Alternative Monies and the Demand for Media of Exchange

PREPAID CARDS, CASH CARDS, electronic purse, smart cards—these are but a few of the elements in the revolution now taking place in monetary systems around the world. At that movement's heart is the emergence of a new value transfer system where alternative monies are offered to consumers through the miracle of electronics. In some cases, the new monies are merely a repackaging of existing media of exchange. For example, stored value cards that have been traditionally available for single vendors now are being offered for multiple merchant use. In other cases, technology and its acceptability are being cross-sold, as with debit cards and the expanded use of ATM cards as point-of-sale vehicles. However, the banking industry is perhaps most enthusiastic about the emerging technology of smart cards—chip-in-cards—that have been making inroads in Europe. With the expanded capability of a memory chip, alternative monies are seen to have the capability of moving to a new plateau of acceptance.

There are many stories in the business press about the general enthusiasm for these new forms of money. For example, Block (1995) and Cutler (1994) herald the new day of electronic money. They forecast the demise of both the greenback and of commercial banks that exploit the float derived from its use. According to the hyperbole from marketing reps from this side of the financial community, currency and demand deposits soon will be endangered species. Consumer acceptance is alleged to be high, and cost efficiency associated with the new technology is expected to be substantial.

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Yet, as Wenninger and Laster (1995) suggest, "to succeed, an electronic purse system will need to offer enough features of value to its three constituencies—consumers, merchants, and issuers—to induce them to bear the cost." To date much has been said about the advantages of alternative monies to the institutions issuing them. In fact, most arguments in favor of these media of exchange can be viewed as a reaction to the rising costs associated with current check clearing and on-line credit card systems. Although hard data are scarce, the cost advantage to the banking system of a movement from paper checks to debit cards is alleged to be in the range of 50 percent.¹

Merchants are perceived to be winners, too. Off-line prepaid cards are expected to reduce the cost of clearing of transactions, handling cash, and reducing fraud. Therefore, the advent of electronic money should be a panacea welcomed by all merchants, large and small. For major retailers, cash management and on-line authorization systems become obsolete issues. For small businesses, inexpensive off-line technologies allow even the local grocery store to accept prepaid cards with integrity assurance and minimal expense.

To read the press, the consumer is equally enthusiastic. A 1992 study by Synergistic Research Corporation found that 30 percent of all consumers surveyed expressed interest in prepaid cards, with 75 percent of this group willing to pay a reasonable level of transaction fee for their use. Smart Card Enterprises found even more support in their 1993 survey of Delaware consumers, with 88 percent expressing willingness to use some form of the new technology.² And this is only the evidence from the United States, where consumer experience is limited and provider experimentation just beginning. NatWest is receiving very positive responses in the United Kingdom from its Mondex product³ and Humphrey, Pulley, and Vesala (1996) report substantial European usage and acceptance of a wide range of noncash payment instruments.

However, the economic rationale for this consumer enthusiasm is a bit unclear. Each application of the emerging electronic technology has been different in important ways. The universal receptivity to alternative monies may be more a good marketing ploy than good economics. These technologies provide alternative transaction vehicles with an array of characteristics that must be evaluated by potential users as to their desirability. These features include transfer fees, acceptability, differential float, different implied yields on balances held in each form, and a wide range of other characteristics. To understand consumer reaction requires a careful analysis of the desirability of any one of these alternative technologies in terms of its impact on consumer cash management costs through changes in transaction patterns, average money holdings, and total transaction costs.

In the current paper we offer a first attempt at such an analysis. We investigate the effect of variations in the number and type of monies on consumer transactions de-

¹. See Saul (1994) for example.
². Again, see Saul (1994) and the studies referenced therein.
mand using a Baumol-Tobin type model of money demand. Our analysis draws on a long literature investigating the transactions demand for money, including, but not limited to, Baumol (1956), Tobin (1956), Santomero (1979), and Romer (1987). 4 We investigate the behavior of a representative agent faced with a choice of monies with which to transact, and ask how variations in the characteristics of these monies will affect the consumer's choice of transaction vehicle, transactions frequency, and average balance in various media.

Interestingly, the results are not transparent. Variations in the cost of transfer, interest rates, and the acceptability of alternative media have surprising effects on consumer choice. In general, the cost of using a medium of exchange determines whether it will be used and for which goods it will be traded. The choice of medium of exchange, then, has a direct effect on both the average holding of different types of money and their transaction frequencies. It therefore suggests that efforts by banks to alter the costs of using a particular form of money may cause representative agents to change their exchange behavior, shifting it sometimes rather dramatically. As a general characterization, the model suggests that consumer monetary choice is anything but simple, and variations in the characteristics of various media of exchange will not necessarily have straightforward effects.

1. THE STATUS OF ALTERNATIVE MONIES

As noted at the outset, there are a number of types of monetary innovations currently underway in the United States. Here, we will merely touch the surface. Interested readers may wish to refer to the references we cite here for a further discussion of current implementation. With all the experimentation, however, it is important to recognize that America is somewhat behind in introducing these types of transaction vehicles. As Humphrey, Pulley, and Vesala (1996) indicate in a companion paper at this conference, and Kokkola and Pauli (1994) report, considerably more innovation has occurred in Europe. Ignoring standard credit card transactions, the Europeans are way ahead of their American brethren in the implementation of noncash, nonpaper-based money utilization. Within Europe, the Scandinavian countries and Northern Europeans clearly have the lead. It is worth noting, however, that Asia is not far behind. Both Japan and Singapore have made substantial inroads in the use of alternative monies in commerce.

