining parameters, we first use ordinary least squares and estimate an AR(1) process for GDP and processes for government spending, the tax rate, and transfers given by equations (5)-(7). The results are used as general guidance for the parameter space of the shocks’ persistence and standard deviations. We also performed prior predictive analysis to ensure the prior space for the shocks’ persistence and standard deviations covers second moments of the data.

For the government spending and the tax adjustment parameters, we form priors for the long-run responses in terms of percentage deviations from the steady state, that is

$$\gamma^{g,L} \equiv \frac{\bar{b}}{\bar{g}} \frac{\gamma^g}{(1 - \rho^g)}, \quad \gamma^{t,L} \equiv \frac{\bar{b}}{\bar{\tau}} \frac{\gamma^\tau}{(1 - \rho^\tau)}$$

These values are more comparable to estimates in the literature. Since determinacy is sensitive to the combination of the $\gamma^{t,L}$ and $\gamma^{g,L}$ parameters, we restrict the lower bound of the $\gamma^{t,L}$ ($\gamma^{g,L}$) prior to a value that ensures determinacy when only $\gamma^{t,L}$ ($\gamma^{g,L}$) finances debt.

We formulate our priors for the standard deviations as a fraction of the relevant steady state variables, instead of in terms of levels. That is, instead of working directly with $\sigma_k$ for $k = \{A, g, \tau, z\}$, we form priors for $\sigma_{k,p} \equiv \sigma_k / \bar{J}$ for $J = \{A, g, \tau, z\}$ and $k = \{a, g, \tau, z\}$. For instance, for the standard deviation on technology, we formulate a prior and estimate the posterior of the standard deviation relative to steady state technology: $\sigma_{a,p} \equiv \sigma_a / \bar{A}$. By dividing each standard deviation by its relative steady state value, we are able to construct and utilize measures that provide a more intuitive comparison across variables.

### 4.2.1 Fiscal Limit

As discussed in section 3.1, the fiscal limit distribution, given by equation (4), is defined by the parameters of $\eta_1$ and $\eta_2$. A property of the logistic function is that for any given two points on the distribution, $(\tilde{s}, \tilde{p})$ and $(\hat{s}, \hat{p})$, the parameters $\eta_1$ and $\eta_2$ can be uniquely determined by

$$\eta_2 = \frac{1}{\tilde{s} - \hat{s}} \log \left( \frac{\hat{p}}{\tilde{p}} \frac{1 - \hat{p}}{1 - \tilde{p}} \right), \quad \eta_1 = \log \frac{\hat{p}}{1 - \hat{p}} - \eta_2 \tilde{s}. \quad (15)$$

$\hat{p}$ and $\tilde{p}$ represent the default probabilities at the debt-to-GDP ratios of $\hat{s}$ and $\tilde{s}$ respectively. It is not possible to identify all four parameters $(\tilde{s}, \tilde{p})$ and $(\hat{s}, \hat{p})$. We must pick either the
debt-to-GDP ratios we are interested in (the $s$’s) or the probabilities we are interested in (the $p$’s). We choose two probabilities we are interested in: $\tilde{p} = 0.3$ and $\hat{p} = 0.999$, one from each tail of the distribution. Ideally, we would estimate both the $\tilde{s}$ and the $\hat{s}$ corresponding to these probabilities. Unfortunately, since defaults are not observed in our data sample, the data is unlikely to be informative about the upper bound of the fiscal limit distribution. Thus, we must pin down $\hat{s}$ as well and estimate only $\tilde{s}$.

To pin down $\hat{s}$, we fix the difference between $\tilde{s}$ and $\hat{s}$ to be 60% of steady-state output, which captures the observation that once risk premia begin to rise, they do so rapidly.\footnote{The difference of 60% of output, albeit ad-hoc, should not change the key estimation results as the data are unlikely to be informative about the upper bound of distribution.} For a given estimate of $\tilde{s}$, this then implies the value of $\hat{s}$ as well. Calibrating the difference between the two fixes the slope of the fiscal limit distribution, while the estimate of $\tilde{s}$ determines the intercept of the distribution. Figure 1 illustrates this concept by plotting the fiscal limit distributions that are associated with various $\tilde{s}$ values. Given the lack of guidance for the parameter $\tilde{s}$, we adopt a diffuse uniform prior over the interval 1.4 to 1.8, implying that the debt level associated with a 30% probability of default ranges from 140 to 180% of GDP.\footnote{Posterior estimates are robust to raising the upper bound of $\tilde{s}$ (results are available from the authors). Decreasing the lower bound of $\tilde{s}$ dramatically increases the parameter region that falls outside of determinacy, as it increases the likelihood of the government encountering a situation where it repeatedly defaults on debt.}

The red dashed line of figure 1 depicts the fiscal limit distribution associated with the lower bound of $\tilde{s}$, while the black dotted line shows the distribution associated with the upper bound of $\tilde{s}$.

### 4.2.2 $\delta$ Identification

To our knowledge, aside from Bi and Traum (2012), this paper is the only attempt to estimate a DSGE model of sovereign default. Thus, prior to estimating the model with real data, we performed several estimations with simulated data.\footnote{The results are available upon request from the authors.} Unfortunately, the results revealed that we cannot jointly identify the default rate ($\delta$) and the fiscal limit parameter ($\tilde{s}$) when the data exclude observed defaults, since various combinations of $\delta$ and $\tilde{s}$ are consistent with the same risk premium. Given this limitation, we estimate our model for two different calibrations: $\delta$ fixed at 0.05 and 0.075. These calibrations imply annualized rates of default ($\delta A$) of 20% and 30% respectively.

Bi (2012) calculates the actual default rate, defined as the share of actual defaulted debt over total public debt, in emerging market economies over the period 1983 to 2005, based on data from Moody’s (March 2009), Panizza (2008), and Sturzenegger and Zettelmeyer (2008). She shows that the actual default rates are between 0 and 0.1 with 70 percent probability.
and between 0 and 0.3 with 90 percent probability. We assume that prior to the Greek
debt restructuring (the period relevant for our estimation), the market’s best expectation of
Greece’s default rate was based off of historical, recent, observed defaults. Calibrating $\delta^A$ to
20% and 30% falls within the range of recent actual default rates and is assumed to reflect
market expectations based on observed defaults.

