Fatigue
(12 : 1, 2, 3, 5, 6, 7, 8, 9, 10, 13, 14, 20, 21)

Fatigue is failure under cyclic loading

- define various parameters (Fig. 12-2):
  \[ \sigma_m - \text{mean stress} \quad \sigma_r - \text{stress range} \quad \sigma_a - \text{stress amplitude} \quad R = \frac{\sigma_{\min}}{\sigma_{\max}} \]

- S - N curves: (Fig.12-3) – S could be \( \sigma_{\text{max}}, \sigma_m \) or \( \sigma_a \)
- fatigue strength
  - endurance-limit (35 - 60% of UTS) (for \( R=-1 \))

\[ \begin{align*}
\text{Ferrous alloys} & \quad \text{Non-ferrous alloys} \\
\text{Fatigue Failures occur at very low stresses, } \sigma & \geq 0.4 \text{ UTS} \quad (\sigma_e \approx 0.4 \text{ UTS}) \\
* \quad \text{Cyclic Deformation:} & \quad \text{stress-controlled } (\Delta \sigma) \\
& \quad \text{strain-controlled } (\Delta \varepsilon) \\
#1 \text{ is the common-type } \text{in-service} \quad (\text{while } #2 \text{ is much easier to perform in lab})
\end{align*} \]

Life Prediction Methodologies:
Note: effects of R-ratio on fatigue life Fig. 12-6 (top) – for constant \( \sigma_{\text{max}} \), as \( R \downarrow N_f \downarrow \)

Effect of mean stress on fatigue life (Fig. 12-6) – for constant \( \sigma_a \), \( N_f \downarrow as \sigma_m \uparrow \)

From these data, note that for each \( \sigma_m \), there exists a range of stresses, \( \sigma_r (=\sigma_{\text{max}}-\sigma_{\text{min}}) \) that can be tolerated (Fig. 12-7) – Goodman diagram

Note here --- when \( \sigma_r=0 \) (i.e., monotonic loading) – (\( \sigma_{\text{min}}=\sigma_{\text{max}} \approx \sigma_{\text{UTS}} \))

Need a ‘master curve’ to illustrate the safe & failure ranges ⇒
• Goodman, Soderberg, Gerber and Parabolic Relations (Eqs. 12-8) - 
define fail-safe regions (for \( N \sim \text{specified} \ N_f \)): 
\[
\sigma_a = \sigma_e [l - \left( \frac{\sigma_m}{\sigma_u} \right)^x] \text{ where } 1 < x < 2
\]

Soderberg: 
\[
\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_{uts}} = 1
\]
\( \uparrow \Rightarrow \text{most conservative} \)

Goodman: 
\[
\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_{uts}} = 1
\]

Gerber: 
\[
\frac{\sigma_a}{\sigma_e} + \left( \frac{\sigma_m}{\sigma_{uts}} \right)^2 = 1
\]

Elliptic: 
\[
\left( \frac{\sigma_a}{\sigma_e} \right)^2 + \left( \frac{\sigma_m}{\sigma_{uts}} \right)^2 = 1 \quad \Leftrightarrow \text{least conservative}
\]

• **Cyclic Stress-Strain Curves**: Strain hardening / softening (Figs. 12-10 to 12-12) 
\[
\Delta \sigma = K' (\Delta \varepsilon)^n' - \text{cyclic hardening vs cyclic softening}
\]
Or, 
\[
\frac{\Delta \varepsilon}{2} = \frac{\Delta \sigma_{E}}{2} + \frac{\Delta \sigma_p}{2E} = \frac{\Delta \sigma}{2E} + \frac{1}{2} \left( \frac{\Delta \sigma}{K'} \right)^n' \quad \text{(Eq. 12-9)}
\]

• Coffin-Manson Equation (for LCF): 
\[
\frac{\Delta \varepsilon}{2} = \varepsilon_f (2N)^c \quad \text{(Eq. 12-10)}
\]

Method of Characteristic Slopes: 
\[
\frac{\Delta \varepsilon}{2} = \frac{\sigma_f}{E} \left( 2N_f \right)^y + \varepsilon_f \left( 2N_f \right)^z \quad \text{(Eq. 12-12)}
\]

Method of Universal Slopes: 
\[
\Delta \varepsilon = 3.5 \frac{S_u}{E} N_f^{-0.12} + \varepsilon_f^{0.6} N_f^{-0.6} \quad \text{(Eq. 12-15)}
\]

where \( S_u \) and \( \varepsilon_f \) are \( f(cw, x, \ldots) \) \( \Leftrightarrow \) can find \( \Delta \varepsilon \) for specific \( N_f \) or vice versa

