INSTRUCTIONS

Please complete this assignment with your assigned group members. The assignment is due in class on Tuesday, October 18, 2005.

1. Exercise 4.8, Page 219, Problem 5. Use MAPLE to plot $y(t)$ for the three cases mentioned in the problem.

2. Exercise 4.9, Page 227, Problem 5. Use MAPLE to plot the solution $y(t)$ in part (c) of the problem.


6. We will use the phenomenon of resonance to pick a radio station that is broadcasting at frequency $\omega$. Consider a series RLC circuit (as shown in Figure 5.31 in Section 5.6 on page 286 of the book), with a resistor of resistance $R$ ohm, an inductor of inductance $L$ henry, a capacitor of capacitance $C$ farad, and an alternating voltage source $E(t) = E_0 \sin \omega t$. The differential equation describing the current $I(t)$ in the circuit is

$$L \frac{d^2 I}{dt^2} + R \frac{dI}{dt} + \frac{I}{C} = \omega E_0 \cos \omega t. \quad (1)$$

Note that the rhs of (1) is $\frac{dE}{dt}$. We know that the solution $I(t) = I_h(t) + I_p(t)$, where $I_h(t)$ is the solution to the homogeneous part of (1) and is a transient current that approaches zero as $t \to \infty$, and a steady state periodic current $I_p(t)$ which is also a particular solution to (1). Please complete parts (a) and (b) of this question. Part (c) only illustrates how these calculations help in picking the right radio station.

(a) Show that

$$I_p(t) = \frac{E_0 \sin(\omega t + \alpha)}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}, \quad (2)$$
with the phase angle

$$\alpha = \arctan \frac{1 - LC\omega^2}{\omega RC}.$$ 

**Hint:** Repeat the calculations in equations (3)-(7) (for the spring-mass system on pages 220-221 of the book) for the RLC circuit.

(b) The amplitude of the steady state periodic current is

$$I_0 = \frac{E_0}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}.$$ 

Let us assume that circuit component values are $E_0 = 1$ V, $R = 0.1$ ohm, $L = 1$ henry, and $C = 1$ farad. Use MAPLE to plot $I_0$ versus $\omega$. Use this plot to confirm that $I_0$ attains its maximum when $\omega_m = \frac{1}{\sqrt{LC}}$.

(c) **Tuning your radio circuit:** Suppose we wanted to pick up a radio station that is broadcasting at a frequency $\omega$, and thereby providing an input voltage $E(t) = E_0 \sin \omega t$ to the tuning circuit of the radio. The resulting steady periodic current $I_p$ in the tuning circuit drives its amplifier, and in turn its loudspeaker, with the volume of sound we hear roughly proportional to the amplitude $I_0$ of $I_p(t)$. To hear our preferred station (of frequency $\omega$) the loudest - and simultaneously tune out stations broadcasting at other frequencies, we choose the capacitance $C$ (the only thing we can vary in the circuit) to maximize $I_0$. So, when we turn the dial and set $C = \frac{1}{L\omega^2}$ we hear only our preferred radio station. Did I say I enjoy jazz music : )