4.2.3 Modeling Alternatives

We assume a closed economy framework for the estimation. In contrast, the literature on
sovereign default in emerging markets often utilizes an open economy specification where all
sovereign debt is held by foreigners, see Eaton and Gersovitz (1981), and Arellano (2008).

In the closed economy, domestic households have incentive to hold debt because of the
negative wealth effects of deficit-financed fiscal policies. Such policies cause households to
increase their current bond holdings (i.e. savings), as they know that future taxes will
increase to finance the accumulated debt. In contrast, foreigners do not face the same
negative wealth effects and, therefore, the interest rate reflects only the probability of default.
Ceteris paribus, in our model the assumption that the economy is open and all debt is held
by foreigners raises the observed risk premium relative to the closed economy environment.
Thus, estimates from our closed economy framework can be thought as the lower bound to
estimates of the fiscal limit associated with certain probabilities of default.

4.3 Posterior Estimates

Figure 2 plots the prior and posterior distributions for parameters when the annualized
default rate is set to 30%. Table 2 compares the medians and 90% credible intervals of the
posterior distributions estimated under both default rate specifications. For comparison,
the means and 90% intervals from the priors are also listed. For comparison, we also list
the estimates implied by a log-linearized version of our model in which the government is
not allowed to default. We estimate this version of the model using the same data, priors,
and calibration as the nonlinear specification (except for setting $\delta^A = 0$). The system of
equations for the log-linearized model is listed in appendix B. We use the Kalman filter to
calculate the likelihood function and initialize the Metropolis-Hastings algorithm using the
posterior mode and inverse Hessian at the posterior mode.$^{17}$

For all specifications, the posterior distributions’ medians tend to differ from the prior
mean ($\sigma_{z,p}$ is an exception). Not all parameters have marginal posterior distributions that
\[\text{171,000,000 draws were made, with the first 500,000 used as a burn-in period and every 50th thinned. The acceptance rate was 39\%.} \]
are tighter than the corresponding priors. The ones that do not include $\sigma_{z,p}$, $\rho_z$, and $\gamma_g$.

The posterior distributions of most parameters are stable across the specifications. However, the estimates of $\tilde{s}$ vary slightly across the two default rate specifications. If the partial default rate, $\delta^A$, is 30%, the debt-to-GDP ratio that is associated with a 30% probability of default is between 1.53-1.59, with the median being 1.56. If the partial default rate is 20%, the debt-to-GDP ratio that is associated with a 30% probability of default is between 1.50-1.56, with the median being 1.53. The lower $\tilde{s}$ estimates for a lower $\delta^A$ calibration are consistent with theory. The model tries to match the interest rate data through the values of $\delta^A$ and $\tilde{s}$. For higher values of $\delta^A$, agents expect to lose more of the face-value of debt following a default, and therefore households demand a higher interest rate to compensate for this risk. For the given risk premium implied by the data, a slightly higher $\tilde{s}$ is needed to offset a higher $\delta^A$ value.

The nonlinear and linear model specifications tend to have similar parameter estimates. Habit persistence is an exception, as $h$ is estimated to be less than 0.5 in the nonlinear specifications and greater than 0.5 in the linear model. In addition, $\gamma^{L,L}$ has higher estimates in the linear model. More formal comparisons of the linear and nonlinear model specifications are provided in the next section.

5. Analysis

5.1 Model Fit

To examine how well the model fits the data, we compute smoothed estimates of model variables using the sequential monte carlo approximation of the forward-backward smoothing recursion, see Doh (2011) for a detailed explanation of the procedure. Figure 3 compares the smoothed values from the nonlinear model with $\delta^A = 0.3$ and from the linearized model without default to the observable variables. For each specification, the fitted values are computed using the corresponding posterior median. The fit for most variables is quite accurate, with the real interest rate being the least precise.

We also compute smoothed estimates of the measurement errors $E(\xi_t | v^T, \theta)$ and report their mean absolute values and the standard deviations of the estimated measurement errors relative to the standard deviation of observables (i.e. relative standard deviation) in table 3. For the estimation, the standard deviation of each measurement error was fixed to be 20% of the standard deviation of the respective observable variable. For most observables, the actual estimated relative standard deviation is less than 20%, suggesting that the measurement error did not introduce many constraints for the model fit. The exception is the measurement error
for the real interest rate in the nonlinear model and for output in the linear model. Table 3 also shows that mean absolute values of measurement error are close to zero.

Given that the different model specifications imply similar smoothed estimates, we perform posterior odds comparisons to determine which model is favored by the data. Bayes factors are used to evaluate the relative model fit of the linear and two nonlinear specifications. Table 4 presents the results. Bayes factors are based on log-marginal data densities calculated using Geweke’s (1999) modified harmonic mean estimator with a truncation parameter of 0.5. The results demonstrate that the data strongly prefer the nonlinear model specifications over the linearized framework. In addition, it appears the data cannot distinguish between the two nonlinear models with alternative calibrated default rates, as the log Bayes factor is close to zero.

5.2 Default Probability and Interest Rate Dynamics

In this section, we use the parameter estimates to evaluate the historical probability of default in Greece and explore the model’s ability to forecast the fiscal deterioration in Greece since 2010Q4, which is the end of our estimation period.

5.2.1 Default Risk

Figure 4 depicts model-implied historical sovereign default probabilities for Greece, based upon the estimated fiscal limit distribution with an annualized default rate of 30%, δₐ = 0.3. Solid lines show the median and 90% posterior interval for the probability of default, calculated using the actual debt-to-GDP ratios from our debt and output observables for the estimated sample period. Figure 4 shows that Greek debt had virtually zero probability of default from 2001 until 2009. Starting in 2009, the probability of default rose steadily, and ranged from 5-10% by the end of the estimated period 2010Q4, reflecting the deterioration in confidence in Greek debt in 2010.