(VG – example of SS – irradiated)

• **Structural Features / Fatigue**: (12-9)
• crack initiation - early development of slip bands / deepening of initial cracks (Fig. 23-15)
• slip band crack growth - on planes of high shear stress - **stage I crack growth**
  (small crack extension - few grain diameters) - PSBs
• crack growth on planes of high tensile stress - **stage II crack growth** -
  due to plastic blunting of crack tip (large crack extension ~ microns per cycle) - fatigue striations (~ cycles, Fig. 12-16) – Paris law / region
• ultimate ductile failure - (crack at ~45° to the loading direction) – eventual mechanical failure of the remaining ligament

Note: Fatigue occurs even at very low temperatures (i.e., not thermally activated)
• Fatigue produces a large number of vacancies (PAS, \( \rho \), stored-energy release, etc)

**Fatigue Crack Growth**: 12-10 (Mechanism – Fig. 12-17)

Stage I - Threshold region  
Stage II - stable / Steady-State  
Stage III - Unstable

Crack-Growth Rate: stage II  
\[
\frac{da}{dN} = C \sigma_m \sigma_a^n, \ n \approx 1 - 2 \text{ and } m \approx 2 \text{ to } 4
\]

\( \leftrightarrow \) CT specimens  
\( K_{\text{min}} = \sigma_{\text{min}} \sqrt{\pi a} \quad \& \quad K_{\text{max}} = \sigma_{\text{max}} \sqrt{\pi a} \) (Generally \( R = 0, \Delta \sigma = \sigma_r \))

\& \( K_{\text{min}} = 0 \) for \( \sigma_{\text{min}} \leq 0 \) (since cracks cannot propagate under compression)

\[
\Delta K = K_{\text{max}} - K_{\text{min}} = Y(\sigma_{\text{max}} - \sigma_{\text{min}}) \sqrt{\pi a} = Y\Delta \sigma \sqrt{\pi a} \quad \text{or} \quad Y\sigma_r \sqrt{\pi a}
\]

• Paris Equation:  
\[
\frac{da}{dN} = A(\Delta K)^p, \ p \approx 2 \text{ to } 4
\]

• Derive Equation for fatigue life, \( N_f \) (Eq. 12-25) \& \( t_f \)

\[
dN = \frac{da}{A(\Delta K)^p} \quad \text{so that} \quad N_f = \int_0^{a_f} \frac{da}{A(Y\sigma_r \sqrt{\pi a})^p},
\]

Or  
\[
N_f = \frac{1}{A Y \sigma_r^p \pi^{p/2}} \int_0^{a_f} (a)^{-p/2} da \quad \text{where} \quad a_f = \frac{K_{IC}^2}{Y^2 \pi \sigma_{max}^2}
\]

\{since \( K_{IC} = Y\sigma_{max} \sqrt{\pi a_f} \}\}

Two Cases: (1) \( p=2 \) and (ii) \( p \neq 2 \) (eq. 12-25)

Note: Fatigue Crack Growth (FCC) tests are performed \( @R=0 \)  
while Fatigue Life Tests (S-N curves) are usually performed \( @R=-1 \) (reverse cycling)

• **Surface Effects**: (12-13)
  - surface roughness (Table 12-3 / Fig. 12-20)
  - surface residual stress - shot-peening / surface rolling
  - changes in surface - surface hardening - carburization / nitriding

• can also be improved by solute addition / strain-aging due to interstitials (Fig.12-23)

• **Corrosion Fatigue**: (12-20) Fig. 12-25 - strong effect on \( \Delta K_{\text{th}} \) \& in stage-II

• **Static Fatigue**: Delayed Frature / Hydrogen embrittlement / Fig. 14-18

• **Thermal Fatigue**: \( \Delta \sigma = \alpha E \Delta T \)
In-Class work

A mild plate (E=207 GPa and $K_c=100 \text{ MPa}\sqrt{\text{m}}$) is subjected to fatigue ($\sigma_{\text{max}}=180 \text{ MPa}$ and $\sigma_{\text{min}}=-40 \text{ MPa}$). If the plate contained an initial through thickness edge crack of 0.5 mm, how many fatigue cycles are required to break the plate? {assume $\frac{da}{dN} (\text{m/cycle}) = 6.9 \times 10^{-12} (\Delta K)^2, (\text{MPa}\sqrt{\text{m}})$, and $Y=1.12$}

(a) Find $a_f =$

(b) Derive equation for $N_f$ and calculate