Within the United States, the battle of electronic monies at the consumer level centers around debit cards and various forms of prepaid plastic and paper vehicles. We have seen more than a decade pass since the banking industry proclaimed the year of the debit card. Yet, despite its slow progress there are continued high expectations and enthusiasm for an increased use in this nonpaper access to a demand deposit account; see Gieseon (1994) or Casey and Sellon (1994). Prepaid cards, long a fixture in Europe, are slowly emerging in the United States as well. Their use

4. Models of this type have recently been reviewed by Milborne (1992).
began with transit cards and has expanded into pay telephone applications recently. The approach taken to increase the use of these prepaid cards appears to be the broadening of the acceptability of existing cards so that they can be made usable at a broader range of retail locations. For example, New York's Metropolitan Transit Authority has broadened the acceptability of its prepaid cards, and universities around the country are following a similar route (Cutler 1994). At the same time, more and more single purpose card applications surface. The most recent one noted in the American Banker is the Blockbuster Video announcement of a stored value card for member rentals this fall (Block 1995).

The frontier application, however, appears to be smart-card technology. Here, a chip-in-card technology substantially expands the memory capacity of prepaid vehicles. We are presented with the capability of carrying a fully electronic identity card that includes both personal information and stored money. This prospect conjures up images of George Orwell's 1984 for some people, but it clearly also represents a unique commercial opportunity. Merchants can access these funds through off-line, relatively inexpensive machines that would transfer ownership to the prepaid balances from the customer to the merchant, for subsequent clearing and collection (Wenninger and Laster 1995). Pilot studies using this technology abound, with the most recent example being a November 15, 1995, announcement of a First Union smart-card pilot in Atlanta (Piskora 1995). To be sure, this technology is part of the emerging payment systems trend and is clearly in the sights of the banking community (Furash and Company 1994).

2. CONSUMER REACTION TO ALTERNATIVE MONIES

Consumer acceptance of any new technology, however, will be key to its success. Yet, little work has been done to examine consumer choice of monetary vehicles and their reaction to changes in the costs associated with value transfer. We now turn to these considerations, looking at a representative agent model of the demand for money for transaction purposes. As all of the proposed monies are offered as a medium of exchange, the demand response to their availability should be determined by the same factors that determine the demand for other forms of money.

Generically, standard models of the demand for the medium of exchange view the driving factors as the volume of payments, transactions costs, yield spreads, uncertainty, illiquidity costs, and so on. These will be considered below. We confine our attention to a world of complete certainty, using the type of model recently reviewed by Milborne (1992). Here, however, we generalize the standard model to include several media of exchange and several goods purchased.

2A. The Theoretical Model

We adopt the usual Baumol-Tobin assumptions, except that (i) we allow consumers to hold inventories of goods, as in Santomero (1974), and (ii) we allow several goods and several media of exchange. We thus ignore the generalizations of the
Baumol-Tobin framework that have been offered in several contributions to the literature over the last ten or fifteen years (for example, Jovanovic 1982; Prescott 1987; and Romer 1986). We do so in the interest of practical tractability. The insights in the recent literature are of second-order importance for the issues we study, almost certainly of negligible empirical magnitude, while including those insights introduces tremendous mathematical complexity that would encumber our analysis severely. In contrast, the analysis within the simpler Baumol-Tobin framework is clean and straightforward. We discuss this issue in more detail below.

**Model Setup.** The household receives a fixed income $Y$ every fixed payments period, and exactly exhausts that income by buying a fixed amount, $X_{g^*}$ of $G$ different goods, such that

$$Y = \sum_{g=1}^{G} X_g.$$  
(1)

Consumption of goods occurs at a constant rate that just exhausts the goods purchased each period. In contrast, consumption *expenditures* (that is, purchases of goods) occur at discrete intervals, to be chosen optimally by the household. We use the standard metaphorical language and describe each of these discrete purchasing events as a "shopping trip." In fact, a purchasing event may be a literal shopping trip, but in general a "trip" may consist of nothing more than a phone call to place an order.

Between shopping trips, the household holds inventories of the various goods, which it gradually consumes until exactly exhausting them at the moment it is time to make another shopping trip. A separate shopping trip is required for each type of good. Each type of commodity inventory pays a unique rate or return, $r_{Xg^*}$, which may be negative, such as a spoilage rate.

There are $L$ media of exchange, $M_i$, available to the household. The household can use any or all of them to buy each type of good. Denote the quantity of good $g$ bought with money $i$ by $X_{gi}$. The household's choice need not be exclusionary, in that it may use one medium, $M_j$, on some shopping trips for good $g$ and another medium, $M_k$, on others. Thus,

$$X_g = \sum_i X_{gi}.$$  
(2)

There are $Z_{gi}$ trips per payments period to purchase good $g$ with money $i$. Each such trip has associated with it the shopping cost $\beta_{gi}$, a lump-sum cost associated with each trip but not depending on the amount spent. This cost may be explicit, such as a delivery charge or a check-cashing fee, or implicit, such as a time cost.

The household spends only a fraction of its income during any one shopping trip. Unspent income is held in a single savings asset, $S$, and in money balances. Savings earn the rate of return $r_s$, and the various kinds of money earn rates of return $r_{M_i}$. We
suppose that \( r_s > r_{Mi} > r_{Xg} \). The household periodically converts some of \( S \) into money by making a "trip to the bank." There are \( T_i \) conversion trips to obtain \( M_i \), and each such trip has associated with it the conversion cost \( \alpha_i \), which is the cost of transferring funds into the medium of exchange. This cost, like shopping trip costs, is modeled as a lump-sum amount paid explicitly or implicitly each time a conversion is made but not depending on the size of the conversion. As with shopping trips to purchase commodities, "trips to the bank" may not be literal trips, and the transactions costs, \( \alpha_i \), may be either explicit or implicit. As the financial and commercial worlds develop new ways to conduct business, trips are more likely to be metaphorical than literal, and trip costs may be dominated by time costs rather than explicit fees.

As in the simple Baumol-Tobin model, optimal conversions are evenly spaced. Shopping trips occur between conversion trips and also are evenly spaced.\(^6\) There are \( N_{gi} \) shopping trips to buy good \( g \) with money \( i \) per conversion of \( S \) into \( M_i \). The total number of shopping trips per income payments period, \( Z_{gi} \), to buy good \( g \) with money \( i \) is thus \( T_i N_{gi} \). Finally, each of the assets, \( S \) and \( M_i \), carries a fixed cost \( F_i \) that must be paid if that asset is held at any time during the payments period. These fixed costs capture such things as monthly account fees.