In addition, the dashed lines provide the median and 90% posterior interval for the model-implied default probabilities for the out-of-sample debt-to-GDP ratios observed in 2011. The probabilities increase dramatically, ranging from 60-80% by 2011Q4. The striking build-up reflects the unsustainability of the Greek fiscal position and suggests imminent default. The predictions are consistent with the debt restructuring arrangements that took place at the

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18 As noted in the literature, marginal likelihoods are subject to caveats (see Gelman, Carlin, H.S., and Rubin (2004) and Del Negro and Schorfheide (2008)).

19 For the estimation, we use data in terms of percentage deviations from the sample average. In contrast, we need level variables to back out model-implied probabilities of default. For model consistency, we convert the percentage deviations of the data to level variables using the steady states of model variables.
beginning of 2012, which can be viewed as an effective default on Greek debt.

5.2.2 Out-of-Sample Interest Rate Forecasts

Over the course of 2011, the long-term nominal interest rate in the secondary market for Greek government bonds rose from 9.1 percentage points in December 2010 to 21.2 in December 2011 based on BIS data, and to 31.2 based on Bloomberg data.\footnote{Quarterly averages of Bloomberg and BIS data differ only in 2011Q3 and 2011Q4, due to differences in pricing such as the PSI introduction.} In this section, we examine the ability of the estimated model to predict this sharp increase in the Greek interest rate.

To examine this issue, we use the posterior median estimates with an annualized default rate of 30% to simulate 10,000 time series for four quarters, starting from the fitted values for model variables in 2010Q4. This gives a distribution for the forecasted real interest rate in 2011. The left panel of figure 5 displays the median (blue, circled line) and 90% interval (blue, dashed lines) of these interest rate forecasts. The figure also plots the observed real interest rate during the same period (black solid line for BIS data and dotted-dashed red line for Bloomberg data).

Figure 5 shows that the surge in the real interest rate in Greece is well within forecast bands of our rational expectations model. The fact that model forecasts can be consistent with the 2011 interest rate path suggests that the actual interest rate surge in 2011 can be explained by macroeconomic fundamentals in a nonlinear rational expectation framework. For comparison, the right panel of figure 5 repeats our forecasting exercise with the estimated log-linear model without default. In this case, the surge in the real interest rate is entirely outside of the forecast bands, suggesting that the nonlinear specification allowing for endogenous default is crucial for accounting for Greece’s recent risk premia.

The model-implied forecast band for 2011 from the nonlinear model specification ranges dramatically from 6 to 40 percentage points. One might be tempted to conclude that the model forecast band is too large to be of much use. To explore this issue, we repeat our forecasting exercise starting from the fitted values for model variables in 2006Q4. That is, we calculate the model-implied distribution for the interest rate path in 2007, assuming that only information through 2006 was known. Figure 6 plots the median (blue, circled line) and 90% interval (blue, dashed lines) of the interest rate forecasts, along with the observed interest rate (black solid line). Again, the actual path is well within the forecast band, and the band, ranging a bit over 2 percentage points, is much tighter than the 2011 band. This is because macroeconomic fundamentals were more stable in 2006Q4 than in 2010Q4, as reflected by the virtually zero probability of default in 2006Q4. Thus, while forecast bands...
appear tight in good times, large forecast bands may emerge at a turning point from a deteriorating fiscal position, which is inherent in the nonlinear model.

The results suggest that the surge in the Greek interest rate premium and the rapid deterioration of confidence in Greek debt in 2011 are consistent with behaviors from a simple rational expectations model, and that Greek debt was not mispriced in 2011.

5.2.3 Greek Fiscal Consolidation

In May 2010, the European Union and International Monetary Fund (IMF) agreed to give a sequence of transfer payments to Greece conditional on the Greek government and the Bank of Greece implementing a sequence of economic and financial reforms. Working in conjunction with policymakers from the troika (the IMF, European Commission, and European Central Bank), Greece developed a plan for immediate economic and financial reforms, as well as reforms to be carried out over the course of 2011-2013. However, all of these reforms and targets have not been subsequently met. In this section, we perform a counterfactual exercise to examine how much of Greece’s interest rate and default risk would have been reduced if the Greek government had carried out the fiscal consolidation reforms laid out by the troika in IMF (2010).

To examine this issue, we use the posterior median estimates with an annualized default rate of 30% to simulate 10,000 time series for three years, starting from the fitted values for model variables in 2010Q4. The simulations are performed conditional on the paths of government spending and the tax rate being in line with the fiscal consolidation measures proposed in Table 1 of IMF (2010). That is, instead of the paths of government spending and the tax rate evolving according to equations (5) and (6), we assume that they follow the processes

\[
\tau_t = \tau_{t-1} + v_{\tau}^t \\
g_t = g_{t-1} - v_{g}^t y_{t-1}
\]

These rules imply that the government spending-to-GDP ratio contracts and the tax revenue-to-GDP ratio increases by the amounts \(v_{\tau}^t\) and \(v_g^t\) respectively each period. Government spending and tax rates are set at the target amounts for each period. Based on the proposed consolidation guidelines in Table 1 of IMF (2010), the values for \(v_{\tau}^t\) and \(v_g^t\) can be computed accordingly and are given in table 5.

We run the forecast simulations by allowing government spending and taxes to evolve according to equations (16)-(17), and the shocks to productivity, transfers and effective fiscal limits to be drawn from their distributions. Figure 7 shows the median and 90% intervals.
for the forecasted paths of percentage changes in the real interest rate, output, government spending-to-GDP ratio, tax rate, and government debt-to-GDP ratio. The planned consolidation measures would have dramatically lowered the government spending-to-GDP ratio while raising the tax rate. These measures imply a drop in output, which in turn fuels an increase in the government debt-to-GDP ratio and an increase in the real interest rate. Interestingly, the median forecasted path of the real interest rate matches the path of the data over the course of 2011.\footnote{Greece officially restructured its debt in the second quarter of 2012, triggering a default that in turn led the interest rate to rapidly decline. The simulations do not force this default to occur, which is why the median forecasted interest rate does not decrease as significantly.}

6. Sensitivity Analysis

We investigate the robustness of our parameter estimates under several alternative model specifications. The results of these robustness checks are summarized in Table 6.\footnote{For these estimations, we sample 500,000 draws from the posterior distribution and discard the first 250,000 draws. Posterior analysis is conducted using every 25th draw from the chains. The acceptance rates for these specifications varies from 22-49%.} For all cases, unless otherwise noted, the calibration, priors, and estimation procedure is the same as that for the model specification with $\delta^A = 0.30$.

