Your Name: __________________________
For purpose of anonymous grading, please do not write your name on the subsequent pages.

This examination consists of 3 problems, which are subdivided into 11 questions, where each question counts for the explicitly given number of points, adding to a total of 45 points. Please write your answers in the spaces indicated, or below the questions (using the back of the sheets if necessary). You are allowed to consult one 8.5in \times 11in sheets with notes, but not your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later. You will have 75 minutes to do this test.

Good luck!

Problem 1 __________

2 __________

3 __________

Total __________

If you are taking the exam later, please sign the following statement:

I, __________________________, affirm that I have no knowledge of the contents of this exam.

________________________
Signature
Problem 1 (20 points, 4 points each part)

(a) Please restate Fibonacci’s famous rabbit problem.

(b) A square matrix is said to be symmetric if $A^T = A$. For an arbitrary matrix $B \in \mathbb{R}^{m \times n}$, is the product $B^T \cdot B$ then always a symmetric matrix? True or false? Please explain.

(c) In Maple, how does one compute the inverse of a matrix? How can one check the result to be the inverse matrix? Please give Maple commands.
(d) An algebraic group \((S; \text{mul}, \text{inv}, \text{id})\), where \(S\) is a set of elements, \(\text{mul}\) is the binary operator, \(\text{id}\) the identity element, and \(\text{inv}\) the inverse operator, is called Abelian if the binary operator satisfies the law of commutativity. Is the set of invertible square matrices of dimension \(n\) with real number entries and matrix multiplication as its binary operator an Abelian group? Please explain.

(e) When performing Gaussian elimination on a coefficient matrix \(A\), it is sometimes useful not only to obtain the row echelon form \(U\) but also the transforming matrix \(T\) such that \(T \cdot A = U\). Please give a scenario where \(T\) becomes useful.
Problem 2 (15 points, 5 points each part): Consider the following system of linear equations in the unknowns $x, y, z$ and with parametric coefficients in $a, b, c$:

$$
-2x + 3y - z = -5, \\
4x - 6y + 3z = a, \\
2x - 3y + bz = c.
$$

(a) Please convert the augmented coefficient matrix of the above system to row echelon form.

\[
\begin{bmatrix}
-2 & 3 & -1 & -5 \\
4 & -6 & 3 & a \\
2 & -3 & b & c
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
-2 & 3 & -1 & -5 \\
0 & 0 & 1 & a - 10 \\
0 & 0 & b - 1 & c - 5
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
-2 & 3 & -1 & -5 \\
0 & 0 & 1 & a - 10 \\
0 & 0 & 0 & c - 5 - (b - 1)(a - 10)
\end{bmatrix}.
\]

(b) Find conditions for $a, b, c$ (equations and/or inequalities) for which the above linear system is consistent.

(c) Assuming that $a, b, c$ satisfy the conditions derived in (b), please give the complete solution of the above linear system.
Problem 3 (10 points, 5 points each part): Consider the following matrix product:

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\cdot
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & -2 & 1
\end{bmatrix}
\cdot
\begin{bmatrix}
1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\cdot
\begin{bmatrix}
0 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 0
\end{bmatrix}
\cdot
\begin{bmatrix}
0 & 2 & -5 \\
0 & 1 & 3 \\
1 & 0 & 0
\end{bmatrix}
\]

Here the matrix \( A \) is multiplied from the left by four elementary matrices.

(a) Please describe in words the elementary row operation that each of \( E_1, E_2, E_3, E_4 \) represents.

(b) Please perform the four matrix products/row operations on \( A \).