The household seeks to maximize the profit from managing its assets over a given payments period. Because all conversion and shopping trips are evenly spaced and consumption proceeds at a constant rate, the profit function of the representative agent can be written in terms of the average values of the respective assets:

\[
\pi = r_s \bar{S} + \sum_{i=1}^{L} r_{Mi} \bar{M}_i + \sum_{g=1}^{G} r_{Xg} \bar{X}_g - \sum_{i=1}^{L} T_i \bar{\alpha}_i - \sum_{i=1}^{L} F_i(I(M_i))
\]

where \( I(X) \) is an indicator function that is 1 if average holdings of asset \( X \) are positive and is 0 otherwise. From the above, one can derive the needed expressions for the average asset holdings. Average total assets can be written as

\[
\bar{A} = \bar{S} + \sum_{i=1}^{L} \bar{M}_i + \sum_{g=1}^{G} \bar{X}_g
\]

\[
= \bar{S} + \sum_{i=1}^{L} \left( \bar{M}_i + \sum_{g=1}^{G} \bar{X}_{gi} \right). \tag{4}
\]

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5. In fact, we do not need to require that all money interest rates exceed all inventory rates of return, but it simplifies the discussion to impose that requirement. In the more general case, it is trivial to show that if the rate of return \( r_{Xg} \) on good \( g \) exceeds the rate of return \( r_{Mi} \) on money \( i \), then money \( i \) will not be used to purchase good \( g \).

6. The proof that optimal trips are evenly spaced is tedious and not given here. See Tobin (1956).
Given that trips are evenly spaced and the rate of consumption is constant, we also have

\[ \bar{A} = \sum_g \frac{X_g}{2} \quad (4') \]

\[ \bar{M}_i + \sum_g \bar{X}_{gi} = \sum_g \frac{X_{gi}}{2T_i} \quad (5) \]

\[ \bar{X}_{gi} = \frac{X_{gi}}{2Z_{gi}} \quad (6) \]

We then have

\[ \bar{M}_i = \left( \bar{M}_i + \sum_g \bar{X}_{gi} \right) - \sum_g \bar{X}_{gi} \]

\[ = \frac{\sum_g \bar{X}_{gi}}{2T_i} - \frac{\sum_g \bar{X}_{gi}}{2Z_{gi}} \quad (5') \]

\[ \bar{S} = \bar{A} - \sum_i \left( \bar{M}_i + \sum_g \bar{X}_{gi} \right) \]

\[ = \sum_g \frac{X_{gi}}{2} - \sum_i \sum_g \frac{X_{gi}}{2T_i}. \quad (7) \]

Substituting (5) through (7) into the profit equation yields

\[ \pi = r_s \left[ \sum_g \left( \frac{X_g}{2} - \sum_i \frac{X_{gi}}{2T_i} \right) \right] + \sum_i r_{M_i} \left[ \sum_g \left( \frac{X_{gi}}{2T_i} - \frac{X_{gi}}{2Z_i} \right) \right] \]

\[ + \sum_g r_{X_g} \left( \sum_i \frac{X_{gi}}{2Z_{gi}} \right) - \sum_i T_i \alpha_i - \sum_i \sum_g Z_{gi} \beta_{gi} \]

\[ - F_g I(S) - \sum_i F_i (M_i). \quad (8) \]
The foregoing profit expression treats income as completely exogenous to cash management. As is well known, this treatment begs the question of what happens to the cash management profit itself. In a fully general model, cash management profit would be part of total income and therefore would affect the quantities of $X_g$ of the various goods purchased. Changes in the interest rates and transactions costs then would have both income and substitution effects on the $X_g$ that are totally absent in the present analysis. Jovanovic (1982), Prescott (1987), and Romer (1986) have addressed these issues. Even their contributions are limited, however, in that labor supply is fixed (in fact, absent from the analyses) and the labor/leisure choice is ignored. The fact that money is required for purchasing consumption but not leisure means that transactions costs have subtle income and substitution effects on work effort and thereby on the quantities of consumption $X_g$. As is usually the case, such income and substitution effects introduce ambiguities into most of the results. However, it is hard to believe that the income and substitution effects arising from any aspect of cash management have empirically significant effects on either the quantities $X_g$ consumed or the labor/leisure decision. Trying to take them into account, however, introduces enormous complexity in the analysis and in any case requires undesirable restrictions to make the analysis feasible. The analytical cost would be considerable, and the substantive gain would be negligible. We therefore simplify the analysis by ignoring the feedback effects of cash management profit on the household's choices.

2B. Optimal Usage Patterns
The household must choose the $T_i$, $Z_{gi}$, and $X_{gi}$. It is easiest to proceed in two steps. First, find the optimal values of $T_i$ and $Z_{gi}$ in terms of the $X_{gi}$, and then find the optimal values of the $X_{gi}$. This procedure is akin to concentrating a likelihood function, except that here we are concentrating the profit function. As usual, we treat $T_i$ and $Z_{gi}$ as continuous variables, even though, in fact, they must be integers. The integer constraints are irrelevant to the later discussion, and it is unnecessary for our purposes to include them in the analysis formally.

The appropriate first-order conditions give the solutions:

\[ T_i = \left( r_s - r_{Mt} \right) \sum_g \frac{X_{gi}}{2\alpha_{ij}} \]  
\[ Z_{gi} = \left( r_{Mt} - r_{Xg} \right) \sum_g \frac{X_{gi}}{2\beta_{gi}} \]  

7. For example, Romer (1986) assumes that trip costs are fixed utility costs rather than financial costs that figure in the budget constraint. That is why his money demand formula is linear in consumption. Prescott (1987) assumes a continuum of goods that the household somehow arranges into equivalence classes. The nature of the household utility function that allows grouping of goods into equivalence classes is not addressed.