6.1 Higher default rate

The default rate $\delta$ cannot be identified given that our data sample excludes default. However, on April 25, 2012, Greece announced that it had successfully negotiated with its private investors a restructuring of Greek debt. In terms of our model specification this restructuring would count as a partial default on Greek sovereign debt. Zettelmeyer, Trebesch, and Gulati (2012) calculates that the present value haircut to private investors from the restructuring was between 55-65\%. However, they note that the haircut excluded the bond holdings of the ECB and other central banks, with the ECB being the single largest bondholder at 22\% of total debt. As such, the actual haircut on total Greek government debt could arguably be placed around 45\%.

To determine how sensitive our estimates and inferences are to a higher default rate calibration, we estimate the model for an alternative calibration with $\delta^A = 0.45$. The results are shown in the ‘$\delta^A = 0.45$’ columns of Table 6. All estimated parameters are trivially affected by the alternative calibration, except that the posterior median for $\tilde{s}$ is raised to 1.61 from 1.56 when $\delta^A = 0.3$. This implies that the default probability for any given debt-to-GDP ratio would only be slightly lower than the case of $\delta^A = 0.3$. In fact, the implied
probability of default by 2010Q4 is between 3-5% and by 2011Q4 is between 45-70%. In addition, the surge in the real interest rate in Greece in 2011 is still within forecast bands of the estimated model.23

6.2 Varying Measurement Error Calibration

For the baseline estimation, the standard deviation of each measurement error was fixed to be 20% of the standard deviation of the respective observable variable. To determine how sensitive our estimates and inferences are to measurement error, we estimate the model for two alternative cases where each measurement error was fixed to be 25 and 30% of the standard deviation of the respective observable variable. The results are shown in the ‘Meas. Err. 30%’ and ‘Meas. Err. 25%’ columns of table 6. Most of the 90% intervals of the estimated parameters are similar to the benchmark case (the ‘δA = 0.3’ column). In addition, the ‘Meas. Err. 30%’ and ‘Meas. Err. 25%’ rows of table 3 lists the mean absolute values of measurement errors and the standard deviations of the estimated measurement errors relative to the standard deviation of observables (i.e. relative standard deviation). For the 25% relative standard deviation case, the actual estimated relative standard deviations are still less than 20% and comparable to the benchmark 20% case. The actual estimated relative standard deviations are slightly larger for the 30% relative standard deviation case, with both the actual estimated relative standard deviations of the measurement errors for the tax revenue-to-GDP ratio and the real interest rate being above 20%.

6.3 Varying γτ,L, γg,L priors

We check whether our results are sensitive to the prior specifications by estimating a version of the model where both γτ,L and γg,L have normal priors centered at 0.5 with a standard deviation of 0.18. The priors were chosen to allow a positive probability mass for negative parameter values, which allows the posterior estimates to potentially encompass zero, while still maintaining a large prior probability mass lying in the determinacy region of the parameter space. We focus on these two parameters as they are essential for the debt dynamics in the model, and as seen in figure 2 the posteriors are similar to the priors, particularly for γg,L. The results are shown in the ‘Alt. γτ,L, γg,L priors’ column of Table 6. The estimates for γτ,L are similar to the benchmark specification, although the 90% credible interval is more diffuse. The posterior for γg,L varies considerably from the benchmark specification, although the posterior median of γg,L is still quantitatively larger than the estimate of γτ,L. The results confirm γg,L is not well identified from the data.

23Details of these results are given in an online appendix.
7. Conclusion

This paper uses Bayesian methods to estimate the fiscal limit distribution and the associated sovereign default probability for Greece. We build a real business cycle model that allows for interactions among fiscal policy instruments, the stochastic fiscal limit, and sovereign default risk. The fiscal policy specification takes into account government spending, lump-sum transfers, and distortionary taxation. We model the fiscal limit distribution with a logistic function, which illustrates the market’s belief about the government’s ability and willingness to service its debt at various debt levels.

Using the particle filter to perform likelihood-based inference, we estimate the full nonlinear model with post-EMU data. We find that the probability of default on Greek debt remained close to zero from 2001 until 2009, and then rose sharply to the range of 5% to 10% by the fourth quarter of 2010. The model also predicts a probability of default between 60% to 80% by the end of 2011, consistent with the debt restructuring arrangements that took place at the beginning of 2012. In addition, the surge in the Greek real interest rate in 2011 is within the forecast bands of our rational expectations model. The results suggest that Greek debt was not mispriced in 2011, as the interest rate path can be accounted for by macroeconomic fundamentals. Finally, model comparisons based on Bayes factors strongly favor our nonlinear model specification with an endogenous probability of default over a linearized specification without default.

In current ongoing research, we are estimating the model for other European countries, so as to allow cross-country comparisons of default probabilities. Although our nonlinear model allows for interactions among fiscal policy instruments and the fiscal limit, it is only a first step in our understanding of default probabilities for developed countries. Exploration of other model features are worthy of attention, including the interaction of monetary and fiscal policies, the interaction of financial sector and sovereign debt, and open economy considerations. In addition, an open question remains as to what types of accommodations for nonlinearities in reduced-form specifications could allow such frameworks to match Greek data.
References


A Estimation

A.1 Data Description

Five observables for Greece over the period 2001Q1-2010Q4 are used for the estimation: real output, government spending to GDP ratio, tax revenue to GDP ratio, government debt to GDP ratio, and 10-year real interest rate. This appendix provides documentation for constructing these series.

