Fatigue

Fatigue is failure under cyclic loading

- define various parameters (Fig. 8.17):
  \[ \sigma_m - \text{mean stress} \quad \sigma_r - \text{stress range} \quad \sigma_a - \text{stress amplitude} \quad R = \frac{\sigma_{\min}}{\sigma_{\max}} \]

- S-N curves: (Fig. 8.19)
  - fatigue strength
  - endurance-limit (35 - 60% of UTS)

- low vs high cycle fatigue: LCF - ductility controlled    HCF - strength controlled
  ⇒ effects of CW / radiation

Coffin-Manson Equation (LCF):

\[ \Delta \varepsilon_p = A (2N)^c, \quad c \sim -0.5 - -0.7, A \sim \varepsilon_f \text{ (Fig.12.13)}; \]

similarly for hcf:

\[ \frac{\Delta \varepsilon_E}{2} = B (2N)^b, \quad b \sim -0.05 - -0.12, B \sim \sigma_f \]

Characteristic slopes:

\[ \frac{\Delta \varepsilon}{2} = \frac{\sigma_f}{E} (2N)^b + \varepsilon_f (2N)^c \]

Universal slopes:

\[ \Delta \varepsilon = 3.5 \frac{S_u}{E} N^{-0.12} + \varepsilon_f^{0.6} N^{-0.6} \]
• mechanism (Fig. 8.22)  
crack initiation and propagation  
(Fig. 8.21)  
stage I (slow / along preferred crystallographic directions)  
stage II (relatively faster / \( \perp \)) to the loading direction  
beachmarks (Fig. 8.23) & fatigue striations (Fig. 8.24)  

• Crack propagation rate (Fig. 8.26)  

![Graph showing crack propagation rate](image)

No. of cycles to failure:  
Paris law:  
\[
\frac{da}{dN} = A(\Delta K)^m  
\Rightarrow \text{Fig. 8.27} 
\]

\[
\therefore N_f = \int_{0}^{N_f} \frac{da}{A(\Delta K)^m} = \frac{1}{AY_m(\Delta \sigma)^m \pi^{m/2}} \int_{a_o}^{a_f} \frac{da}{a^{m/2}}  
\]

\[
N_f = \frac{1}{A\pi^{m/2}(\Delta \sigma)^m} \int_{a_o}^{a_f} \frac{da}{Y_m a^{m/2}} \quad \text{(Eq. 8.21)} \quad \Leftrightarrow \begin{cases} 
m \neq 2 \\
m = 2 
\end{cases}  
\]

Factors Affecting Fatigue Life (& solutions to improve fatigue life)  
1. Mean Stress (Fig. 8.29)  
2. Surface Effects (polished vs machined)  
3. Design Factors (Fig. 8.30)  
4. Surface Treatments  
   • shot peening  
   (surface compressive stresses)  
   • case hardening  
   (carburization / nitriding - Fig. 8.31)
Group Work

A steel has the following properties:
Yield stress $\sigma_0 = 700$ MPa
Toughness $K_{IC} = 165$ MPa

A plate of this steel containing an edge crack was tested in fatigue under conditions where $a_o = 2$ mm, $R = 0.5$ and $\sigma_{max} - \sigma_{min} = 140$ MPa.

This steel follows Paris law: $da/dN = 0.66 \times 10^{-8} (\Delta K)^{2.25}$ where $da/dN$ is in m/cycle and $\Delta K$ is in MPa$\sqrt{m}$.

a. Show that the critical crack size at failure $a_f$ is 110 mm.
   First find $\sigma_{max}$ from $R$ and $\Delta \sigma$ ⇒ use relation between $K_{IC}$, $\sigma$ and $a_f$

b. Compute the fatigue life $N_f$
   In Eq. 8.21 we know $A$, $m$, $\Delta \sigma$, $a_o$, $a_f$ and $Y^1$

c. Compute $t_f$ at a cyclic frequency $\nu = 15$ Hz
   \[ t = \frac{N}{\nu} \]