Substituting (9) and (10) into (5) through (7) yields the following expressions for average balances:

\[
\bar{X}_{gi} = \left[ \frac{X_{gi} \beta_{gi}}{2(r_{Mi} - r_{Xg})} \right]^{1/2}.
\] (11)

\[
\bar{M}_i = \left[ \frac{\alpha_i}{2(r_S - r_{Mi})} \sum_g X_{gi} \right]^{1/2} - \sum_g \left[ \frac{\beta_{gi} X_{gi}}{2(r_{Mi} - r_{Xg})} \right]^{1/2}.
\] (12)

\[
\bar{S} = \sum_g \frac{X_g}{2} - \sum_i \left[ \frac{\alpha_i}{2(r_S - r_{Mi})} \sum_g X_{gi} \right]^{1/2}.
\] (13)

Substituting these expressions into the profit function, doing some algebra, and using the adding-up identity (2) yields

\[
\pi = r_S \sum_g \frac{X_g}{2} - \sum_i \left[ \sum_g \frac{2\alpha_i(r_S - r_{Mi})X_{gi}}{2} \right]^{1/2}
\]

\[
- \left[ 2\alpha_i(r_S - r_{ML}) \sum_g \left( X_g - \sum_i X_{gi} \right) \right]^{1/2}
\]

\[
- \sum_g \left\{ \sum_i \frac{2\beta_{gi}(r_{Mi} - r_{Xg})X_{gi}}{2} \right\}^{1/2}
\]

\[
+ \left[ 2\beta_{gi}(r_{ML} - r_{Xg}) \left( X_g - \sum_i X_{gi} \right) \right]^{1/2}
\]

\[
- F_{gi} \bar{S} - \sum_i F_i \bar{M}_i.
\] (14)

We next consider the first-order condition for the kth good purchased with money i, \( X_{ki} \):

\[
\frac{\partial \pi}{\partial X_{ki}} = -\frac{1}{2} \left[ 2\alpha_i(r_S - r_{Mi}) \right]^{1/2} \left[ \sum_g X_{gi} \right]^{-1/2}
\]

\[
+ \frac{1}{2} \left[ 2\alpha_i(r_S - r_{ML}) \right]^{1/2} \left[ \sum_g \left( X_g - \sum_i X_{gi} \right) \right]^{-1/2}
\]
\[\begin{align*}
&- \frac{1}{2} [2\beta_{ki}(r_{M_{i}} - r_{X_{k}})]^{1/2} (X_{ki})^{-1/2} \\
&+ \frac{1}{2} [2\beta_{kl}(r_{M_{l}} - r_{X_{k}})]^{1/2} \left( X_{k} - \sum_{i}^{L-1} X_{ki} \right)^{-1/2} \\
&= 0 \quad (15)
\end{align*}\]

There are \(G(L - 1)\) such conditions to be solved simultaneously for the \(G(L - 1)\) different \(X_{ki}\).

Not much insight can be gained from the general case, so we restrict attention to a simpler model where \(G = L = 2\).\(^9\) In this case, only \(X_{11}\) and \(X_{21}\) need to be chosen; \(X_{12}\) and \(X_{22}\) are determined by the adding-up identity. The profit function then can be written as

\[\pi = r_{S} \frac{X_{1} + X_{2}}{2} - [2\alpha_{1}(r_{S} - r_{m_{1}})(X_{11} + X_{21})]^{1/2}\]

\[\quad - [2\alpha_{2}(r_{S} - r_{m_{2}})(X_{1} - X_{11}) + (X_{2} - X_{21})]^{1/2} \]

\[\quad - (2\beta_{11}(r_{M_{1}} - r_{X_{1}})X_{11})^{1/2} + (2\beta_{21}(r_{m_{1}} - r_{X_{1}})X_{21})^{1/2}\]

\[\quad + (2\beta_{12}(r_{m_{2}} - r_{X_{1}})(X_{1} - X_{11}))^{1/2} + (2\beta_{22}(r_{M_{2}} - r_{X_{2}})(X_{2} - X_{21}))^{1/2}\]

\[\quad - F_{S}(S) - F_{1}(M_{1}) - F_{2}(M_{2}) \quad (14')\]

The first-order conditions are

\[\frac{\partial \pi}{\partial X_{11}} = - \frac{1}{2} [2\alpha_{1}(r_{S} - r_{M_{1}})]^{1/2}[X_{11} + X_{21}]^{-1/2}\]

\[+ \frac{1}{2} [2\alpha_{2}(r_{S} - r_{m_{2}})]^{1/2}(X_{1} - X_{11}) + (X_{2} - X_{21})^{-1/2}\]

\[\quad - \frac{1}{2} [2\beta_{11}(r_{M_{1}} - r_{X_{1}})]^{1/2}(X_{11})^{-1/2}\]

\[+ \frac{1}{2} [2\beta_{12}(r_{m_{2}} - r_{X_{1}})]^{1/2}(X_{1} - X_{11})^{-1/2}\]

\[= 0. \quad (15')\]

\[\frac{\partial \pi}{\partial X_{21}} = - \frac{1}{2} [2\alpha_{1}(r_{S} - r_{M_{1}})]^{1/2}[X_{11} + X_{21}]^{-1/2}\]

\[+ \frac{1}{2} [2\alpha_{2}(r_{S} - r_{m_{2}})]^{1/2}(X_{1} - X_{11}) + (X_{2} - X_{21})^{-1/2}\]

9. Explorations of higher-order models suggest that this restriction in fact loses no generality.
\[ -\frac{1}{2} [2\beta_{21}(r_{M1} - r_{X2})]^{1/2}(X_{21})^{1/2} \]
\[ + \frac{1}{2} [2\beta_{22}(r_{M2} - r_{X2})]^{1/2}(X_{2} - X_{21})^{-1/2} \]
\[ = 0 . \]  
\( (15^\prime) \)