**Real GDP.** Constructed by dividing the nominal quarterly gross domestic product from the OECD quarterly National Accounts (using the expenditure approach, series B1_GE) by the gross domestic product deflator (constructed using the expenditure approach, series B13).

**Real Gov. Spending.** Constructed using general government final consumption expenditure from the OECD quarterly National Accounts (series P3S13) divided by the gross domestic product deflator (constructed using the expenditure approach, series B13).

**Real Tax Revenue.** A quarterly tax revenue series is not provided by the OECD. Thus, we construct a quarterly measure in the following way. First, we construct a measure of total tax revenue by combining Eurostat quarterly series for tax receipts on income/wealth, production and imports, capital taxes, and social contributions. We seasonally adjust this series using Demetra+ and the tramo-seat RSA4 specification. Next, using an annual nominal tax revenue series from the OECD volume 90 (consisting of indirect and direct taxes and social security contributions, TIND + TY + SSRG), we interpolate a quarterly frequency series using the method of Chow and Lin (1971)\(^{24}\) and the seasonally adjusted quarterly Eurostat tax revenue series for the interpolation. Finally, we construct a quarterly real tax revenue series by dividing the interpolated series by the OECD’s gross domestic product deflator (constructed using the expenditure approach, series B13).

**Real Gov. Debt.** A quarterly government debt series is not provided by the OECD. Thus, we construct a quarterly measure in the following way. First, we seasonally adjust using Demetra+ and the tramo-seat RSA4 specification the Eurostat quarterly series for nominal gross government consolidated debt. Next, using the annual nominal gross public debt series (under the Maastricht criterion) from the OECD volume 90, we interpolate a quarterly frequency series using the method of Chow and Lin (1971) and the seasonally adjusted quarterly Eurostat debt series for the interpolation.\(^{25}\) Finally, we construct a quarterly real debt series by dividing the interpolated series by the OECD’s gross domestic product deflator.

\(^{24}\)Forni, Monteforte, and Sessa (2009) use a similar approach.

\(^{25}\)We directly use the seasonally adjusted Eurostat debt series for the forecast exercise in section 5.2.1, as OECD debt data is not available after 2010Q4.
Real Interest Rate. To construct a 10-year real interest rate measure, we use data for the nominal interest rate, $i_t$ (taken from the BIS) and the expected inflation rate, $\pi^e_t$. Our measure of expected inflation for Greece is the expected inflation series from the Survey of Professional Forecasts EU-area five year ahead expected inflation series. The gross real interest rate is constructed using the relation,

$$R_t = \frac{1 + i_t}{1 + \pi^e_t}.$$ 

Data for Estimation. We calculate the government spending to GDP ratio, tax revenue to GDP ratio, and government debt to GDP ratio by taking each real fiscal series described above and dividing by our real GDP series. The estimation uses these three series, along with the real GDP and real interest rate series described above. For each series, we transform the series into percentage deviations from its mean value over the period 2001Q1-2010Q4. In addition, the real GDP series is linearly detrended. The black solid lines of figure 3 graphs the observables.

A.2 Particle Filter Algorithm

Let $v^T$ denote $\{\hat{v}_t\}_{t=1}^T$, which evolves according to equations (13) and (14) in the text. To evaluate the likelihood function $L(\theta|v^T)$, we use a sequential Monte Carlo filter, specifically, the sequential importance resampling filter of Kitagawa (1996). The algorithm is as follows:

- Step 1. Initialize the state variable $x_0$. Instead of drawing particles of $x_0$ from the unconditional distribution $p(x_0|\theta)$, we utilize pre-sample data to construct values $x_0$. We use data for the observables in 2000Q3 and 2000Q4 to back out the model state vector in 2000Q4, assuming no measurement error.\(^{26}\)

Denote these particles by $x_0^i$ for $i = 1, ..., 60,000$. Draw 60,000 values from standard normal distributions for each of the structural shocks ($\epsilon^A, \epsilon^g, \epsilon^i, \epsilon^z$) and 60,000 values from a standard uniform distribution for fiscal limit probabilities. Denote the vector of these particles by $u^i$. By induction, in period $t$ these are particles $u^{t|t-1,i}$.

- Step 2. Construct $x^{t|t-1,i}$ using equation (13) in the text. Assign to each draw ($u^{t|t-1,i}$, $x^{t|t-1,i}$) a weight defined as:

$$w_i^t = \frac{1}{(2\pi)^{d/2}|\Sigma|^{1/2}} \exp \left[ -\frac{1}{2} \left( y_t - Ax^{t|t-1,i} \right)' \Sigma \left( y_t - Ax^{t|t-1,i} \right) \right] \quad (A.1)$$

\(^{26}\)Fixing the initial state vector $x_0$ with the data improves the accuracy of parameter estimates, which is especially useful given our short sample period.
Step 3. Normalize the weights:
\[ \tilde{w}_t^i = \frac{w_t^i}{\sum_{i=1}^{N} w_t^i} \]

Update the values of \( x_{t-1,i} \) by sampling with replacement 60,000 values of \( x_{t-1,i} \) using the relative weights \( \tilde{w}_t^i \) and the residual resampling algorithm.

Repeat steps 2-3 for \( t \leq T \).

The log-likelihood function is approximated by
\[ L(\theta|v^T) \simeq \sum_{t=1}^{T} \ln \left( \frac{1}{60,000} \sum_{i=1}^{60,000} w_t^i \right) \] (A.2)

### A.3 MCMC Algorithm

The random walk Metropolis-Hastings algorithm used for estimation works as follows:

- Step 1. Compute the posterior log-likelihood for 500 draws from the priors. Call the draw with the highest posterior log-likelihood value \( \theta^* \).

- Step 2. Starting from \( \theta^* \), generate a MCMC chain using the following random-walk proposal density
  \[ \theta_{j+1}^{\text{prop}} = \theta_j^{\text{prop}} + c \mathcal{N}(0, \Lambda), \quad j = 1, \ldots, 100,000 \]
  where \( \Lambda \) is the covariance matrix of 500 draws from the priors and \( c > 0 \) is a tuning parameter set to determine the acceptance ratio.

