I. Introduction

Cellular base stations may make use of an antenna array to achieve diversity gains or antenna gains so as to improve the system capacities. While considerable progress has been made on the use of receiver arrays for the uplink, comparatively little progress has been made for the downlink communications, where instead transmitter arrays are exploited at the base stations and only one receiver antenna is used at each mobile handset in practice. Since uplinks and downlinks are used in duplex mode, it is possible to apply the principle of reciprocity, which implies that the channel is identical on both links as long as the channel is measured at the same frequency and time instant. In Time Division Duplex (TDD) systems the principle of reciprocity can be applied if the dwell (“Ping-Pong”) time is short compared to the channel coherence time. In Frequency Division Duplex (FDD) systems (FDD is adopted in most current wireless systems and most probably in next generation systems), the separation between the uplink and downlink carrier frequencies is large enough to reject the reciprocity principle. However, if the frequency separation is not too large, the uplink and downlink will still share many common features, among which are the number of multipaths, their delays and angles, the large scale path loss and shadowing, and the variance of small-scale fading [13], [14]. Nevertheless, the instantaneous small-scale fading of the two links is uncorrelated, which makes the downlink problem more difficult for FDD systems. The signal received at the base station provides a means for directly estimating the uplink, not the downlink channel. While such information could be available via a feedback channel from the mobile, we will assume that no such channel exists. The fact that the array response is also frequency dependent further complicates the problem. In this study, we focus on array techniques to improve the cellular CDMA downlink, which is foreseen to be of crucial importance for the 3rd generation communication systems supporting wireless Internet, video on demand, and multimedia services.

Perhaps the simplest form of spatial processing is open loop transmit diversity, which will be used as the performance baseline in this study [10], [8]. Sectorization, which can be interpreted as fixed beam transmission, has been shown an effective way to improve the system capacity [25]. Another simple transmitter array processing is the so-called beam steering, which assumes the knowledge of mobile position and forms a beam in the direction of line-of-sight. The performance of beam steering is expected to degrade in multipath channel with angle spread. A more sophisticated use of the array is to determine the antenna weighting vectors that maximize SNR at the mobiles. Alternatively, one can borrow the idea from uplink receiver array processing and come up with a maximum SIR solution for weighting vector design, i.e., maximizing the ratio of the received power of one signal at the desired user and that leaked to the other users. The key element that comes into play of the max SNR or max SIR scheme is the spatial covariance matrix, more details of which will be given later [2], [11], [14], [21]. Please note here that, compared to the counterpart of the uplink processing, there are two differences for max SIR scheme: 1) the interference term is what this signal contributes to the other users, not that seen at the desired mobile; 2) the power levels of transmitted signals are not available at this stage (it is decided at the power allocation step discussed in the following), so we can not do max SINR as uplink processing.

Power control was used basically as a mechanism to deal with the near far problem, but more general emerging view is that it is a flexible mechanism to provide different quality-of-service to users with heterogeneous requirements [7]. For downlink transmission, power control is also important for energy conservation and interference mitigation. Standard power control
algorithms have been reported in [4], [5], [6]. When we do the above downlink transmission array processing together with power control, we do it in two steps: 1) array weighting vector is determined (not needed for transmit diversity) and SINR calculated for each mobile receiver; 2) transmitted power is allocated among users so as to minimize the total transmitted power from base station while keeping the SINR of all links above a certain threshold.

As we mentioned, downlink communication scenario is different from that of uplink. While in the uplink the weighting vector designs for different users are de-coupled, optimal beamforming for the downlink will have to be considered jointly, because the weighting vector for one user will impact the interference received by other users as well as the useful signal power received by the desired user. Farrokhi came up with the idea of joint power control and downlink beamforming in [15], [16]. The algorithm in [16] was later modified in [18] to give the optimal transmit beamforming vectors. These algorithms require knowledge of the downlink channel. We make modifications so that only information available from the uplink measurements is used. The result of the original algorithm will serve as the bound for performance comparison of various techniques.

Besides the lack of direct downlink channel information, the limited number of available antennas may also hamper the algorithms. When the number of antennas is small compared to the number of mobiles (as in a circuit-switched system), there are an insufficient number of degrees of freedom to produce simultaneous nulls for each user. However, in packet-switched systems, where users are delay tolerant, the base can also control the number of simultaneous transmissions. This points out that the performance tradeoffs between these algorithms depend on the nature of the traffic. In the packet-switched case, we do the rate control instead of power control, so we assume the base will transmit at its maximum power. Thus, the max SIR scheme will be replaced by max SINR in packet-switched system case.