As mentioned earlier, the household is free to use more than one medium of exchange to buy a given good by using one medium on some shopping trips and the other medium on the remaining trips. Given the compound nonlinearities in the first-order conditions, one might expect the general solution to have this characteristic. In fact, however, the solution always is in a corner. The household always uses only one medium to buy a given good. This result emerges from the second-order conditions for the problem, which the reader can verify easily:

\[ \frac{\partial^2 \pi}{\partial X_{ij} \partial X_{ij}} > 0 \quad \text{for } i, j = 1, 2 \]
\[ \det H > 0 \]  
\( (16) \)

where \( H \) is the Hessian. These conditions show that the interior extremum is a profit minimum so that the maximum occurs at a corner. In such a situation, the household merely compares the profit functions associated with each possible pattern of usage and picks the pattern with the highest profit. The first-order conditions of the previous paragraph do not apply at corner solutions; indeed, they are not defined because some of the \( X_{ij} \) appearing in the denominators of the various terms are zero. We thus ignore them henceforth.

Assuming the agent finds it profitable to use the savings asset as a cash management store of value, there are four possible solutions:

1. \( (S > 0, X_{11} = X_1, X_{21} = X_2) \) that is, hold \( S \), use \( M_1 \) to buy \( X_1 \) and \( X_2 \);
2. \( (S > 0, X_{11} = X_1, X_{21} = X_2) \) that is, hold \( S \), use \( M_1 \) to buy \( X_1 \) and \( M_2 \) to buy \( X_2 \);
3. \( (S > 0, X_{11} = 0, X_{21} = X_2) \) that is, hold \( S \), use \( M_2 \) to buy \( X_1 \) and \( M_1 \) to buy \( X_2 \);
4. \( (S > 0, X_{11} = 0, X_{21} = 0) \) that is, hold \( S \), use \( M_2 \) to buy \( X_1 \) and \( X_2 \).

If the agents choose not to use the savings asset, there are four additional possibilities:

5. \( (S = 0, X_{11} = X_1, X_{21} = X_2) \) that is, do not use \( S \), use \( M_1 \) to buy \( X_1 \) and \( X_2 \);
6. \( (S = 0, X_{11} = X_1, X_{21} = 0) \) that is, do not use \( S \), use \( M_1 \) to buy \( X_1 \) and \( M_2 \) to buy \( X_2 \);
7. \( (S = 0, X_{11} = 0, X_{21} = X_2) \) that is, do not use \( S \), use \( M_2 \) to buy \( X_1 \) and \( M_1 \) to buy \( X_2 \);
8. \( (S = 0, X_{11} = 0, X_{21} = 0) \) that is, do not use \( S \), use \( M_2 \) to buy \( X_1 \) and \( X_2 \).

Given the use of savings as a cash management asset, the choice among these four usage patterns of transactions assets is determined by examining the relevant
### TABLE 1

**Profit Functions**

\[
\begin{align*}
\Pi_{0,1,1} &= r_0 \frac{X_1 + X_2}{2} - [2\alpha_1(r_m - r_d)(X_1 + X_2)]^{1/2} \\
&\quad - [2\beta_{11}(r_m - r_X)X_1]^{1/2} - [2\beta_{21}(r_m - r_X)X_2]^{1/2} \\
&\quad - F_s - F_i \\
\Pi_{0,1,2} &= r_0 \frac{X_1 + X_2}{2} - [2\alpha_1(r_s - r_d)X_1]^{1/2} - [2\alpha_2(r_s - r_d)X_2]^{1/2} \\
&\quad - [2\beta_{11}(r_m - r_X)X_1]^{1/2} - [2\beta_{21}(r_m - r_X)X_2]^{1/2} \\
&\quad - F_s - F_i - F_d \\
\Pi_{0,2,1} &= r_0 \frac{X_1 + X_2}{2} - [2\alpha_1(r_s - r_d)X_2]^{1/2} - [2\alpha_2(r_s - r_d)X_1]^{1/2} \\
&\quad - [2\beta_{11}(r_m - r_X)X_1]^{1/2} - [2\beta_{21}(r_m - r_X)X_2]^{1/2} \\
&\quad - F_s - F_d \\
\Pi_{0,2,2} &= r_0 \frac{X_1 + X_2}{2} - [2\alpha_1(r_s - r_d)(X_1 + X_2)]^{1/2} \\
&\quad - [2\beta_{11}(r_m - r_X)X_1]^{1/2} - [2\beta_{21}(r_m - r_X)X_2]^{1/2} \\
&\quad - F_s - F_d \\
\Pi_{1,1,1} &= r_1 \frac{X_1 + X_2}{2} - [2\beta_{11}(r_m - r_X)X_1]^{1/2} - [2\beta_{21}(r_m - r_X)X_2]^{1/2} \\
&\quad - F_i \\
\Pi_{1,1,2} &= r_1 \frac{X_1}{2} + r_2 \frac{X_2}{2} - [2\beta_{11}(r_m - r_X)X_1]^{1/2} - [2\beta_{21}(r_m - r_X)X_2]^{1/2} \\
&\quad - F_i - F_d \\
\Pi_{1,2,1} &= r_1 \frac{X_2}{2} + r_2 \frac{X_1}{2} - [2\beta_{11}(r_m - r_X)X_2]^{1/2} - [2\beta_{21}(r_m - r_X)X_1]^{1/2} \\
&\quad - F_i - F_d \\
\Pi_{1,2,2} &= r_1 \frac{X_2}{2} + r_2 \frac{X_1}{2} - [2\beta_{11}(r_m - r_X)X_1]^{1/2} - [2\beta_{21}(r_m - r_X)X_2]^{1/2} \\
&\quad - F_i - F_d \\
\end{align*}
\]

**Note.** The subscripts in the profit expression \( \pi_{ik} \) have the following meanings:

- \( i = 0 \) if the saving asset is used, \( 1 \) otherwise.
- \( j = 1 \) or \( 2 \) as \( M_1 \) or \( M_2 \) is used to buy good 1.
- \( k = 1 \) or \( 2 \) as \( M_1 \) or \( M_2 \) is used to buy good 2.

profit functions given in Table 1. In that table, the subscripts of the profit functions \( \pi_{ik} \) take the following values:

- \( i = 0 \) if the saving asset is used, \( 1 \) otherwise;
- \( j = 1 \) or \( 2 \) as \( M_1 \) or \( M_2 \) is used to buy good 1;
- \( k = 1 \) or \( 2 \) as \( M_1 \) or \( M_2 \) is used to buy good 2.
The important question is how the household’s choice depends on income, expenditure patterns, interest rates, and both transactions and fixed costs.

