SHOW YOUR WORK. NO WORK = NO CREDIT. NO CALCULATORS.

1. (20%) Let \( A = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & -1 \\ 3 & 2 & 1 & 1 \end{pmatrix} \). Find a basis for the \( NS(A) \) and a basis for \( NS(A^T A) \). Determine \( \text{rank}(A) \) and \( \text{rank}(A^T A) \).

2. (16%) Find the equation of the line that best fits the points \((-1, 1), (0, 1)\) and \((2, -1)\) in the least-square sense.

3. (20%) Let \( v_1 = (0, 1, 0)^T; v_2 = (-4, 0, 3)^T; v_3 = (3, 0, 4)^T \).
   
   (a) Show that \( \{v_1, v_2, v_3\} \) is an orthogonal, but not an orthonormal set.
   
   (b) Find an orthonormal basis \( \{u_1, u_2, u_3\} \) for the subspace \( S = \text{span}\{v_1, v_2, v_3\} \) and write \( v = (1, 1, 1)^T \) as a linear combination of \( \{u_1, u_2, u_3\} \).

4. (24%) Let \( A = \begin{pmatrix} -1 & -1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \), \( \overline{b} = \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix} \).
   
   (a) Find an orthonormal basis for the column space \( C(A) \).
   
   (b) Find the \( QR \)- decomposition of \( A \).
   
   (c) Find the least square solution to \( AX = \overline{b} \).

5. (20%) Let \( A = \begin{pmatrix} 2 & -5 & 5 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{pmatrix} \).
   
   (a) Find all eigenvalues of \( A \).
   
   (b) Find a basis for each eigenspace of \( A \).