To summarize, the essential question we are addressing is: for a given number of available transmit antennas, should we use transmit diversity, sectorization, simple directional beam steering, max SNR beamforming, max SIR/SINR beamforming, or the joint beamforming and power control scheme? Which choice is best for circuit and packet systems? All these questions are addressed in the context of a system that does not have explicit feedback of the downlink channel measurements.

The paper is organized as follows: in Section II we give the multipath channel model and talk about the spatial covariance matrix approximation for downlink in FDD system. Standard power control algorithm is addressed in Section III and in Section IV various transmit array-processing techniques are discussed. Section V proposes a speed-up technique for cellular network simulation. Section VI gives the numerical comparison results for circuit-switched and packet-switched system. Section VII gives the conclusions.

II. System Model

1. Multipath Channel

We mainly talk about the model for array processing here. The model for transmit diversity will be discussed in the end. In out system setting, each mobile user employs a single antenna, and communicates with a base station having an $M$-element antenna array. The physical channel
between the mobile and the base station is assumed to be WSSUS multipath frequency-selective fading. The uplink received signal vector at the base station is given by

$$x(t) = \sum_{k=1}^{K} \sqrt{P_k} \sum_{l=1}^{L_k} \alpha_{kl}^U(t) a^U(\theta_k) s_k(t - \tau_k) + n(t)$$

(1)

where $P_k$ is the transmitted power from the $k$-th user, $\alpha_{kl}^U(t)$ and $\tau_k$ are the complex gain and delay of the $l$-th path of the $k$-th user respectively. The $k$-th user signal (including spreading, data modulation, etc.) is denoted as $s_k(t)$. The path complex gain can be modeled as a random process of the form:

$$\alpha_{kl}^U(t) = \frac{C}{d_k^{\eta}} \sqrt{S_k} F_{kl}^U(t)$$

(2)

Where $C$ is a constant, $d_k$ is the distance between the mobile and the base station, and $\eta$ is the path loss parameter. $S_k$ denotes the log-normal shadowing, which is assumed to be quasi-time invariant within the period of interest and frequency independent. $F_{kl}^U(t)$ describes the Raleigh fading random process, which is frequency dependent and largely uncorrelated for uplink and downlink. $\theta_k$ is the arrival angle of the $l$-th path of the $k$-th user. In our model, we assume $\theta_{kl}$, $1 \leq l \leq L$, is Gaussian distributed around $\theta_k$, the direction from line-of-sight of user $k$. $a(\theta)$ is the array response to a wave impinging from an azimuth direction $\theta$. With the assumption of planar wave and uniform linear array, the frequency dependent array response is given by

$$a(\theta, f) = [1, e^{-j2\pi f \frac{d}{c} \sin(\theta)}, \cdots, e^{-j2\pi f (M-1) \frac{d}{c} \sin(\theta)}]^T$$

(3)

Where $d$ is the inter-element spacing of the antenna array. So

$$a^U(\theta_k) = [1, e^{-j2\pi f \frac{d}{c} \sin(\theta_k)}, \cdots, e^{-j2\pi f (M-1) \frac{d}{c} \sin(\theta_k)}]^T$$

(4)

$n(t)$ is an $M$-dimensional complex Gaussian vector with i.i.d. components of zero mean and variance $\sigma^2$.

For the downlink, after joint transmission of the weighted signals bounded for different users from the base station, the baseband signal received by the mobile $i$ is given by

$$x_i(t) = \sum_{k=1}^{K} \sqrt{P_k} w_k^H \sum_{l=1}^{L_i} \alpha_{kl}^D(\theta_k) a^D(\theta_k) s_k(t - \tau_k) + n_i(t)$$

