Lab 2

Topic: Implicit plots/parametric plots

Let us explore the various plotting techniques we can use on MAPLE. We can easily plot the graphs of functions, recall that these graphs will satisfy the vertical line test. What about getting the graphs of implicitly defined functions? A simple example is the circle $x^2 + y^2 = 1$. The following are just three methods we might try.

(i) If possible, get some explicit solutions, which are called branches. Here we have:

$$y = \sqrt{1-x^2} \quad \text{and} \quad y = -\sqrt{1-x^2},$$

and so we can ask for,

```maple
> plot({sqrt(1-x^2),-sqrt(1-x^2)},x=-1..1);
```

This does not look like a circle unless we constrain it, then it looks fine.

(ii) If possible, use parametric equations. For our circle we have $x = \cos(t)$ and $y = \sin(t)$, where $t = 0..2*\Pi$, and the appropriate MAPLE command is,

```maple
> plot([cos(t),sin(t),t=0..2*Pi]);
```

This particular method has the advantage that we can conveniently plot pieces of the circle. For example,

```maple
> plot([cos(t),sin(t),t=Pi/2..2*Pi]);
```

(iii) We can use the non-sporting method and use the command "implicitplot". This is under "with(plots)" and so here we have,

```maple
> with(plots);
> implicitplot(x^2+y^2=1,x=-1..1,y=-1..1);
```

(One quirk of implicitplot is that it sometimes gives "shaky" graphs. This can usually be minimized by appropriate choices of ranges for both $x$ and $y$. Failing this look at the grid option)

Assignments:

(a) Duplicate all of the above methods for the ellipse $4*x^2 + 9*y^2 = 16$.

(b) Duplicate any two of the above methods for $x^3 + y^3 = 8$ on an interval of $x=-5..5$. Do your results indicate that this relationship defines an explicit function? If so what is it?

(c) Now examine $(x^2 + y^2)^2 = 4*x*y$. Say why the graph of this relationship is confined to the first and third quadrants. Verify that a parametric representation is:

$$x = \sqrt{2*\sin(2*t)}*\cos(t) \quad \text{and} \quad y = \sqrt{2*\sin(2*t)}*\sin(t).$$

Plot, using MAPLE, the graph of this function two different ways.
Plot, using MAPLE, the portion of the graph in the first quadrant where \( y < x \) and \( 0 \leq x \leq 1 \).