Curve fitting is a very important topic in mathematics and in all areas which use mathematics, such as engineering and science. The situation frequently arises in which we given a set of data which we would like to link through some given function. As with all topics some cases will be easy and some hard.

(a) Let's first look at a simple problem: given three points - find a parabola which passes through them. To see how we can do this let's suppose the points are (1,4), (2,6) and (4,2). Any parabola has the form \( y = ax^2 + bx + c \). In rough terms then we think that having three points will determine the three constants \( a, b \) and \( c \). Maple V does this as follows:

\[
\begin{align*}
  f := x &\rightarrow ax^2 + bx + c \\
  u := \text{solve}([f(1)=4,f(2)=6,f(4)=2],[a,b,c]); \\
  y := \text{subs}(u,f(x));
\end{align*}
\]

Suppose now that we asked to fit a straight line through three or more points which are not in a straight line. In particular suppose we want to find the "best fit" line through the points (1,1), (2,5/2), (3,3), (4,3), (5,6), (6,5), (7,9) and (8,8). One approach to this problem would be to take the line \( y = mx + b \), say, and to find \( m \) and \( b \) so as to minimize the sum of the vertical distances between the given points and the corresponding points on the line. The following Maple V commands do this. The first thing you should do is to enter these commands into a Maple V worksheet and execute them and then figure out the mathematical steps which have been used.

\[
\begin{align*}
  > &\text{xl:=\{1,2,3,4,5,6,7,8\};} \\
  > &\text{yl:=\{1,5/2,3,3,6,5,9,8\};} \\
  > &\text{dist:=sum((m*xl[r]+b-yl[r])^2,r=1..8);} \\
  > &\text{with(student);} \\
  > &\text{u:=\text{simplify(dist)};} \\
  > &\text{complete_square(u,m);} \\
  > &\text{complete_square((28/17)*b^2+(9/17)*b+1379/204);} \\
  > &\text{line:=(107/102-(3/17)*(-9/56))*x-9/56;} \\
  > &\text{l:=xl[1],yl[1];} \\
  > &\text{for n from 2 to 8 do l:=l,xl[n],yl[n]: od;} \\
  > &\text{a:=\text{plot([l],style=point)};} \\
  > &\text{b:=\text{plot(line,x=0..8)};} \\
  > &\text{with(plots);} \\
  > &\text{display([a,b]);}
\end{align*}
\]

Assignment:
(a) Follow the above procedure and find the best fit line through the points: (1,2), (2,2), (3,3), (4,3), (5,4), (5,5), (6,7) and (7,7).
(b) By writing a similar procedure find the best fit parabola through the points in (a).