(5)

where $w_k$ is the transmit beamforming weight vector for user $k$ and $P_k$ is the power assigned to the user $k$ signal, which are the two parameters we want to design. $n_i(t)$ is a complex white Gaussian process. Due to the reciprocity of uplink and downlink, other elements of the equation (5) are self-explained. As we said before, although uplink and downlink share many common features, the instantaneous fading coefficient and the steering vector are different for FDD system. To be specific, the downlink fading coefficients are given by
\[
\alpha_{kl}^D(t) = \frac{C}{d_k^{q/2}} \sqrt{S_{kl}} F_{kl}^D(t)
\]  
(6)

where all the scale parameters are identical to the uplink, but the Rayleigh fading is drawn from an independent instantiation. The downlink steering vector is given by

\[
a^D(\theta_{kl}) = [1, e^{-j2\pi d_k \sin(\theta_k)}, \ldots, e^{-j2\pi d_k (M-1) \sin(\theta_k)}]^T.
\]  
(7)

While the antenna elements should be closely spaced (e.g., half wavelength) for beamforming to get coherent signals across the antenna array, they are separated far apart (e.g., 10 wavelength) to get diversity gain for transmit diversity scheme. Rather than combined with a steering vector, the signals coming from different elements of antennas exploiting transmit diversity experience uncorrelated fading. Assume we have \(M\) transmit antennas exploiting code transmit diversity, which transmit the data of \(K\) users simultaneously. Then the transmitted signal from the \(m\)-th antenna can be modeled as

\[
s_m(t) = \frac{1}{\sqrt{M}} \sum_{k=1}^{K} \sqrt{P_k} b_k c_{km}(t)
\]  
(8)

where \(c_{km}(t)\) is the spreading code for the \(k\)-th user in the \(m\)-th antenna with the spreading gain \(N\). The channel between the \(m\)-th antenna and the \(i\)-th receiver can be modeled as

\[
h_{mi}^j(t) = \sum_{l=1}^{t_i} \alpha_{ml}^j \delta(t - \tau_i^j)
\]  
(9)

where \(\alpha_{ml}^j\) is the complex path gain, and \(\tau_i^j\) is the path delay. The received signal at the \(i\)-th mobile is given by

\[
r_i(t) = \frac{1}{\sqrt{M}} \sum_{m=1}^{M} \sum_{k=1}^{K} \sqrt{P_k} b_k \sum_{l=1}^{t_i} \alpha_{ml}^j c_{km}(t - \tau_i^j) + n_i(t)
\]  
(10)

2. FDD Framework

The essential element in antenna array beamforming design is the spatial covariance matrix, which, according to (5), is given by

\[
R_{true,k}^D = \sum_t |\alpha_{kl}^D|^2 |\alpha_{kl}^D|^2 \ a^D(\theta_{kl}) a^D(\theta_{kl})^H
\]  
(11)

Unfortunately, neither the instantaneous fading coefficients nor steering vectors are known at the base stations, so estimation and approximation are necessary. Although the fading is uncorrelated for uplink and downlink, their average strength is assumed to be insensitive to small changes in frequency [9], [14], i.e.,
\[ E(\{ \alpha^D_k \} \mid \alpha^U_k) = E(\{ \alpha^U_k \} \mid \alpha^D_k) \]  

(12)

which can be estimated through time average from uplink data. To estimate the downlink steering vectors, several approaches exist. One idea (the matched array) is to design two separate closely located arrays which are scaled versions of each other in proportion to the ratio of the uplink and downlink wavelengths, thus making the uplink and downlink steering vectors same [14]. The drawbacks of this approach are expensive cost, imperfect array matching and near field uneven scattering. A witty log-periodic array configuration is proposed in [9], which overlaps the two subarrays of \( M \) elements mentioned above into one \( M+1 \) array with \( d_m / d_{m-1} = \lambda_u / \lambda_d \), where \( d_m \) is the spacing between the \( m \)-th and the \( (m+1) \)-th element. The drawbacks above are alleviated but still exist. Another approach (the duplex array) is to use a single array for both the uplink and downlink, and transpose the array response from the uplink to the downlink via a linear transformation [1], [14]. However, some constraints are imposed to make the linear transformation tractable, e.g., small frequency shift assumption in [14] and circular array geometry in [1]. In our work, we exploit the approach to estimate the downlink array response from the uplink data through high resolution DOA estimation methods or training sequences. We also ignore the estimation errors, which deserves further study.