**Usage versus Average Balances.** For some of the possible solutions, only one medium of exchange is used, so its average balance in the second is obviously zero. Nonusage necessarily implies zero average balances. The converse, however, need not be true. Average balances can be zero even if an asset is used. Consider an asset, say $M$, that is used. From equation (5'), average balances of $M$ will be zero if all the $Z_{g}$ equal $T$. The household puts some of its income into $M$ but then immediately spends it and lets no wealth reside in it for any duration of time. Consequently, average balances are zero even though the medium is used.

If there is no external requirement that the household use an asset, it will not be optimal for the household to use any asset but hold no balances in it, as a comparison of the profit functions shows. If the household did use the asset but chose to hold no balances, it would pay the transactions and fixed costs associated with obtaining the asset but would derive no interest earnings from it.

### 2C. Comparative Static Results

Given that the optimal behavior is characterized by a selection from the profit functions of Table 1, this section will evaluate the behavior of these functions. To do so we examine the effect of various shifts in income, expenditure patterns, interest rates, and transaction fees on the solution. Of interest is the effect on the average balances of the chosen medium and how such changes might cause the household to select an alternative payments scheme.

(1) **Level of Income.** To see the effect of household income, $X_{1} + X_{2}$, on the usage pattern of media of exchange, consider the difference between any two profit functions, such as $\pi_{511}$ and $\pi_{512}$:

\[
\pi_{5,1,1} - \pi_{5,1,2} = -[2\alpha_{1}(r_{S} - r_{M1})(X_{1} + X_{2})^{1/2} - [2\beta_{21}(r_{M1} - r_{X2})X_{2}]^{1/2} + [2\alpha_{1}(r_{S} - r_{M1})X_{1}]^{1/2} + [2\alpha_{2}(r_{S} - r_{M2})X_{2}]^{1/2} + [2\beta_{22}(r_{m2} - r_{X2})X_{2}]^{1/2} + F_{2} = \omega(X_{1}, X_{2}) + F_{2} \quad (17)
\]

where $\omega$ is the sum of the first five terms in the right side of the first line. In (17), all expenditure terms $X_{1} + X_{2}$, $X_{1}$ and $X_{2}$ appear under the square root sign, so $\omega$ is homogeneous of degree one-half in $(X_{1}, X_{2})$. The effect of a change in income depends to some extent on whether or not both goods are superior. We consider two cases: (i) both goods are superior and respond equiproportionally to an increase in income, and (ii) one good, $X_{2}$, is inferior. In case (i), an increase in income raises $X_{1} + X_{2}, X_{1}$ and $X_{2}$ equiproportionally and therefore preserves the sign of $\omega$. The effect on the choice of media of exchange usage then can be divided into three possibilities:
(a) $\omega > 0$. In this case, (17) is positive, and an increase in income keeps it so.
(b) $\omega < 0$ and $\omega + F_2 > 0$. In this case, (17) is positive because the cost term $F_2$
outweighs the transactions cost terms collected in $\omega$, but an increase in income
increases the magnitude of $\omega$ and so possibly reverses the sign of (17)
and thereby leads to a change in the usage pattern.
(c) $\omega < 0$ and $\omega + F_2 < 0$. In this case, (17) is negative because $\omega$ outweighs $F_2$,
and an increase in income keeps (17) negative.

Which of these cases holds depends on the magnitudes of the various transactions
costs $\alpha$ and $\beta$, and also of the fixed cost $F_2$. Note that, if $\omega < 0$, the usage pattern
depends on household income. Compared to high-income households, low-income
households will have fewer negative values of $\omega$, will consequently be more likely
to have a positive value for (17), and therefore will be more likely to choose to use
just one medium of exchange.

In case (ii), results are more ambiguous. An increase in income does not necessarily
preserve the sign of $\omega$. Consequently, (17) may change sign irrespective of the
original relation between $\omega$ and $F_2$.

Although the foregoing results were derived for the specific profit difference
in (17), similar results apply to the difference between any other two profit
expressions.

(2) Relative Expenditures. Consider now the effect of changing the composition
of expenditures while holding income (that is, total expenditure) constant. The decision
on the use of the saving asset is independent of expenditure composition because all profit differences depend only on the sum $X_1 + X_2$ and not on the division
of that total between $X_1$ and $X_2$. However, the choice of usage pattern for media of exchange does depend on the expenditure composition. To analyze this dependence, suppose we start with a provisional choice of not using $M_1$ at all and then consider
whether it would be better to make a different choice.

