PART I

(i) = X knight, Y knight
(ii) = X knave, Y knave
(iii) = X knave, Y knight
(iv) = X knight, Y knave
noa = none of the above

In each case, circle all the situations from the choices (i)-(iv) that are consistent with the information, or circle only ‘noa’.

(a) X says: Y is a knave.
    Y says: X is a knight.
    (i) (ii) (iii) (iv) noa

(b) X says: X and Y are alike.
    Y says: X and Y are alike.
    (i) (ii) (iii) (iv) noa

(c) X says: Y is a knight.
    Y says: X is a knight.
    (i) (ii) (iii) (iv) noa

(d) X says: One of us lies.
    Y says: Both of us lie.
    (i) (ii) (iii) (iv) noa

PART II

S = Sometimes true, sometimes false
A = Always true
N = Never true

Circle the correct answer.

(a) If the set of statements making up an argument is consistent, then the argument is valid.
    S A N

(b) If the set of statements making up an argument is inconsistent, then the argument is invalid.
    S A N

(c) If \( \frac{A}{B} \) is valid, then \( \frac{\neg B}{\neg A} \) is valid too.
    S A N

(d) If \( \frac{A}{B} \) is valid, then \( \frac{\neg A}{\neg B} \) is valid too.
    S A N

(e) If \{A, B, C\} is consistent, then \{not A, not B, not C\} is inconsistent.
    S A N

PART III

Which of the following are (syntactically unambiguous) statement forms of SL?
Which of the syntactically ambiguous expressions of SL are semantically unambiguous?

(a) \( P \land Q \)
(b) \( P \neg Q \)
(c) \( (P \land Q \lor R) \)
(d) \( (P \land Q) \rightarrow R \)
(e) \( \neg (P \leftrightarrow \neg Q) \lor R \)
(f) \( (P \leftrightarrow ((P \land (Q \lor \neg R)) \lor ((P \land (\neg Q \land R)) \lor (\neg P \land (Q \land R)))) \)
Exercise Set #1 — 2

(g) \( (P \rightarrow (Q \lor R)) \rightarrow S \)  
(h) \( (P \land Q \land (R \lor \neg S)) \)  
(i) \( \neg \neg (\neg P \lor \neg \neg R) \)  
(j) \( P \lor Q \land R \lor S \)

**PART IV**

Each of the following is a semantically unambiguous SL statement form. Circle the symbols that could be the main logical operator in each case (there may be more than one to circle in some cases).

Determine the truth value of each statement form for these truth value assignments:

(TV1) P is True, Q is False, R is True, S is False
(TV2) P is False, Q is True, R is True, S is False

(a) \( \neg \neg \neg P \)  
(b) \( \neg \neg \neg P \rightarrow Q \)  
(c) \( \neg \neg \neg (P \rightarrow Q) \land R \)  
(d) \( \neg \neg \neg (P \rightarrow (Q \lor R)) \)  
(e) \( \neg (\neg (P \lor Q) \rightarrow (\neg (R \land S))) \)  
(f) \( (P \rightarrow (Q \rightarrow (R \rightarrow S))) \)  
(g) \( ((P \rightarrow Q) \rightarrow (R \rightarrow S)) \)  
(h) \( ((P \land Q) \land (Q \rightarrow R) \land (R \leftrightarrow S)) \)

**PART V**

First use truth tables to determine which of these sets of SL statement forms are consistent and which are inconsistent. Then use the tableau method. Make a completed tableau. In the case of a consistent set, list the truth value assignments that make all the statement forms true.

(a) \{ P \leftrightarrow (Q \lor R), \neg (Q \land P) \}  
(b) \{ P \lor Q, P \rightarrow R, Q \rightarrow R, \neg (R \lor S) \}  
(c) \{ \neg (P \rightarrow (P \rightarrow (P \rightarrow Q))) \}  
(d) \{ \neg ((P \rightarrow Q) \leftrightarrow (Q \lor \neg P)) \}  
(e) \{ ((P \lor Q) \land (R \lor S)) \rightarrow T, \neg P \land \neg R, \neg (Q \rightarrow T) \}  

**PART VI**

Symbolize — that is, describe the logical form of the following statements using SL statement forms.

Use this key:

P: The Legislature wants to raise teachers’ salaries.
Q: The NC state income tax rate will go up.
R: The House and the Senate will agree on a highway plan.
S: The Governor and Lt. Governor both oppose a tax increase.

(a) If the Legislature wants to raise teachers’ salaries then the tax rate will go up.
(b) If the Legislature wants to raise teachers’ salaries or the House and Senate agree on a highway plan then the tax rate will go up.
(c) If the Governor and Lt. Governor both oppose a tax increase then the tax rate won’t go up.
(d) If the Governor and Lt. Governor both oppose a tax increase then the tax rate will go up; and if the Legislature wants to raise teachers’ salaries, the House and Senate will agree on a highway plan.

(e) Unless the House and Senate agree, the tax rate will go up if the legislature wants to increase teachers’ salaries.

(f) If neither the Governor and Lt. Governor both oppose a tax increase nor the House and Senate agree on a highway plan then the tax rate will go up.

(g) The tax rate will go up if and only if either the Legislature wants to raise teachers’ salaries and the House and Senate agree on a highway plan or the Governor and Lt. Governor don’t both oppose a tax increase.

(h) The Governor and Lt. Governor both oppose a tax increase only if the House and Senate will agree on a highway plan.

(i) It is not the case that if the House and Senate won’t agree on a highway plan, the tax rate will go up.

(j) If the tax rate will go up if the Legislature wants to increase teachers’ salaries, then the House and Senate won’t agree on a highway plan if the Legislature wants to increase teachers’ salaries.

**PART VII**
Symbolize each argument using SL statement forms. Be sure to display the key in addition to the statement forms. Then use the tableau method to check for validity. In each case in which the tableau method reveals an invalid argument form, determine at least one counterexample truth value assignment. Also determine whether the original argument is really invalid or not.

(a) If the Legislature wants to increase teachers’ salaries then the tax rate will go up. So if the tax rate won’t go up then the Legislature doesn’t want to increase teachers’ salaries.

(b) If the primary cooling system fails then the core will overheat unless the emergency cooling system is activated. The only ways to activate the emergency cooling system are the automatic cut–in and the manual over–ride. The reactor operator will use the manual over-ride only if the monitoring instruments indicate overheating. So if the monitoring instruments do not indicate overheating, then the core will overheat if the primary cooling system fails.

(c) At least one of Alice, Betty, and Carol will be at the store. So if Alice isn’t at the store, either Betty or Carol will be.

(d) Knights always tell the truth, knaves always lie, and normals sometimes lie and sometimes tell the truth. Mordred is either a knight, a knave, or normal. Mordred says that he is not a knight. So Mordred is normal.