To sum up, we assume the perfect knowledge of downlink array response and calculate the array response for the downlink as (7). Then the approximated downlink spatial covariance matrix is given by

\[ R^D_{\text{approx},k} = \sum_1^L \sum_1^K E(\{ \alpha^U_{kl} \} \mid \alpha^D_{kl}) a^D(\theta_{kl}) a^D(\theta_{kl})^H \]  

(13)

3. Cellular System

We consider a cellular geometry as shown in Fig. 1. It consists of two tiers of surrounding cells around the cell of interest. Each cell is divided into three sectors of 120 degrees. Because we exploit CDMA, the sector of out interest will get interference from adjacent sectors of the same cell, as well as out of cell interference, as indicated in Fig. 1.

The six-sector case is similar and thus omitted here. We assume that each cell and sector is identical. They are loaded with the same number of users with same behavior, and at the base stations same operations are exploited. All the cells and sectors operate at the same time. Hopefully this will reflect the average performance of the real system in the long run. Based on this assumption, a new speedup technique is proposed in the Section V for the whole system simulation.

III. Power Control Algorithms

In different application scenarios, power control may have different meanings. The commonly used criterion is formulated as follows:

\[ \min \sum_{k=1}^K P_k \text{ subject to } \text{SINR}_k \geq \gamma_k, 1 \leq k \leq K. \]  

(14)
That is, to minimize the total transmitted power with the constraints that each link obtains a SINR above a certain threshold. This is the optimization criterion we adopt for circuit-switched system. For the packet-switched system, we always assume to transmit at the maximum power, and we are concerned with the throughput of the network. We can simply equally allocate power among the active users, or we can allocate power in some optimal way. An optimal power assignment scheme is proposed in [20] and formulated as follows:

$$\max \text{ SINR}_{\text{min}} \quad \text{subject to} \quad \sum_{k=1}^{K} P_k \leq P_{\text{max}}$$

(15)

That is, to maximize the minimum link SINR with the total transmitted power constraint. This scheme tries to be fair to the all users, which is not necessarily a good strategy for maximum throughput. It would be better to combine the study of physical layer transmission with the data link budget and network schedule, which is beyond the scope of this paper.

Before we go into the details of the power control schemes, we will talk about their underlying theorem.

1. **Perron-Frobenius Theorem and its Applications**

*Theorem:* Suppose $T$ is an $n \times n$ non-negative irreducible matrix. Then there exists an eigenvalue $r$ such that:

(a) $r$ real, $> 0$;
(b) with $r$ can associated strictly positive left and right eigenvectors;
(c) \( r = \max \{ \| \lambda_i \| \} = \rho(T) \), where \( \lambda_i, 1 \leq i \leq n \) are eigenvalues of matrix \( T \), and \( \rho(T) \) denotes its spectral radius;
(d) \( r \) has algebraic multiplicity 1;
(e) \( \min \sum_{i=1}^{n} t_{ij} \leq r \leq \max \sum_{i=1}^{n} t_{ij} \) with equality on either side implying equality throughout. A similar result holds for columns.

Application 1: A necessary and sufficient condition for a non-negative (non-trivial) solution \( x \) to the equations \( (sI - T)x = c \) to exist for any nonnegative (non-trivial) vector \( c \) is that \( s > r \). In this case there is only one strictly positive solution given by \( (sI - T)^{-1}c \).

Application 2: If a non-negative (non-trivial) vector \( y \) satisfies \( Ty \leq sy(s > 0) \), Then a) \( y > 0 \); b) \( s \geq r \); c) \( s = r \) if and only if \( Ty = sy \).

In the above, non-negative \((\geq 0)\) means all the elements of a vector or matrix is nonnegative; and trivial means all-zero vector or matrix.

2. General Form of Power Control Problem

For the sake of simplicity, we only talk about the general form of power control here. In the next section, exact SINR formula will be given and can be fit in this general setting without difficulty. The power control criterion of (14) is related to application 1 of the theorem as follows. The general form of power control problem is given by [7]

\[
\min \sum_{k=1}^{K} P_k \text{ subject to } (I - DF)p = u
\]

(16)

where \( I \) is an \( K \) by \( K \) identity matrix, \( D \) is a diagonal matrix of with entries \( \gamma_1, \cdots, \gamma_K \), \( F \) is an non-negative irreducible matrix (interference term), and \( u \) is a positive vector (noise term). So we have a feasible (positive) solution for power allocation vector if and only if the spectral radius of \( DF \) is less than one, otherwise we will claim an outage occurs. We call it type-I outage and call the case that we do get a positive solution but the total transmitted power exceeds a threshold, i.e., \( p^T1 > P_{max} \), type-II outage. If existing, the solution is given by \( (I - DF)^{-1}u \) or alternatively by Jacobi iteration

\[
p^{(n+1)} = u + DFp^{(n+1)}
\]

(17)

which is bound to converge for any initial value.

