Your Name: ________________________
For purpose of anonymous grading, please do not write your name on the subsequent pages.

This examination consists of 3 problems, which are subdivided into 11 questions, where each question counts for the explicitly given number of points, adding to a total of 47 points. Please write your answers in the spaces indicated, or below the questions (using the back of the sheets if necessary). You are allowed to consult one 8.5in × 11in sheet with notes, but not your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have 75 minutes to do this test.

Problem 1   _____

            2   _____

            3   _____

Total   _____

If you are taking the exam later, please sign the following statement:

I, ____________________ affirm that I have no knowledge of the contents of this exam.

_________________________________________ Signature
Problem 1 (20 points)

(a, 3 pts) Please name the countries of birth for the mathematicians C. F. Gauss, M.-E. C. Jordan, and Fibonacci.

Germany, France, Italy

(b, 5 pts) This is a variant of Fibonacci’s famous rabbit problem: Suppose you start out with one newly born pair of rabbits, but each pair needs 2 months to reach maturity but thereafter gives birth to 2 pairs (2 male and 2 female) after every month. How many pair of rabbits are there after 10 months?

\[ f_{n+3} = f_{n+2} + 2f_n. \]

<table>
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<th>2</th>
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<td>1</td>
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<td>13</td>
<td>23</td>
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</tbody>
</table>

(c, 4 pts) True or false: for any matrices \( A, B, C \in \mathbb{R}^{n \times n} \) we have \( (A \cdot B \cdot C)^T = C^T \cdot B^T \cdot A^T \). Please explain.

True.

\( (A \cdot B)^T = B^T \cdot A^T \) hence

\[ (A \cdot B \cdot C)^T = ((A \cdot B) \cdot C)^T = C^T \cdot (A \cdot B)^T = C^T \cdot (B^T \cdot A^T) = C^T \cdot B^T \cdot A^T. \]

(d, 4 pts) In Maple, how does one compute the reduced row echelon form of a matrix? Please give Maple commands.

```
with(linalg); rref(A);
```

or

```
linalg[rref](A);
```

(e, 4 pts) Please define the notion that a binary operation on a set is commutative. For the set of \( n \times n \) matrices over the reals, is matrix addition commutative?

Let \( \circ \) be the operation. \( \forall a, b \in S: a \circ b = b \circ a. \)

Yes, \( + \) on \( \mathbb{R}^{n \times n} \) is commutative.
Problem 2 (15 points): Consider the following augmented matrix of a system of linear equations, where the first 3 columns correspond to the variables $x, y, z$ and where $a$ and $b$ are real parameters.

$$
\begin{bmatrix}
2 & 0 & 1 & : & 1 \\
0 & a & a & : & a \\
0 & b & b & : & 0
\end{bmatrix}
$$

(a, 5pts) For which values of $a$ and $b$ is the matrix (1) in row echelon form? Please give all the conditions.

$b = 0$.

(b, 5pts) Please perform the back-substitution for those values given in part (a).

Case $a = 0$: $2x + z = 1 \implies x = 1/2 - 1/2z$ $z = z$ $y = y$.

Case $a \neq 0$: $ay + az = a \implies y = 1 - z$ $z = z$ $x = 1/2 - 1/2z$.

(c, 5pts) Please solve the corresponding system for the values of $a$ and $b$ for which matrix (1) is not in row echelon form. Please also indicate for which values there is no solution.

$b \neq 0$:

Case $a = 0$: $z = z$ $by + bz = 0 \implies y = -z$ $x = 1/2 - 1/2z$.

Case $a \neq 0$: $z = z$. No solution.
**Problem 3** (12 points): Suppose you have a matrix $A \in \mathbb{R}^{3 \times 4}$.

(a, 4 pts) Please write (explicitly in form of a matrix) an elementary matrix $E$ that effects the subtraction of the double of each entry in row 1 from the corresponding entry in row 4 by performing the product $E \cdot A$.

$$
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
-2 & 0 & 0 & 1
\end{bmatrix}
$$

(b, 4 pts) What elementary operation is performed by the product $A \cdot E$, where $E$ is as in part (a)?

Subtract 2 times column 4 from column 1.

(c, 4 pts) Please write $E^{-1}$, where $E$ is as in part (a), in form of an elementary matrix, that is in the form $E_{III}(\ldots)$.

$E_{III}(4, 1, 4, 2)$