- Step 3. Compute the acceptance ratio \( \varphi = \min \left\{ \frac{p(\theta_{j+1}^{\text{prop}}|v^T)}{p(\theta_j|v^T)}, 1 \right\} \). Given a draw \( u \) from the standard uniform distribution, \( \theta_{j+1} = \theta_{j+1}^{\text{prop}} \) if \( u < \varphi \); and \( \theta_{j+1} = \theta_j \) otherwise. Repeat for \( j = 1, \ldots, 100,000 \).

- Step 4. Update the random walk proposal density in the following way. Update \( \Lambda \) to be the covariance matrix from the previous draws \( \{\theta_j\}_{1}^{100,000} \). Update \( \theta^* \) to be the mean of previous draws \( \{\theta_j\}_{1}^{100,000} \). Starting from the new \( \theta^* \), proceed through steps 2 and 3 for 1,000,000 draws from the new MCMC chain.

We burn the first 500,000 draws from the final MCMC chain and thin every 50 draws.
B Log-Linearized Model Equations

The log-linearized system of equations for the basic variant of the model without default are:

\[
\hat{c}_t - \frac{1}{1+h}E_t\hat{c}_{t+1} + \frac{1-h}{1+h}\hat{R}_t = \frac{h}{1+h}\hat{c}_{t-1} \quad (B.1)
\]

\[
\frac{1}{1-h}\hat{c}_t + \frac{n}{1-n}\hat{n}_t - \hat{A}_t + \frac{\tau}{1-\tau}\hat{\tau}_t = \frac{h}{1-h}\hat{c}_{t-1} \quad (B.2)
\]

\[
\frac{c}{y}\hat{c}_t + \frac{g}{y}\hat{g}_t = \hat{A}_t + \hat{n}_t \quad (B.3)
\]

\[
\frac{b}{y}\hat{b}_t - \frac{g}{y}\hat{g}_t - \frac{z}{y}\hat{z}_t + \tau(\hat{\tau}_t + \hat{A}_t + \hat{n}_t) = R \times \frac{b}{y}(\hat{R}_{t-1} + \hat{b}_{t-1}) \quad (B.4)
\]

\[
\hat{g}_t = (1-\rho^g)\hat{g}_{t-1} - \gamma^g(L)(1-\rho^g)b_{t-1} + \sigma_{g,p}\epsilon_t^g, \quad \epsilon_t^g \sim N(0,1) \quad (B.5)
\]

\[
\hat{\tau}_t = (1-\rho^\tau)\hat{\tau}_{t-1} + \gamma^\tau(L)(1-\rho^\tau)b_{t-1} + \sigma_{\tau,p}\epsilon_t^\tau, \quad \epsilon_t^\tau \sim N(0,1) \quad (B.6)
\]

\[
\hat{z}_t = (1-\rho^z)\hat{z}_{t-1} + \sigma_{z,p}\epsilon_t^z, \quad \epsilon_t^z \sim N(0,1) \quad (B.7)
\]

\[
\hat{A}_t = (1-\rho^a)\hat{A}_{t-1} + \sigma_{a,p}\epsilon_t^a, \quad \epsilon_t^a \sim N(0,1) \quad (B.8)
\]
Figure 1: The fiscal limit distribution associated with various $\tilde{s}$ values. Red dashed line: $\tilde{s} = 1.4$; blue solid line: $\tilde{s} = 1.6$; black dotted line: $\tilde{s} = 1.8$.

Figure 2: Prior distributions (black solid lines) versus posterior distributions (blue dashed lines) for the nonlinear model with annualized default rate of 30%, $\delta^A = 0.3$. 
Figure 3: Fitted values for various estimations for Greece. Black, solid lines: data. Blue, dashed lines: nonlinear model with annualized default rate of 30%, $\delta_A = 0.3$. Red, dotted-dashed lines: linear model.

Figure 4: Model-implied sovereign default probabilities for Greece. Solid lines denote the median and 90% confidence interval probabilities for in-sample debt-to-GDP ratios. Dashed lines denote the median and 90% confidence interval probabilities for out-of-sample debt-to-GDP ratios.
Figure 5: Data versus fitted and forecast values for the Greek interest rate. The median (blue, circled line) and 90% interval (blue, dashed lines) of the interest rate forecasts for 2011 are calculated based on the posterior median parameter estimates. The black solid line shows the real interest rate based on BIS data, and red dotted-dashed line shows the rate based on Bloomberg data. Left panel for the nonlinear model with $\delta^A = 0.3$; right panel for log-linear model without default.

Figure 6: Data versus fitted and forecast values for the Greek interest rate. The median (blue, circled line) and 90% interval (blue, dashed lines) of the interest rate forecasts for 2007 are calculated based on the posterior median parameter estimates. The black solid line shows the real interest rate based on BIS data.
Figure 7: The median (blue, circled line) and 90% interval (blue, dashed lines) of model variable forecasts for 2011-2013 assuming the 2010 planned Greek fiscal consolidation measures are enacted. The black solid line shows the real interest rate based on BIS data.