The power control criterion of (14) is related to application 2 of the theorem as follows. It can easily be shown that such effort results in equal SINR=\( \gamma \) for all links. The objective functions then become

\[
P = \gamma(FP + h) \text{ and } P^T \cdot 1 = P_{max}
\]

(18)

with \( h_i = u_i / \gamma_i \). Let \( y = [P^T,1]^T \), we can rewrite the equation above as
\[ Ty = \gamma Q y \]  
(19)

with

\[ T = \begin{bmatrix} I_{K \times K} & 0_{K \times d} \\ 1^T & -P_{\text{max}} \end{bmatrix} \quad \text{and} \quad Q = \begin{bmatrix} F & h \\ 0_{d \times K} & 0 \end{bmatrix} \]  
(20)

Alternatively, we can write it as

\[ R y = \frac{1}{\gamma} y \]  
(21)

with

\[ R = T^{-1} Q = \begin{bmatrix} F & h \\ 1^T F / P_{\text{max}} & 1^T h / P_{\text{max}} \end{bmatrix} \]  
(22)

It is easily shown that \( R \) is a non-negative irreducible matrix. So we always have a unique positive solution for \( p \) and the SINR margins are the reciprocal of the largest eigenvalue of \( R \).

**IV. Array Signal Processing**

In this section, various array signal processing techniques are discussed on details, among which are transmit diversity, sectorization, and beamforming techniques including beam steering, max SNR, and max SIR or SINR. We assume that the mobile receiver can learn the fading channel and perform the RAKE combining. So the instantaneous SINR is obtained for each scheme, based on which the power control of Section III is applied. The joint power control and beamforming algorithm is also addressed, whose optimality is verified.

**1. Transmit Diversity**

We exploit code transmit diversity for downlink CDMA communications [10]. The data streams of all users are transmitted simultaneously. For each user the data symbol are transmitted with equal power from every antenna using mutually orthogonal spreading code. A total of \( K M \) Walsh codes are required in a straightforward design. To conserve codes, techniques such as space-time spreading can be used [8], however the \( E_b / N_0 \) performance achieved is no different than in the simple case above. Diversity gain is achieved because the receiver can separate the transmitted signals based on knowledge of the spreading codes.

We can rewrite the discretized signal model of (8) as

\[ s_n = \frac{1}{\sqrt{M}} \sum_{k=1}^{K} \sqrt{P_k} b_k e_{kn} \]  
(23)
The channel between the $m$-th antenna and the $i$-th receiver of (9) is rewritten to emphasize the small-scale time-varying part

$$h_m^i(t) = \sqrt{G_i} \sum_{l=1}^{L} \alpha_{ml}^i \delta(t - \tau_l^i)$$  \hspace{1cm} (24)

where $G_i$ is the path gain from the transmit antennas to the $i$-th user which combines the effect of path loss and shadowing, $\alpha_{ml}^i$ is the instantaneous Rayleigh fading factor of the $l$-th path from the $m$-th antenna to the $i$-th user, and $\tau_l^i$ is the path delay. Although the antenna separation causes large variation of fading for different elements, the path delay is almost the same. If the path delay is within a few chips which is much smaller than the symbol interval, we can ignore the ISI and the received signal at the $i$-th mobile is given by

$$r_i = \frac{G_i}{M} \sum_{m=1}^{M} \sum_{l=1}^{K} \sqrt{P_k} b_k \mathbf{C}_{km} h_m^i + n_i$$  \hspace{1cm} (25)

where $\mathbf{C}_{km} = [c_{km}^1, \cdots, c_{km}^L]$, which are delayed versions of $\mathbf{c}_{km}$; $h_m^i = [\alpha_{m1}^i, \cdots, \alpha_{ml}^i]^T$; and $n_i$ is the complex Gaussian noise. Denote $\mathbf{b} = [b_1, \cdots, b_K]^T$, $\mathbf{I}_{km}^i = \mathbf{C}_{km} h_m^i$, $\mathbf{I}_k^i = \sum_{m=1}^{M} \mathbf{I}_{km}^i$, and $\mathbf{L} = [\mathbf{I}_1^i, \cdots, \mathbf{I}_K^i]$, $\mathbf{P} = \sqrt{\frac{G_i}{M}} \text{diag}(\sqrt{P_1}, \cdots, \sqrt{P_K})$, we have

$$r_i = \mathbf{L}^1 \mathbf{P} \mathbf{b} + n_i$$  \hspace{1cm} (26)

