SHOW YOUR WORK. NO WORK = NO CREDIT. NO CALCULATORS.

1. (20%) Which of the following are subspaces? Justify your answer.
   (a) \( \{(x, y, z) \mid z = 2x + y\} \).
   (b) \( \{(x, y, z) \mid z \geq 0\} \).

2. (20%) Consider the vectors \( v_1 = (1, 1, 0, 0)^T, v_2 = (0, 0, 2, 2)^T, v_3 = (3, 1, 4, 1)^T \).
   (a) Determine whether the vectors \( v_1, v_2, \) and \( v_3 \) are linearly independent or dependent.
   (b) Is \( v = (1, 1, 1, 1)^T \in \text{span}\{v_1, v_2, v_3\} \)? Justify.

3. (20%) Determine whether the following set of vectors form a basis for \( \mathbb{R}^3 \). Justify your answer.
   (a) \( \{v_1 = (1, 1, 0)^T, v_2 = (1, 0, 1)^T, v_3 = (0, 1, -1)^T\} \).
   (b) \( \{v_1 = (3, -2, 1)^T, v_2 = (2, 3, 1)^T, v_3 = (2, 1, -3)^T\} \).

4. (20%) Let \( A = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & -1 \\ 3 & 2 & 1 & 1 \end{pmatrix} \).
   Find a basis and determine the dimensions of the: (i) row space \( R(A) \), (ii) column space \( C(A) \) and
   (iii) null space \( N(A) \).

5. (20%) Consider the order bases \( B = \{(1, 1)^T, (1, 0)^T\} \) and \( C = \{(2, 3)^T, (4, 2)^T\} \) of \( \mathbb{R}^2 \). Let \( X \in \mathbb{R}^2 \)
   with \( [X]_B = (-1, 2)^T \).
   (a) Find the transition matrix from the basis \( B \) to the basis \( C \).
   (b) Find the coordinate matrix \( [X]_C \).