Table 1: Calibration and Priors for Greece

<table>
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<th>Calibration</th>
<th>Function</th>
<th>Mean</th>
<th>St. Dev.</th>
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<td>( \beta )</td>
<td>Beta</td>
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<td></td>
</tr>
<tr>
<td>( \bar{\eta} )</td>
<td></td>
<td>0.75</td>
<td></td>
</tr>
<tr>
<td>( \bar{\gamma}/\bar{\bar{y}} )</td>
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<td>0.181</td>
<td></td>
</tr>
<tr>
<td>( \bar{b}/\bar{\bar{y}} )</td>
<td></td>
<td>1.095*4</td>
<td></td>
</tr>
<tr>
<td>( \tau )</td>
<td></td>
<td>0.333</td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Priors</th>
<th>Function</th>
<th>Mean</th>
<th>St. Dev.</th>
</tr>
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<tbody>
<tr>
<td>( h )</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Uniform</td>
<td>1.6</td>
<td>0.013</td>
</tr>
<tr>
<td>( \gamma_{\tau,L} )</td>
<td>Gamma</td>
<td>0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>( \gamma_{g,L} )</td>
<td>Gamma</td>
<td>1.1</td>
<td>0.3</td>
</tr>
<tr>
<td>( \rho^{a} )</td>
<td>Beta</td>
<td>0.8</td>
<td>0.1</td>
</tr>
<tr>
<td>( \rho^{g} )</td>
<td>Beta</td>
<td>0.8</td>
<td>0.1</td>
</tr>
<tr>
<td>( \rho^{\tau} )</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>( \rho^{z} )</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>( \sigma_{a,p} )</td>
<td>Gamma</td>
<td>0.005</td>
<td>0.01</td>
</tr>
<tr>
<td>( \sigma_{g,p} )</td>
<td>Gamma</td>
<td>0.02</td>
<td>0.015</td>
</tr>
<tr>
<td>( \sigma_{\tau,p} )</td>
<td>Gamma</td>
<td>0.005</td>
<td>0.01</td>
</tr>
<tr>
<td>( \sigma_{z,p} )</td>
<td>Gamma</td>
<td>0.5</td>
<td>0.1</td>
</tr>
</tbody>
</table>
Table 2: Greece Estimates.

<table>
<thead>
<tr>
<th></th>
<th>Prior</th>
<th>Posterior: $\delta^A = 0.3$</th>
<th>Posterior: $\delta^A = 0.2$</th>
<th>Posterior: Linear</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$</td>
<td>0.5 [0.17, 0.83]</td>
<td>0.13 [0.04, 0.28]</td>
<td>0.11 [0.04, 0.21]</td>
<td>0.73 [0.59, 0.83]</td>
</tr>
<tr>
<td>$\delta$</td>
<td>1.6 [1.42, 1.78]</td>
<td>1.56 [1.53, 1.59]</td>
<td>1.53 [1.50, 1.56]</td>
<td>-</td>
</tr>
<tr>
<td>$\gamma_{r,L}$</td>
<td>0.4 [0.14, 0.78]</td>
<td>0.22 [0.08, 0.45]</td>
<td>0.20 [0.08, 0.42]</td>
<td>0.61 [0.30, 1.05]</td>
</tr>
<tr>
<td>$\gamma_{g,L}$</td>
<td>1.1 [0.66, 1.64]</td>
<td>1.18 [0.77, 1.71]</td>
<td>1.20 [0.73, 1.62]</td>
<td>1.07 [0.66, 1.62]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.8 [0.61, 0.94]</td>
<td>0.97 [0.96, 0.98]</td>
<td>0.97 [0.96, 0.98]</td>
<td>0.95 [0.94, 0.96]</td>
</tr>
<tr>
<td>$\rho^g$</td>
<td>0.8 [0.61, 0.94]</td>
<td>0.94 [0.90, 0.97]</td>
<td>0.94 [0.90, 0.98]</td>
<td>0.96 [0.91, 0.98]</td>
</tr>
<tr>
<td>$\rho^r$</td>
<td>0.8 [0.17, 0.83]</td>
<td>0.58 [0.29, 0.82]</td>
<td>0.44 [0.23, 0.75]</td>
<td>0.63 [0.39, 0.82]</td>
</tr>
<tr>
<td>$\sigma_{a,p}$</td>
<td>0.01 [0.003, 0.02]</td>
<td>0.022 [0.017, 0.028]</td>
<td>0.022 [0.017, 0.027]</td>
<td>0.019 [0.015, 0.024]</td>
</tr>
<tr>
<td>$\sigma_{g,p}$</td>
<td>0.02 [0.003, 0.05]</td>
<td>0.041 [0.034, 0.049]</td>
<td>0.038 [0.032, 0.047]</td>
<td>0.041 [0.034, 0.051]</td>
</tr>
<tr>
<td>$\sigma_{z,p}$</td>
<td>0.5 [0.35, 0.68]</td>
<td>0.49 [0.35, 0.63]</td>
<td>0.54 [0.46, 0.64]</td>
<td>0.50 [0.38, 0.66]</td>
</tr>
<tr>
<td>$\sigma_{\tau,p}$</td>
<td>0.01 [0.003, 0.02]</td>
<td>0.026 [0.022, 0.032]</td>
<td>0.025 [0.021, 0.032]</td>
<td>0.022 [0.018, 0.027]</td>
</tr>
</tbody>
</table>

Table 3: Smoothed estimates of measurement error.

<table>
<thead>
<tr>
<th>Measure</th>
<th>$y_t$</th>
<th>$\hat{y}_t$</th>
<th>$\hat{y}_t$</th>
<th>$\hat{y}_t$</th>
<th>$\hat{y}_t$</th>
<th>$\hat{y}_t$</th>
<th>$\hat{y}_t$</th>
<th>$\hat{y}_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nonlinear $\delta^A = 0.3$</td>
<td>mean absolute value</td>
<td>0.01</td>
<td>0.004</td>
<td>0.002</td>
<td>0.005</td>
<td>0.001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Linear</td>
<td>mean absolute value</td>
<td>0.12</td>
<td>0.09</td>
<td>0.11</td>
<td>0.14</td>
<td>0.24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Meas. Err. 25%</td>
<td>mean absolute value</td>
<td>0.01</td>
<td>0.006</td>
<td>0.002</td>
<td>0.01</td>
<td>0.0003</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Meas. Err. 30%</td>
<td>mean absolute value</td>
<td>0.09</td>
<td>0.13</td>
<td>0.08</td>
<td>0.29</td>
<td>0.10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Model Fit Comparisons

<table>
<thead>
<tr>
<th>Model Specification</th>
<th>Bayes Factor</th>
<th>Rel. to M1</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1: Nonlinear Model w/ $\delta^A = 0.3$</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>M2: Nonlinear Model w/ $\delta^A = 0.2$</td>
<td>exp[0.63]</td>
<td></td>
</tr>
<tr>
<td>M3: Linear</td>
<td>exp[55.5]</td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Fiscal Consolidation Experiment. Government spending-to-GDP ($s^g_t$) reductions and tax revenue-to-GDP ($s^r_t$) increases assumed for fiscal consolidation experiment. We assume that the annual target consolidations from table 1 of IMF (2010) are uniformly distributed within the year’s quarters.

