Stress Fields Around a Screw Dislocation ($\vec{b} = b\hat{k}, \hat{t} = \hat{k}$)
(from Hull & Bacon, pp. 75-76)

Note that no displacements along x and y directions: $u_x = u_y = 0$
While $u_z$ increases uniformly from 0 to $b$ as $\theta$ increases from 0 to $2\pi$:

$$u_z = \frac{b\theta}{2\pi} = \frac{b}{2\pi} \tan^{-1}\left(\frac{y}{x}\right)$$

We can now evaluate the various strain components ($\varepsilon_{ij}$):

$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{zz} = \varepsilon_{xy} = \varepsilon_{yx} = 0$$

$$\varepsilon_{xz} = \varepsilon_{zx} = -\frac{b}{4\pi} \frac{y}{(x^2 + y^2)} = -\frac{b}{4\pi} \frac{\sin \theta}{r}$$

$$\varepsilon_{yz} = \varepsilon_{zy} = \frac{b}{4\pi} \frac{x}{(x^2 + y^2)} = \frac{b}{4\pi} \frac{\cos \theta}{r}$$

The stress components are found using: $\sigma_{xy} = 2G\gamma_{xy}$ etc. (theory of elasticity)

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \sigma_{xy} = \sigma_{yx} = 0$$

$$\sigma_{xz} = \sigma_{zx} = -\frac{Gb}{2\pi} \frac{y}{(x^2 + y^2)} = -\frac{Gb \sin \theta}{2\pi} \frac{1}{r}$$

$$\sigma_{yz} = \sigma_{zy} = \frac{Gb}{2\pi} \frac{x}{(x^2 + y^2)} = \frac{Gb \cos \theta}{2\pi} \frac{1}{r}$$

In cylindrical polar coordinates, they are much simpler:

$$e_{0z} = e_{z0} = \frac{b}{4\pi r}, \quad \text{and} \quad \sigma_{0z} = \sigma_{z0} = \frac{Gb}{2\pi r}$$
Elastic Strain-Energy Density

\[ U^{el} = \frac{1}{2} \sigma_{ij} \varepsilon_{ij} = \frac{G b^2}{8 \pi^2 r^2} \]

is the elastic strain energy density at ‘r’ due to a screw dislocation.

Similarly for an edge dislocation:

\[ U^{el} = \frac{1}{2} \sigma_{ij} \varepsilon_{ij} = \frac{G b^2}{8 \pi^2 (1-\nu) r^2} \]

The elastic strain energy in the volume element at \( r \) and \( r+dr \) per unit length of the screw dislocation:

\[ dU^{el} = (2\pi dr) \frac{G b^2}{8 \pi^2 r^2} = \frac{G b^2}{4\pi} \frac{dr}{r} \]

**Line Tension (\( \Gamma \)) of a Dislocation**: total strain energy per unit length

\[ \Gamma = \int_{r_c}^{\Omega} U^{el} d\Omega = \int_{r_c}^{\Omega} (2\pi dr) \frac{G b^2}{8 \pi^2 r^2} = \int_{r_c}^{\Omega} \frac{G b^2}{4\pi} \frac{dr}{r} = \frac{G b^2}{4\pi} \ln \left( \frac{\Omega}{r_c} \right) \]

for an edge dislocation: \( \Gamma_\perp = \frac{G b^2}{4\pi(1-\nu)} \ln \left( \frac{\Omega}{r_c} \right) \)

Or, \( \Gamma = \alpha G b^2 \), with \( 0.5 \leq \alpha \leq 1 \)

Note:

1. minimize line tension – \( b \) smallest (along cpd)
2. Frank’s rule: \( b^2 \) minimized (dislocation reactions)
3. Dislocations want to be straight to minimize length (line tension)

An external force is needed to keep a dislocation curved ⇒