8.1 S vs e curve : Definitions -
Tensile Strength (Eq. 8-3)
Yield Strength (0.2%)
Ductility : \( e_f = \frac{l_f - l_o}{l_o} \) (depends on \( l_o \))
\[
q = \frac{A_o - A_f}{A_o} \Rightarrow (RA)
\]
volume constancy \( \Rightarrow e_f = \frac{q}{1-q} = e_o \) \( \Rightarrow \) zero gauge length ductility ---- Eq. 8-7

Resilience : modulus of resilience , \( U_R = \frac{1}{2} s_o e_o = \frac{1}{2} \frac{s_o}{E} \) \( \Rightarrow U_R \uparrow \) if \( E \downarrow \) or \( S_o \uparrow \) Eq. 8-7

• large elastic strain (low \( E \)) before yielding with high yield stress

Toughness : Energy to Fracture 
\[
J = \int \sigma \, d\varepsilon \approx S_{flow} \, e_f = K \frac{\varepsilon_f^{n+1}}{n+1}, \{n=\varepsilon_u\}
\]

True \( \sigma \) vs \( \varepsilon \) : need necking-correction beyond UTS \( (P_{max}) \) - Bridgmann, etc. (sec 8:3)
\[
\sigma_{TS} = S_u \, e^{\varepsilon_u} \text{ (Eq. 8-16)}; \quad e_f = \ln \left( \frac{1}{1-q} \right) \text{ Eq. 8-18}; \quad \varepsilon_u = n = \ln \left( \frac{A_o}{A_u} \right)
\]

In general, \( \sigma = K \, \varepsilon^n \) (plastic stress and strain) \( \leftarrow \) but \( @ \, \varepsilon = 0, \sigma \neq 0 \) !!!

often \( \Rightarrow \) Ludwik Relation holds better \( \sigma = \sigma_0 + K \varepsilon^n \) and other relations exist

\( \Rightarrow \) In general, strain-hardening exponent varies with strain / different values in different strain ranges

Instability in Tension : Necking or geometrical softening occurs in tension

Necking Strain : \( \varepsilon_n = \ln \left( \frac{A_u}{A_f} \right) \); \( \varepsilon_t = \varepsilon_u + \varepsilon_n \)

Where does necking start or Instability
Criterion : \( @ \, P_{max} \, \varepsilon = \varepsilon_u \) and \( dP=0 \)

since \( P = \sigma \, A \Rightarrow \frac{d\sigma}{d\varepsilon} = \sigma \)

or in terms of \( \sigma \) and \( \varepsilon \) :
\[
\frac{d\sigma}{d\varepsilon} = \frac{\sigma}{1+\varepsilon}
\]

\( \Rightarrow \) Considere’s Construction

Localized vs Diffuse Neck (Fig. 8-9)
Effect of Strain-Rate (8-6):

Fig. 8-12: \( \dot{\varepsilon} = \frac{\dot{x}}{L} \) where \( \dot{x} \) is the cross-head speed

Flow Stress vs Strain-Rate: \( \sigma_{T,\varepsilon} = C\dot{\varepsilon}^m \Rightarrow m = \left( \frac{\partial \ln \sigma}{\partial \ln \dot{\varepsilon}} \right)_{T,\varepsilon} \leftarrow \text{SRS} \)

Refer to Table 8-5 for range of strain-rates & techniques
(Creep, Tensile, Dynamic Compression or Tension, Impact, Shock Wave (Hopkinson-Bar))

\[ m \uparrow \varepsilon_t \uparrow \text{ (vs } n \uparrow \varepsilon_u \uparrow) \quad m_{\text{max}} = 1 \quad \text{---- Newtonian Viscous} \]

In general, \( m \) is low (<0.1) for many metals at ambient while it approaches 1 at very high temperatures and very low strain-rates

Higher the \( m \) value, lower the rate of decrease on the cross-sectional area in the neck ---(Fig. 8-14) and when \( m = 1 \), the rate of area decrease becomes independent of the cross-sectional area.

For superplastic materials ---- \( m \sim 0.5 \) at normal strain-rates and temperatures ---- leads to large (in excess of 1,000% strains) !!!

Eq. 8-50: \[ -\frac{dA}{dt} = \left( \frac{P}{C} \right)^{1/m} \frac{1}{A^{1-m/m}}, \begin{cases} m < 1 \quad \sigma = C\dot{\varepsilon}^m \\ \end{cases} \]

For \( m < 1 \), \( \frac{dA}{dt} \downarrow \text{ as } A \downarrow \text{ or } \text{smaller the } A, \text{ more rapid is decrease in } A \)

while for \( m = 1 \), \( \frac{dA}{dt} \neq f(A) \quad \text{------ Or, higher } m \text{ resists smaller } A \text{ from rapid reduction} \)

Conditions for Superplasticity:
\begin{align*}
\text{small equiaxed grains} \\
\text{stable grain size} \\
\text{suppress matrix flow} \\
\text{high } m
\end{align*}

---- thus Eutectics and Eutectoids with dominant grain boundary sliding (GBS) exhibit superplasticity ---- advantages in superplastic forming

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Effect of Temperature (8-7): (fig. 8-17 ---note fcc vs bcc – NDT in bcc)

\[ \sigma = f(\dot{\varepsilon},\varepsilon,T) \quad \text{rate } \propto e^{-Q/RT} \text{ or } \sigma \propto e^{Q/RT} \]

So that \( \sigma = Ae^{n\dot{\varepsilon}^m}e^{Q/RT} \)

