Your Name: SOLUTION
For purpose of anonymous grading, please do not write your name on the subsequent pages.

This examination consists of 4 problems, which are subdivided into 10 questions, where each question counts for the explicitly given number of points, adding to a total of 46 points. Please write your answers in the spaces indicated, or below the questions (using the back of the sheets if necessary). You are allowed to consult two 8.5in × 11in sheets with notes, but not your book or your class notes. If you get stuck on a problem, it may be advisable to go to another problem and come back to that one later.

You will have 75 minutes to do this test.

Problem 1 ______

2 ______

3 ______

4 ______

Total ______
**Problem 1** (14 points) Consider the following mathematical formula:

\[
\frac{a}{(b - c \times d + e) \times f}
\]  

(a, 5pts) Please draw an expression tree for (1) that complies with the usual operator precedence rules and left-to-right tie-breaking for operators of equal precedence.

(b, 5pts) Please draw the parse tree for (1) using the context-free grammar given in class.

(c, 4pts) Please give a postfix string of operators and variables, but with no parentheses, that represents the tree given under part (a).

\[abcd * - e + / f *\]
Problem 2 (10 points): Consider the following graph:

(a, 5pts) Please draw the depth-first search tree for the above graph, processing the neighboring vertices of each vertex in numerical order, starting at vertex 1.

(b, 5pts) Using the tree in part (a), find a one-way street assignment for the above graph, i.e., orient the edges so that the resulting digraph is strongly connected.
Problem 3 (12 points):
Consider the following graph (here the vertices are pairs of integers):

(a, 5pts) Is the above “butterfly” graph planar? Please explain.

Yes.

(b, 2pts) What is the chromatic number of the above “butterfly” graph? Please justify your answer.

\[ \chi = 2: \ (0, i) \rightarrow R, \ (1, i) \rightarrow G, \ (2, i) \rightarrow R. \]
Since there are edges, 2 is the minimum.

(c, 5pts) True or false: if a graph has \( n \) vertices and chromatic number \( \chi = n \) it must be the complete graph \( K_n \). Please justify your answer.

TRUE: Suppose \( G \) is not complete. Assign a different color to each vertex. Now pick a vertex with fewer than \( n - 1 \) neighbors. Such a vertex must exist because \( G \) is not complete. Replace the color by the one that a vertex has that is not a neighbor. Therefore \( \chi(G) \leq n - 1 \).
Problem 4 (10 points):
Consider the following fractal polygonal line.

Here you start out at the origin of the plane and move in the x-axis direction by $+1$ unit. Then you move in the y-axis direction by $+\frac{3}{4}$ units, then in the x-axis direction by $-(\frac{3}{4}) \cdot (\frac{3}{4})$ units, then in the y-axis direction by $-(\frac{3}{4})^3$ units, then in the x-axis direction by $+(\frac{3}{4})^4$ units, and so on.

(a, 5pts) Please determine the x- and y-coordinates of the point on the plane to which this polygonal line is converging.

\[
\begin{align*}
x &= 1 - \frac{9}{16} + \frac{81}{256} - \cdots + \left(-\frac{9}{16}\right)^i + \cdots \\
&= 1/(1 + 9/16) = 16/25 = 0.64. \\
y &= \frac{3}{4} - \frac{27}{64} + \frac{243}{1024} - \cdots + \frac{3}{4} \left(-\frac{9}{16}\right)^i + \cdots \\
&= 3/4 \cdot 16/25 = 48/100 = 0.48.
\end{align*}
\]

(b, 5pts) Please give a Lindenmeyer system that would draw the above polygonal line. In addition to the productions, please also give the semantics, i.e., the interpretation, of each variable.

\[
\begin{align*}
X_+: \text{ draw a line from current point of length } L \text{ in positive } x\text{-coordinate direction. Then set } \\
L = 3/4 \cdot L. \\
X_-: \text{ Same but in negative direction. } Y_+, Y_-: \text{ Same as } X_+, X_- \text{, but in } y\text{-coordinate direction.}
\end{align*}
\]

The Lindenmeyer system is with start variable $A$:

Variables $\ A \ B \ C \ D \ X_+ \ Y_+ \ X_- \ Y_-$ 
Productions $\ X_+ B \ Y_+ C \ X_- D \ Y_- A \ X_+ Y_+ X_- Y_-$