1. We will consider a model that describes the population of the passenger pigeon which, unfortunately, is now extinct. Consider the differential equation
\[
\frac{dp}{dt} = p(p - 1)(2 - p)
\]
for the population \( p \) (in thousands) of the passenger pigeon at time \( t \).

(a) Sketch the direction field using the \textit{dfield} command in MAPLE. Remember to start your MAPLE session with the commands \textit{with(DEtools)} and \textit{with(plots)}.

(b) If the initial population of the pigeon is 3000, i.e. \( p(0) = 3 \), what can you say about the limiting population \( \lim_{t \to \infty} p(t) \)?

(c) If \( p(0) = 1.5 \), what is \( \lim_{t \to \infty} p(t) \)?

(d) If \( p(0) = 0.5 \), what is \( \lim_{t \to \infty} p(t) \)?

(e) Can a population of 900 ever increase to 1100?

(f) Using the separation of variables, solve the differential equation with the initial condition \( p(0) = 3 \). \textbf{Hint:} You will need partial fraction expansions to evaluate the integral. Plot the solution \( p(t) \) in MAPLE.

(g) You will be using MATLAB 7 for this portion of the assignment: Please review Kartik’s sample session with \textit{euler.m} in MATLAB on the course webpage. Also, download Kartik’s codes \textit{euler.m} and \textit{myfunc.m} from this website. Use the code \textit{euler.m} to find approximations to the solution of the differential equation at \( t = 5 \), with the initial condition \( p(0) = 3 \), taking 1, 2, 4, 8, and 16 steps. Also, plot \( p \) versus \( t \) using the \textit{plot} command in MATLAB. Type \textit{help plot} in MATLAB for help with the \textit{plot} command.

(h) Finally, solve the differential equation with the initial condition \( p(0) = 3 \) using MATLAB’s ODE function \textit{ode45}. Type \textit{help ode45} in MATLAB for help with \textit{ode45}. Plot the solution in MATLAB. Compare the solution with the various solutions obtained with Euler’s method for different steps?. What do you observe?
(i) Based on the various observations you have tabulated so far, can you offer any suggestions on why the passenger pigeon probably became extinct? Although a reasonably large number of birds remained alive in the late 1880s, the last survivor among carrier pigeons died in 1914.

2. Exercise 2.2, Page 46, Problem 27. Do parts (a)-(c).


5. Exercise 2.3, Pages 57-58, Problem 36.