MEOS ---- \( f(\dot{\varepsilon},\varepsilon,T,\sigma) = 0 \) (similar to Thermo ---- PV=nRT) but while the gas law is path-independent, the mechanical properties and plastic deformation do depend on the path unless
(a) at very low temperatures where there is no recovery at all, and
(b) at high temperatures where there is complete recovery such as during high temperature creep
As will be seen later, during HT deformation (or creep),
\[ \sigma|_{\dot{\varepsilon}} = f(Z) = f(\dot{\varepsilon} e^{Q/RT}) \]
where \( Z \) is known as Zener-Holloman Parameter

In general, \( \dot{\varepsilon} = Af(\sigma)e^{-Q/RT} \) where \( f(\sigma) = \begin{cases} e^{B\sigma} & \text{at high stresses} \\ \sigma^n & \text{at low stresses (n = 1/m, m = SRS)} \end{cases} \)

Or \( \dot{\varepsilon} = A(SinhB\sigma)^n e^{-Q/RT} \)

-----

**Effect of Test Machine (8-8):** Machine Stiffness ----- \( \Delta x = \dot{v} t = \frac{P}{K} + \frac{\sigma}{E} L + \varepsilon_p L \)

so that \( \dot{\varepsilon}_p = \frac{\dot{v} t}{L} - \frac{\sigma}{E} - \frac{P}{KL} \) and applied strain-rate \( \dot{\varepsilon} = \frac{v}{L} = \dot{\varepsilon}_p + \frac{\dot{\sigma}}{E} \left( \frac{AE}{KL} + 1 \right) \)

--- difference between Rigid vs Soft Machine (A rigid machine like a screw driven machine gives distinct yield points characteristic of the material)

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**Stress Relaxation (8-11):** At P (see Fig. 8-19) \( \varepsilon = 0 = \varepsilon_p + \varepsilon_E \)

Or \( \dot{\varepsilon}_p = -\dot{\varepsilon}_E = -\frac{\dot{\sigma}}{E} \) (if rigid machine where K is very large)

In general, \( \dot{\varepsilon}_p = -\left( \frac{A}{KL} + \frac{1}{E} \right)\dot{\sigma} \) (8-80)

By knowing machine stiffness (K), and materials’ Young’s modulus (E), one can find \( \dot{\sigma} \) vs t or \( \sigma \) during stress relaxation from which one can evaluate: \( \dot{\varepsilon}_p \) vs \( \sigma \) covering a large range of strain-rates so that the SRS (m) can be evaluated: \( m = \frac{d \ln \sigma}{d \ln \dot{\varepsilon}} = \frac{d \ln \sigma}{d \ln \dot{\sigma}}. \)
The Hardness Test

(Ch. 9 : all sections)

• Hardness is defined as the resistance of a material to surface penetration
• scratch hardness - useful to mineralogists and now to thin films in semiconductors
• dynamic hardness - (rebound) of an indenter dropped on to the surface (impact energy)
• indentation hardness \( \sqrt{\text{ }} \)

Recall - Rockwell (depth measurement of a spherical indenter) - arbitrary (\( \propto \) DPH)

DPH (VHN) - measures size of impression of right diamond with 135°

\[
\text{VHN} = \frac{\text{load}}{\text{surface area of indentation}} = \frac{\sin(\theta/2) P}{d^2} = \frac{1.854P}{d^2}
\]

\( d = \text{mean diagonal of indentation} \)

• Good for research but slow, needs good surface preparation, operator errors in d-measurement \( \Leftarrow \) Industry uses Rockwell more often

KHN (Knoop Hardness) - Variation of VHN - diamond with one long diagonal - close to plane strain in the deformed region - much lower loads/microhardness

• can use to measure hardness in close proximity to each position - advantage in measuring gradient properties (such as in welds, etc.) & close to the surface

• adaptable to thin layers & brittle materials (since area or depth \( \sim 15\% \) of VHN)

From Plasticity Analyses (ignoring work-hardening)

\( H = C \sigma_{TS} \) recall (MAT 201) \( \Leftarrow \)
• Thus, can use H vs T to evaluate \( \sigma(T) \): see Fig. 9-5 ⇔

H is much easier to determine than \( \sigma \)

BHN (Brinell Hardness) - measure indentation diameter of a ball indenter

\[
\text{BHN} = \frac{\text{Load}}{\text{surface area of indentation}}
\]

\[= \frac{P}{\pi D t} \quad \text{(Eq.9-1)} \quad t \text{ is depth}
\]

• Eq.9-1 not very satisfactory since it does not give the mean pressure over the surface of the indentation

Since \( d = D \sin \phi \) (see Fig 9-1),

\[
\text{BHN} = \frac{P}{(\pi/2) D^2 (1-\cos \phi)} \quad \text{(Eq.9-2)}
\]

• Geometric similarity when \( 2\phi \) is kept constant for nonstandard load(s) or ball diameters

• Better approach is to use Meyer hardness:

use projected area \( \frac{1}{4} \pi d^2 \) of impression (rather than surface area)

\[\Rightarrow \text{mean pressure (} p_m \text{) or Meyer Hardness} = \frac{4P}{\pi d^2} \quad \text{(Eq. 9-4)}
\]

Meyer’s law: \( P = k d^n \)’ where \( n' \approx n + 2 \), \( n \) = work-hardening parameter (\( \sigma = K\varepsilon^n \))

• elastic-plastic analysis:

mean pressure or Meyer Hardness, \( p_m = C \sigma_f \approx 3 \sigma_f \) \( \sigma_f \) is flow stress (Eq. 9-6)

• Relation Between Meyer Hardness and flow (\( \sigma-\varepsilon \)) curve (Fig. 9-3):

\[
\text{Tabor} \Leftrightarrow \varepsilon = 0.2 \frac{d}{D}
\]

and relate \( p_m \) to \( \sigma (\varepsilon) \) ⇔

• ABI (Automated Ball Indentation)