Typical space-time RAKE receiver yields

$$\text{Re}(\mathbf{I}_m^i)^H r_i = \sqrt{\frac{G_i}{M}} \sqrt{P_k} \text{Re}(\mathbf{I}_m^i)^H \mathbf{I}_k^i b_k + \sum_{k 
eq i}^{K} \sqrt{\frac{G_i}{M}} \sqrt{P_k} \text{Re}(\mathbf{I}_m^i)^H \mathbf{I}_k^i b_k + \text{Re}(\mathbf{I}_m^i)^H n_i$$  \hspace{1cm} (27)

With the assumption of

$$\langle c_{km1}^{l1}, c_{k2m2}^{l2} \rangle = \begin{cases} 1 & l1 = l2, k1 = k2, m1 = m2 \\ 0 & l1 = l2, (k1,m1) \neq (k2,m2) \\ \beta & l1 \neq l2 \end{cases}$$ \hspace{1cm} (28)

where $\beta$ is a random variable with

$$E(\beta) = 0 \text{ and } E(\beta^2) = \frac{1}{N},$$ \hspace{1cm} (29)

and denote

$$A^i = \sum_{m=1}^{M} \sum_{l=1}^{L} |\alpha_{ml}^i|^2$$ \hspace{1cm} (30)

$$C^i = \frac{1}{N} \sum_{m} \sum_{m'} \sum_{l} \sum_{l'} |\alpha_{ml}^i|^2 |\alpha_{m'l'}^i|^2$$ \hspace{1cm} (31)
the SINR for user $i$ is given by

$$\text{SINR}_i = \frac{\frac{G_i}{M} P_i (A'_i)^2}{\sum_k \frac{G_k}{M} P_k C_k' + \sigma_i^2 A'_i}$$  \hspace{1cm} (32)$$

where $\sigma_i^2$ is the noise power at the user $i$ receiver.

The power control formula (16) is exemplified here with

$$D = \text{diag}(\gamma_1, \ldots, \gamma_K)$$  \hspace{1cm} (33)$$

$$F_{ij} = \begin{cases} 0 & i = j \\ \frac{C_i}{Na'} & i \neq j \end{cases}$$  \hspace{1cm} (34)$$

$$u_i = \frac{\gamma_i \sigma_i^2 M}{G_i A'_i}$$  \hspace{1cm} (35)$$

2. **Sectorization**

The co-channel interference in a cellular system may be decreased by replacing omni-directional antennas with directional antennas, each radiating within a specified sector. Sectorization usually increases users’ SINR or equivalently increases the system capacity, at the expense of increased number of antennas and decrease in trunking efficiency. We adopt an antenna radiation pattern given in [19] formulated as follows:

$$G_s(\theta) = \begin{cases} 1 - \frac{(1 - b)}{(\pi / S)^2} \theta^2 & |\theta| \leq \sqrt{\frac{1 - a}{1 - b} \frac{\pi}{S}} \\ a & \text{elsewhere} \end{cases}$$  \hspace{1cm} (36)$$

where $a$ denotes front-to-back ratio, $b$ denotes the attenuation at sector crossover, and $S$ is the number of sectors per cell. The antenna gain pattern for three and six sectors are given in Fig. 2 with $10 \log a = -15$ dB and $10 \log b = -3$ dB.

In our study, all techniques are employed in three-sector cells. We only study the transmit diversity scheme in six-sector cell case.

