Some Trig Basics

- \( \sin \theta = \frac{\text{opp}}{\text{hyp}} \)
- \( \cos \theta = \frac{\text{adj}}{\text{hyp}} \)
- \( \tan \theta = \frac{\text{opp}}{\text{adj}} \) → \( \tan \theta = \frac{\sin \theta}{\cos \theta} \)

\( x^2 + y^2 = 1 \) unit circle

If \( x = \cos \theta \) and \( y = \sin \theta \) we get

\( \cos^2 \theta + \sin^2 \theta = 1 \)

The unit circle is helpful for determining the sign of sine, \( \cos \theta \), and \( \cos \theta \). Google it for a more filled in version.

Notice! \( \sin \frac{\pi}{4} = \sin \frac{3\pi}{4} \) (y values are the same)

but \( \cos \frac{\pi}{4} = -\cos \frac{3\pi}{4} \) (x values are different)

* To convert from degrees to radians multiply by \( \frac{\pi}{180} \)
To convert from radians to degrees multiply by \( \frac{180}{\pi} \)
Use these triangles to find \( \sin \theta \) and \( \cos \theta \) of common angles.

Note: \( \frac{\pi}{4} = 45^\circ \)  
\( \frac{\pi}{3} = 60^\circ \), \( \frac{\pi}{6} = 30^\circ \)

\[
\cos \frac{\pi}{4} = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{2}}{2} \\
\text{or} \quad \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}
\]

\[
\sin \frac{\pi}{4} = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{2}}{2} \\
\tan \frac{\pi}{4} = \frac{\text{opp}}{\text{adj}} = 1
\]

\[
\cos \frac{\pi}{3} = \frac{1}{2} \\
\tan \frac{\pi}{3} = \frac{\sqrt{3}}{1} = \sqrt{3}
\]

\[
\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2} \\
\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \\
\sin \frac{\pi}{6} = \frac{1}{2} \\
\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}
\]

What do we think about radicals in the denominator? Liberating!

* The unit circle is also good for finding \( \sin \) and \( \cos \) of \( 0, \frac{\pi}{2}, \frac{3\pi}{2}, \pi, (90^\circ), (180^\circ), (270^\circ), (360^\circ) \)

Reciprocals: \( \sin \theta = \frac{1}{\csc \theta} \), \( \cos \theta = \frac{1}{\sec \theta} \), \( \tan \theta = \frac{1}{\cot \theta} \)

I think it is easier to remember like this:

\[
\begin{array}{c|c|c|c|c|c}
\hline
\sin & \cos & \tan & \csc & \sec & \cot \\
\hline\end{array}
\]
**Derivatives:**

\[
\frac{d}{dx}(\sin x) = \cos x
\]

\[
\frac{d}{dx}(\cos x) = -\sin x
\]

\[
\frac{d}{dx}(\tan x) = \sec^2 x
\]

\[
\frac{d}{dx}(\cot x) = -\csc^2 x
\]

\[
\frac{d}{dx}(\sec x) = \sec x \tan x
\]

\[
\frac{d}{dx}(\csc x) = -\csc x \cot x
\]

**Graphs:**

For every value of \(x\),

\[-1 \leq \sin x \leq 1\]

\[-1 \leq \cos x \leq 1\] for all \(x\)

We can see \(\sin 0 = 0\)

\(\sin \pi = 0\), \(\sin \frac{\pi}{2} = 1\), etc

\(\frac{1}{2}\) Angle Identities:

\[
\cos^2 x = \frac{1}{2}(1 + \cos(2x))
\]

\[
\sin^2 x = \frac{1}{2}(1 - \cos(2x))
\]