We assume for exposition ease that $r_{M1} > r_{M2}$. Because the use of $S$ does not depend on expenditure composition, we can simplify the mathematics by restricting attention to the case where the asset $S$ is not used. We begin by making the two profit comparisons $\pi_{012} - \pi_{022}$ and $\pi_{012} - \pi_{022}$, which tell us whether we should start using $M_1$ to buy at least one good:

\[
\pi_{0,1.2} - \pi_{0,2.2} = (r_{M1} - r_{m2}) \frac{X_1}{2} - \{[2\beta_{11}(r_{M1} - r_{X1})]^{1/2}
- [2\beta_{11}r_{M2} - r_{X1}]^{1/2} [X_1]^{1/2} - F_1' \},
\]  

\[
\pi_{0,2.1} - \pi_{0,2.2} = (r_{M1} - r_{m2}) \frac{X_2}{2} - \{[2\beta_{21}(r_{M1} - r_{X2})]^{1/2}
- [2\beta_{22}(r_{m2} - r_{X2})]^{1/2} [X_2]^{1/2} - F_1 \}. \]
As $X_1$ rises and $X_2$ falls, (18) tends to become positive and (19) tends to become negative (for example, (19) $< 0$ if $X_2 = 0$) and vice versa if $X_1$ falls and $X_2$ rises. Thus, the larger the share of total expenditure that a particular good commands, the more likely it is that the household uses the high-interest medium of exchange to buy it. This relation reflects the opportunity cost associated with holding a medium of exchange. The more one spends on a good, the larger the average balances held. Therefore, the larger the interest foregone compared with holding the savings asset, and so the greater the importance of using a medium paying a high rate of interest. Similar intuition underlies many of our subsequent results.

Suppose (18) is positive and (19) is negative. Then the household will make a new provisional usage decision and begin using $M_1$ to buy $X_1$ while continuing to use $M_2$ to buy $X_2$. Next, the household must decide whether to use $M_1$ for both goods, which it does comparing $\pi_{011}$ and $\pi_{012}$:

$$\pi_{0,1,1} - \pi_{0,1,2} = \frac{(r_{M1} - r_{M2})X_2}{2} - [2\beta_{21}(r_{M1} - r_{X2})X_2]^{1/2} - [2\beta_{22}(r_{M2} - r_{X2})X_2]^{1/2} + F_2.$$  \hspace{1cm} (20)

This expression can be positive even if (19) is negative because of the different fixed cost terms in (19) and (20). The household thus may choose to use only $M_1$ in making purchases. However, this outcome is less likely when the share of total expenditure commanded by $X_2$ is smaller because then (20) is less likely to be positive.

We thus have established two results. The household tends to use the high-interest medium to buy the larger-share good, and a split use of media of exchange is more likely to occur the more unequal are the expenditures $X_1$ and $X_2$. Clearly, households with the same income and facing the same interest rates and transactions costs still may choose different usage patterns of media of exchange solely because their tastes in consumption goods differ. Equations (11) through (13) show that average asset holdings also depend on the allocation of income among different goods.

(3) Interest Rates. There are three kinds of interest rates in the model: the rate on savings $r_s$, the rates on media of exchange $r_{M1}$, and the rates on commodity inventories $r_{X_i}$. Their effects on usage patterns are subtle and not easily discerned.

A change in $r_s$ affects the usage pattern for media of exchange. In the most general case, this effect is of ambiguous sign, but if $\alpha_1 = \alpha_2 = \alpha$ and $r_{M1} = r_{M2} = r_M$, we can obtain an interesting result. Consider profit differences of the form $\pi_{Sii} - \pi_{Sij}$ or $\pi_{Sii} - \pi_{Sji}$ and see how they are affected by an increase in $r_s$. For example, consider the derivative of $\pi_{S11} - \pi_{S12}$ when $\alpha_1 = \alpha_2 = \alpha$ and $r_{M1} = r_{M2} = r_M$:

$$\frac{\partial(\pi_{S1,1} - \pi_{S1,2})}{\partial r_s} = \frac{\alpha}{2(r_s - r_m)} \left[ X_1^{1/2} + X_2^{1/2} - (X_1 + X_2)^{1/2} \right]^2$$  \hspace{1cm} (21)

which is positive because the square root is a concave function. Thus, an increase in $r_s$ tends to lead to use of just one medium of exchange to buy both goods.
A change in one of the interest rates on money, \( r_M \), or \( r_{M^2} \), has ambiguous effects on the use of both the savings asset \( S \) and the media of exchange \( M_1 \) and \( M_2 \). The effects of \( r_M \) on the use of media of exchange are governed by profit differences, and the derivatives of these differences with respect to \( r_M \) are always of ambiguous sign. For example, the derivative of \( \pi_{S1} - \pi_{S2} \) is

\[
\frac{\partial (\pi_{S,1,1} - \pi_{S,1,2})}{\partial r_S} = - \left[ \frac{\alpha_1}{2(r_M - r_{M1})} \right]^{1/2} [X_1^{1/2} - (X_1 + X_2)^{1/2}]
\]

\[
- \left[ \frac{\beta_{11} X_2}{2(r_M - r_{M2})} \right]^{1/2} .
\] (22)

The first term is positive and the second negative, leading to an ambiguous sign for the derivative as a whole.

The rates of return on commodities, \( r_{X1} \) and \( r_{X2} \), also affect the usage pattern for the media of exchange. For expository ease, suppose \( r_{M1} > r_{M2} \) and \( r_{X1} = r_{X2} \). Consider the effect of changing good \( 1 \)'s rate of return \( r_{X1} \) on the decision of which medium of exchange to use in purchasing good \( 1 \). The derivatives of the relevant profit difference with respect to \( r_{X1} \) are of ambiguous sign, but if \( \beta_{11} = \beta_{12} = \beta \), then we can obtain definite results. For example, suppose \( \beta_{11} = \beta_{12} = \beta \). Consider the derivative of \( \pi_{S1} - \pi_{S2} \) with respect to \( r_{X1} \):

\[
\frac{\partial (\pi_{S,1,1} - \pi_{S,2,1})}{\partial r_{X1}} = \left( \frac{\beta_1 X_1}{2} \right)^{1/2} [r_{M1} - r_{X1}]^{-1/2} - (r_{M2} - r_{X1})^{1/2}]
\] (23)

which is negative because \( (r_{M2} - r_{X1})^{-1/2} \) exceeds \( (r_{M1} - r_{X1})^{-1/2} \) when \( r_{M1} > r_{M2} \). Thus, an increase in the return on good \( 1 \) tends to induce the household to use the medium with the lower return to buy good \( 1 \). More generally, the household tends to use the lower-return medium to buy the higher-return good.

