1. (10 pts) EVALUATE the following iterated integral:

\[
\int_0^1 \int_x^{x^2} \int_0^{\sqrt{y}} 1 \, dz \, dy \, dx
\]

**SOLUTION:**

\[
\int_0^1 \int_x^{x^2} \int_0^{\sqrt{y}} 1 \, dz \, dy \, dx = \int_0^1 \int_x^{x^2} \sqrt{y} \, dy \, dx = \int_0^1 \frac{1}{2} \left( x^3 - x^{3/2} \right) dx = \frac{4}{9} - \frac{1}{10} = -\frac{1}{10}
\]

4 points first integral, 3 points for second and third

2. (10 pts) Consider the function \( f(x, y) = xy^2 - 6x^2 - 3y^2 \).

(a) Find all critical points of \( f \). [Hint: There are exactly three!]

**SOLUTION:**

\[ f_x = y^2 - 12x, \quad f_y = 2xy - 6y. \]

Setting \( f_y = 0 \) yields 0 = 2y(x - 3) so y = 0 or x = 3. Putting y = 0 into \( f_x = 0 \) yields x = 0 so one critical point is \( P_1 = (0, 0) \). Substituting x = 3 into \( f_x = 0 \) yields y^2 = 36 \( \Rightarrow y = \pm 6 \). Hence two more critical points are \( P_2 = (3, -6) \) and \( P_3 = (3, 6) \).

2 points for each critical point

(b) Use the second derivative test to determine if each critical point corresponds to a local extreme value or a saddle point of the function \( f \).

**SOLUTION:**

\[ D(x, y) = f_{xx}f_{yy} - (f_{xy})^2 = 24(3-x) - 4y^2. \]

Substituting \( P_1 = (0, 0) \) yields \( D(0, 0) > 0 \). Then using \( f_{xx} = -12 < 0 \) \( \Rightarrow P_1 = (0, 0) \) corresponds to a local max. For \( P = (3, \pm 6) \) we find \( D(3, \pm 6) = -4 \cdot 36 < 0 \), so both \( P_2 \) and \( P_3 \) correspond to saddle points.

2 points

3. (20 pts) Set up (BUT DO NOT EVALUATE) the explicit double iterated integral in **Cartesian coordinates** needed to find the volume of the solid in the first octant that is bounded on the top by the plane \( x + y + z = 10 \) and that lies above the region in the first quadrant in the xy-plane bounded by \( y = 0, x = 0 \), and the \( y = 2 - x^2 \).

**SOLUTION:**

There are two possible solutions to this problem:

\[
\text{VOLUME} = \int_0^{\sqrt{2}} \int_0^{2-x^2} (10 - x - y) \, dy \, dx = \int_0^{\sqrt{2}} \int_0^{2-x^2} (10 - x - y) \, dxdy
\]

limits 4 points each, integrand worth 4 points

4. (20 pts) Rewrite the iterated integral

\[
\int_0^1 \int_0^{\sqrt{1-x^2}} e^{x^2+y^2} \, dy \, dx
\]

as an iterated integral in **polar coordinates** (DO NOT EVALUATE).

**SOLUTION:**

There are two possible solutions to this problem:
\[
\int_0^1 \int_0^{\sqrt{1-x^2}} e^{x^2+y^2} \, dy \, dx = \int_0^{\pi/2} \int_0^1 r e^{r^2} \, dr \, d\theta = \int_0^1 \int_0^{\pi/2} r e^{r^2} \, d\theta \, dr
\]

limits 4 points each, integrand worth 4 points

5. (20 pts) Set up (BUT DO NOT EVALUATE) the explicit triple iterated integral in Cartesian coordinates needed to evaluate the triple integral
\[
\iiint_E \sqrt{x^2 + y^2} \, dV
\]
where \( E \) is the region bounded by the paraboloid \( z = x^2 + y^2 \) and the plane \( z = 4 \). Set up the iterated integral so that the order of integration is in the order \( dz \, dy \, dx \).

SOLUTION:
\[
\text{VOLUME} = \int_{-2}^2 \int_{\sqrt{4-x^2}}^{\sqrt{9-x^2}} \int_{x^2+y^2}^{\sqrt{x^2+y^2}} dV
\]
limits 4 points each for "inside" integral, other limits 3 points each

6. (10 pts) Let \( E \) be the region in first octant that lies between the two spheres \( x^2 + y^2 + z^2 = 4 \) and \( x^2 + y^2 + z^2 = 9 \).

(a) Use set notation to describe the region \( E \) in cylindrical coordinates.

SOLUTION: This region cannot be described as a single set in cylindrical coordinates. Rather one needs to divide the region into at least two sets.
\[
E = E_I \cup E_{II}
\]
where
\[
E_I = \{(r, \theta, z) \mid 0 \leq \theta \leq \frac{\pi}{2}, 0 \leq r \leq 2, \sqrt{4-r^2} \leq z \leq \sqrt{9-r^2}\}
\]
and
\[
E_{II} = \{(r, \theta, z) \mid 0 \leq \theta \leq \frac{\pi}{2}, 2 \leq r \leq 3, 0 \leq z \leq \sqrt{9-r^2}\}
\]
limits 4 points each for \( E_I \), other limits 3 points each

(b) Use set notation to describe the region \( E \) in spherical coordinates.

SOLUTION:
\[
E = \{(\rho, \theta, \phi) \mid 0 \leq \theta \leq \frac{\pi}{2}, 0 \leq \phi \leq \frac{\pi}{2}, 2 \leq \rho \leq 3\}
\]
5 points

7. (10 pts) Suppose you are seated at a workstation in a computer lab, with a Maple worksheet in front of you. Write the three Maple command(s) that will plot \( x^2 + y^2 + z^2 = 1 \) in green and \( x + 3y - z = 1 \) in red on the same plot with "boxed" coordinate axes. Use for the domain \(-2 \leq x \leq 2, -2 \leq y \leq 2, 0 \leq z \leq 2\). Assume that "with(plots):" has already been executed in your worksheet.

SOLUTION:
\[
a := \text{implicitplot3d}(x^2 + y^2 + z^2 = 1, x = -2..2, y = -2..2, z = 0..2, \text{color} = \text{green}, \text{axes}=\text{boxed}); \quad \text{4 points}
\]
\[
b := \text{implicitplot3d}(x + 3y - z = 1, x = -2..2, y = -2..2, z = 0..2, \text{color} = \text{red}, \text{axes}=\text{boxed}); \quad \text{4 points}
\]
\[
\text{display}([a, b]); \quad \text{OR} \quad \text{display}\{a, b\}; \quad \text{OR} \quad \text{display}\{a, b, \text{axes}=\text{boxed}\}; \quad \text{2 points}
\]

REMARK: There are other variations of these commands. First note that "axes=boxed" only needs to occur once. Also, the second command above for the surface \( x + 3y - z = 1 \) can be given in the form
\[
\text{plot3d}(x + 3y - 1, x = -2..2, y = -2..2, \text{color} = \text{red}, \text{axes}=\text{boxed});
\]
with or without the z range.