Solutions to Test #1, Fall 2004 MA 242.601

1. \[ -5 \sqrt{46} \in (-1, 3, 6] \]

2. A vector valued function is continuous at a point if each of its component functions is continuous at the point. So we consider the component functions one at a time.

(a) \( \cos(t) \) is continuous everywhere and hence is continuous at \( t = 0 \).

(b) \( \exp(t^2) \) is the composition of \( \exp(x) \) and the polynomial \( t^2 \). Both are continuous everywhere and since the composition of continuous functions is continuous, \( \exp(t^2) \) is continuous at \( t = 0 \).

(c) \( \frac{t^2 + 1}{t^2 - 1} \) is a rational functions and \( t = 0 \) is in the domain of this rational function. Rational functions are continuous on their domains and hence \( \frac{t^2 + 1}{t^2 - 1} \) is continuous at \( t = 0 \).

Hence \( \langle \cos(t), \exp(t^2), \frac{t^2 + 1}{t^2 - 1} \rangle \) is continuous at \( t = 0 \).

3. \( \vec{r}(t) = \langle t, t^2, t^2 \rangle \).

(a) \( \vec{T}(1) = \langle 1/3, 2/3, 2/3 \rangle \)

(b) Use the tangent vector \( \langle 1, 2, 2 \rangle \) at \( t = 1 \) as the direction vector of the line. Then \( x = 1 + t, y = 1 + 2t, z = 1 + 2t \).

(c) \( a_T(1) = \frac{\vec{r}'(1) \cdot \vec{r}''(1)}{v(1)} = 8/3 \)

\( a_N(1) = \frac{|\vec{r}'(1) \times \vec{r}''(1)|}{v(1)} = \frac{\sqrt{3}}{3} \)

(d) \( \kappa = \frac{\sqrt{3}}{27} \)

4. \(-21(x - 1) + 0(y + 1) + 7(z - 1) = 0.\)

5. The equation for the 1-level curve of the function \( f(x, y) = -\sin(x) + y \) is \( -\sin(x) + y = 1 \). Rewrite it as \( y = 1 + \sin(x) \). This is the graph of the \( \sin(x) \) function shifted up the \( y \) axis 1 unit.

6. The level surface of the given function passing through the point \( (1, 1, 1) \) is

\[ f(x, y, z) = f(1, 1, 1) \]
Since $f(1,1,1) = 6$, the equation is

$$2x^2 + 3y^2 + z = 6$$

To sketch this surface rewrite it in the form $z = 6 - 2x^2 - 3y^2$ which is clearly a paraboloid centered on the $z$-axis, opening downward, with the heightest point at $z = 6$. 