From (11) through (13), household average balances of each asset are positively related to their own rates of return, provided the household already is using the asset and is not induced by a change in an interest rate to abandon it. However, as we have seen, usage patterns can change in response to interest rate changes. For the media of exchange, changes in own rates of return have ambiguous effects on usage. If usage turns out to be negatively related to own rates, then aggregate demand for average balances of those assets might fall in response to an increase in the own rate. This is because the decline in the number of households using the medium may more than offset the higher balances of those that continue to use it.

(4) Transactions and Fixed Costs. The effects of all costs on usage are straightforward for the representative agent. An increase in any transaction or fixed cost reduces the use of the associated asset. These results can be derived easily from the relevant profit differences; the derivation is left to the reader. The effects of costs on average balances also are straightforward and follow from (11) through (13). For the household, given use of an asset, the effects of costs show the adjacency property.
discovered by Santomero (1974). An increase in the cost of transferring funds into a particular medium, \( \alpha_i \), reduces average holdings of \( S \), raises average holdings of \( M_i \), and does not affect holdings of commodity inventories. An increase in the cost of using that medium to purchase goods, \( \beta_{ij} \), does not affect holdings of \( S \), but reduces average holdings of \( M_j \), and raises average holdings of \( X_i \). Given use, changes in fixed costs do not affect average holdings of any asset.

For aggregate average balances, we have some ambiguities. An increase in the cost of transfer into a particular medium, \( \alpha_e \), unambiguous reduces aggregate demand for \( S \) because it reduces household use and holdings. The effect on aggregate demand for \( M_j \) is unclear. Use is reduced, but average balances rise for those households that continue to use \( M_j \). In contrast, an increase in the cost of usage, \( \beta_{ij} \), unambiguously reduces aggregate demand for \( M_j \) because it reduces household use and holdings, and it also unambiguously raises aggregate demand for \( X_i \). Finally, an increase in any fixed cost unambiguously reduce aggregate demand for the associated money because it reduces household use and does not affect household holdings.

(5) Integrated Media. We have explored models in which media of exchange are more fully integrated than in the foregoing model. In particular, we have examined models where (i) the financial transfer costs, \( \alpha_i \) are all the same and one “trip to the bank” allows transfers among all financial assets, (ii) the shopping costs \( \beta_{ij} \) of all media are all the same, and (iii) the \( \alpha_e \) and the \( \beta_{ij} \) are all equal, as might be the case with a “smart card.” Although some details change, the conclusions are essentially the same as those reported above, so we do not dwell on such alternative models here.

3. CONCLUSIONS FROM THE REPRESENTATIVE AGENT MODEL

The foregoing results have several interesting implications. Here we first examine some of the broad outcomes that spring from the analysis. We have seen that the range of asset use decreases as household income falls. We also have seen that the usage patterns of media of exchange differ among households with the same income but different allocations of income among consumption goods. Given that consumption patterns differ across socioeconomic groups, it should not be surprising to find different money usage patterns across different types of consumers.

The dependence of media of exchange usage patterns on income levels and consumption patterns also suggests that we should expect different reactions across economic groups to the newly emerging monetary mechanisms. Various households offered a new medium of exchange with its own implied interest rate, transactions costs, and fixed cost will react differently. A cross-section of consumers will be expected to demand different combinations of media of exchange. To put it another way, demand for a medium of exchange will vary across households, just as the demand for automobiles does. Thus, there appears to be room for transfer mechanisms with different combinations of fees and interest rates. In transaction schedules, one size does not fit all. Our theory suggests that a whole set of instruments may be viable with different demanders, depending on the features offered with the instrument.
Consider, in particular, the implications of these results on the emerging stored-value card, or smart-card technology, and its competitive position vis-à-vis the two most common media of exchange. This transfer medium offers some of the features of demand deposits such as ease of transfer and general acceptability. At the same time, it avoids the fixed costs of a checking account while offering no return on average balances. We have seen that households tend to use the higher-interest medium of exchange to buy the good that constitutes the larger share of its income. Given the relative rates of return on stored-value cards and checking account balances, the former is unlikely to dislodge the use of checks for purchases. Similarly the stored value card represents a credible threat for cash transactions. Here, rates of return are both zero, although it can be argued that cash may have a negative return due to theft. Given their new ease of use, smart cards could also be competitive from a transfer fee perspective. In fact, given the lower cost of transfer into the card, it may dominate cash in the near future. This might explain why promoters have sought the early successes of such spending vehicles at convenience stores, transit stations, and the like.

The foregoing results apply to the individual household. Aggregate results often are more difficult to obtain. As we have demonstrated, many variables affect usage decisions and average balances in opposite ways. For example, an increase in the cost of obtaining medium \( i \), \( \alpha \), reduces the number of households using \( M_i \) but raises average balances of \( M_i \) for those households that continue to use it. Thus, the change in aggregate holdings of \( M_i \) is ambiguous. Therefore, efforts by banks to lower the costs of using a particular form of medium of exchange may well reduce the average holdings of that medium. Also, interest rates generally have ambiguous effects on money holdings. Determining whether an increase in a particular interest rate will raise or lower holdings of a particular form of money requires a case-by-case analysis, and the conclusions can change from one day to the next with changes in other parameters of concern to the household. For example, simply changing the relative amounts of the kinds of goods bought can change the average holdings of various types of money and also the response of those monies to changes in interest rates and transactions costs.

In general, our results are intriguing in both their complexity and sensitivity. Affecting monetary behavior is no simple matter. As providers of different monies move from experimentation to implementation, these results offer a warning. The choice of money or monies to be used for transactions purposes is a complex decision. It does not lend itself to simple extrapolation from consumer surveys, and, in fact, may result in substantially different outcomes than had been presumed. The innovators would do well to proceed slowly. We would not want to "bet the bank" on any one emerging technology.

LITERATURE CITED


