Test 1 Review

MA 231-005
Spring 2015

Test 1 will cover sections 7.1-7.6 in *Calculus & Its Applications*, Goldstein, Lay and Schneider, 13th Edition.

**Topics**

- Constructing a function of multiple variables.
- Level curves.
- Partial derivatives.
  - First partial derivatives.
  - Second partial derivatives.
  - Interpretation of partial derivatives.
  - First derivative test of a function of two variables.
  - Second derivative test of a function of two variables.
- Lagrange Multiplier method of optimization.
- Least-Squares.
  - Minimizing the least-square error using partial derivatives.
  - Finding regression line for large data sets.
- Double Integrals.

**Practice Problems**

1. (7.1.8) Find a formula $C(x, y)$ that gives the cost of materials for a rectangular enclosure with width $x$, length $y$, and height $z$ (in feet), with a back, two sides and a top, if material for the top costs $3 per square foot and material for the back and two sides costs $5 per square foot.

2. (7.2.17) Let $f(x, y, z) = xze^{yz}$. Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, and $\frac{\partial f}{\partial z}$.

3. (7.2.24) Let $f(x, y) = xe^{y} + x^{4}y + y^{3}$. Find $\frac{\partial^{2} f}{\partial x^{2}}$, $\frac{\partial^{2} f}{\partial y^{2}}$, and $\frac{\partial^{2} f}{\partial x \partial y}$. 
4. (7.2.30) The demand for a certain gas-guzzling car is given by \( f(p_1, p_2) \), where \( p_1 \) is the price of the car and \( p_2 \) is the price of gasoline. Explain why \( \frac{\partial f}{\partial p_1} < 0 \) and \( \frac{\partial f}{\partial p_2} < 0 \).

5. (7.3.5) Find all points at which \( f(x, y) = x^3 + y^2 - 3x + 6y \) has a possible relative minimum or maximum.

6. (7.3.23) Let \( f(x, y) = x^3 - y^2 - 3x + 4y \).
   
   (a) Find all points at which \( f(x, y) \) has a possible relative minimum or maximum.
   
   (b) For each point found in part (a) use the second derivative test to characterize it. If the second-derivative test is inconclusive, say so.

7. (7.4.11) Use the method of Lagrange Multipliers to find the dimensions of the rectangle of maximum area that can be inscribed in the unit circle. (Hint: the equation for the unit circle is \( x^2 + y^2 = 1 \)).

8. (7.5.5) Use partial derivatives to find the best least-squares fit to the points (1,2), (2,5), (3,11).

9. (7.5.15) An ecologist wishes to know whether certain species of aquatic insects have their ecological range limited by temperature. He collected the data in the table, relating the average daily temperature at different portions of a creek with the elevation of that portion of the creek.

<table>
<thead>
<tr>
<th>Elevation (km)</th>
<th>Average Daily Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.7</td>
<td>11.2</td>
</tr>
<tr>
<td>2.8</td>
<td>10</td>
</tr>
<tr>
<td>3.0</td>
<td>8.6</td>
</tr>
<tr>
<td>3.5</td>
<td>7.5</td>
</tr>
</tbody>
</table>

   (a) Find the straight line that provides the best least-squares fit to the data.
   
   (b) Use the function found in part (a) to estimate the average daily temperature for this creek at altitude 3.2km.

10. Evaluate
    
    \[ \int_{-2}^{0} \int_{-1}^{1} xe^y dy dx. \]

11. (7.6.14) Evaluate
    
    \[ \int_{0}^{1} \int_{0}^{\sqrt{x}} x^2 + y^2 dy dx. \]