<table>
<thead>
<tr>
<th>Measure</th>
<th>11Q1</th>
<th>11Q2</th>
<th>11Q3</th>
<th>11Q4</th>
<th>12Q1</th>
<th>12Q2</th>
<th>12Q3</th>
<th>12Q4</th>
<th>13Q1</th>
<th>13Q2</th>
<th>13Q3</th>
<th>13Q4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s^g_t$</td>
<td>0.275</td>
<td>0.275</td>
<td>0.275</td>
<td>0.275</td>
<td>0.425</td>
<td>0.425</td>
<td>0.425</td>
<td>0.425</td>
<td>0.575</td>
<td>0.575</td>
<td>0.575</td>
<td>0.575</td>
</tr>
<tr>
<td>$s^r_t$</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>-0.075</td>
<td>-0.075</td>
<td>-0.075</td>
<td>-0.075</td>
</tr>
</tbody>
</table>
Table 6: **Sensitivity analysis.** The table reports posterior medians and 90% credible intervals (in brackets) for various models.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\delta^A = 0.3$</th>
<th>$\delta^A = 0.2$</th>
<th>$\delta^A = 0.45$</th>
<th>Meas. Err. 30%</th>
<th>Meas. Err. 25%</th>
<th>Alt. $\gamma^{r,L}, \gamma^{g,L}$ priors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$</td>
<td>0.13</td>
<td>0.11</td>
<td>0.12</td>
<td>0.13</td>
<td>0.11</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>[0.04, 0.28]</td>
<td>[0.04, 0.21]</td>
<td>[0.04, 0.27]</td>
<td>[0.04, 0.32]</td>
<td>[0.04, 0.25]</td>
<td>[0.05, 0.39]</td>
</tr>
<tr>
<td>$\tilde{s}$</td>
<td>1.56</td>
<td>1.53</td>
<td>1.61</td>
<td>1.58</td>
<td>1.58</td>
<td>1.58</td>
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<tr>
<td></td>
<td>[1.53, 1.59]</td>
<td>[1.50, 1.56]</td>
<td>[1.58, 1.66]</td>
<td>[1.54, 1.62]</td>
<td>[1.55, 1.61]</td>
<td>[1.53, 1.63]</td>
</tr>
<tr>
<td>$\gamma^{r,L}$</td>
<td>0.22</td>
<td>0.20</td>
<td>0.21</td>
<td>0.24</td>
<td>0.24</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
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<td>[0.08, 0.42]</td>
<td>[0.08, 0.46]</td>
<td>[0.09, 0.49]</td>
<td>[0.09, 0.54]</td>
<td>[0.02, 0.72]</td>
</tr>
<tr>
<td>$\gamma^{g,L}$</td>
<td>1.18</td>
<td>1.20</td>
<td>1.13</td>
<td>1.10</td>
<td>1.10</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
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<td>[0.73, 1.62]</td>
<td>[0.72, 1.75]</td>
<td>[0.72, 1.67]</td>
<td>[0.71, 1.63]</td>
<td>[0.25, 0.81]</td>
</tr>
<tr>
<td>$\rho^a$</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>[0.96, 0.98]</td>
<td>[0.96, 0.98]</td>
<td>[0.95, 0.98]</td>
<td>[0.96, 0.98]</td>
<td>[0.95, 0.98]</td>
<td></td>
</tr>
<tr>
<td>$\rho^g$</td>
<td>0.94</td>
<td>0.94</td>
<td>0.92</td>
<td>0.92</td>
<td>0.93</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>[0.90, 0.97]</td>
<td>[0.90, 0.98]</td>
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<td>[0.87, 0.96]</td>
<td>[0.88, 0.97]</td>
<td>[0.82, 0.95]</td>
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<tr>
<td>$\rho^z$</td>
<td>0.58</td>
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<td>0.57</td>
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<tr>
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<td>[0.29, 0.81]</td>
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<td>[0.28, 0.84]</td>
<td>[0.30, 0.80]</td>
</tr>
<tr>
<td>$\rho^\tau$</td>
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<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
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<td>[0.91, 0.96]</td>
<td>[0.90, 0.96]</td>
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<td>[0.89, 0.97]</td>
</tr>
<tr>
<td>$\sigma_{a,p}$</td>
<td>0.022</td>
<td>0.022</td>
<td>0.016</td>
<td>0.016</td>
<td>0.018</td>
<td>0.014</td>
</tr>
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<td>[0.017, 0.027]</td>
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<td>[0.012, 0.21]</td>
<td>[0.013, 0.224]</td>
<td>[0.011, 0.018]</td>
</tr>
<tr>
<td>$\sigma_{g,p}$</td>
<td>0.041</td>
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<td>0.036</td>
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<td>[0.029, 0.046]</td>
<td>[0.029, 0.045]</td>
<td>[0.030, 0.048]</td>
<td>[0.027, 0.044]</td>
</tr>
<tr>
<td>$\sigma_{z,p}$</td>
<td>0.49</td>
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<td>0.48</td>
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</tr>
<tr>
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<td>[0.35, 0.64]</td>
<td>[0.35, 0.67]</td>
<td>[0.35, 0.65]</td>
</tr>
<tr>
<td>$\sigma_{r,p}$</td>
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<td>0.026</td>
<td>0.019</td>
<td>0.023</td>
<td>0.014</td>
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</tr>
<tr>
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<td>[0.014, 0.025]</td>
<td>[0.018, 0.030]</td>
<td>[0.010, 0.019]</td>
</tr>
</tbody>
</table>