3. **Beamforming Techniques**

Unlike transmit diversity scheme, where antennas are separated far apart to get diversity gain, for beamforming scheme, antennas are closely spaced so that signals coming at or going from the array elements are correlated. Taking advantage of the coherent signals and underlying uncorrelated noise, antenna gain is obtained through judicious design of weighting vectors to combine or pre-
apply to the antenna array signals. Before we talk about the various beamforming techniques, let us first assume generally a set of transmit weighting vectors of \{w_i\}_{i=1}^K are adopted for the K users’ signals at the base stations. Again ignore ISI and separate the long-term fading with short-term fading, the discretized signal model of (5) can be rewritten as

\[ r_i = \sqrt{G_i} \sum_{k=1}^K \sqrt{P_k} b_k \omega_i^H \sum_{l=1}^L \alpha_d^{(l)} \alpha_d^{(l')} c_i^{(l')} + n_j \]  

(37)

let \( C_i = [c_i', \ldots, c_i''] \), \( h_i^D = [\alpha_d^{(1)}, \ldots, \alpha_d^{(L)}]^T \), and \( l_i = C_i h_i^D \), then typical space-time RAKE receiver of user \( i \) yields (we ignore the superscript \( D \) for simplicity)

\[ z_i = l_i^H r_i = \sqrt{P_i b_i \omega_i^H} \sqrt{G_i} (\sum_{l} \alpha_d^{(l)} \alpha_d^{(l')} c_i^{(l')} c_i^H a_d + \sum_{k=1}^K \sqrt{P_k b_k \omega_k^H} \sqrt{G_k} (\sum_{l} \alpha_d^{(l)} \alpha_d^{(l')} c_i^{(l')} c_i^H a_d)) + \sum_{j=1}^K \alpha_d^{(j)} (c_i^{(j)})^H n_j \]

(38)

With the assumption of

\[ \langle c_i^{(l_1)}, c_i^{(l_2)} \rangle = \begin{cases} 
1 & l_1 = l_2, k_1 = k_2 \\
0 & l_1 = l_2, k_1 \neq k_2 \\
\beta & l_1 \neq l_2 
\end{cases} \]

(39)

where \( \beta \) is a random variable defined in (28), and denote

\[ R_i = G_i \sum_{l} |\alpha_d^{(l)}|^2 a_d a_d^H \]  

(40)

\[ Q_i = \frac{1}{N} G_i \sum_{l \neq l'} |\alpha_d^{(l)}|^2 a_d a_d^H \]

(41)

the instantaneous SINR for user \( i \) is given by
The power control formula (16) is exemplified here with

\[
D = \text{diag}(\gamma_i \gamma_j \cdots \gamma_k) \text{diag}(w_i R_i w_i, \cdots, w_k R_k w_k)
\]  

(43)

\[
F_{ij} = \begin{cases} 
0 & i = j \\
\frac{w_j^H Q_i w_j}{w_i^H R_i w_i} & i \neq j 
\end{cases}
\]  

(44)

\[
u_i = \frac{\gamma_i \sigma^2 \sum_{j=1}^{K} |\alpha_{ij}|^2}{w_i^H R_i w_i}
\]  

(45)

Since the downlink fading coefficients are not known at the base station, approximation of Section II.2 is adopted and given as follows:

\[
\overline{R}_i = G_i \sum_i \sum_j E(|\alpha_{ij}|^2 | \alpha_{ij}|^2) a_i a_j^H
\]  

(46)

\[
\overline{Q}_i = \frac{1}{N} G_i \sum_i \sum_j E(|\alpha_{ij}|^2) E(|\alpha_{ij}|^2) a_i a_j^H
\]  

(47)

Based on these matrices, various beamforming schemes are illustrated below. (42)-(45) are adjusted accordingly.

1. **Beam Steering**

   This is a simple beamforming technique where transmit antenna array forms a beam in the direction of line-of-sight of the desired user, as shown in the following:

   \[
w_j = \frac{a^D(\theta_{i,\text{los}})}{\|a^D(\theta_{i,\text{los}})\|} \]

   (48)

   where \(\theta_{i,\text{los}}\) denotes the azimuth angle of the line-of-sight of the \(i\) user with antenna arrays.

2. **Maximum SNR**

   This scheme try to maximize the SNR at the \(i\) user. According to (42), it is equivalent to
It is well known that such $\mathbf{w}_i$ is given by the principal eigenvector of matrix $\mathbf{R}_i$.

3. Maximum SIR/SINR

The Max SIR scheme try to transmit as much energy as possible to the desired user while minimizing its interference to other users. It is formulated as

$$\arg\max_{\mathbf{w}_i} \frac{\mathbf{w}_i^H \mathbf{R}_i \mathbf{w}_i}{\mathbf{w}_i^H \mathbf{T}_i \mathbf{w}_i} \quad \text{with} \quad \mathbf{T}_i = \sum_{k \neq i} \mathbf{Q}_k$$

(50)

Such $\mathbf{w}_i$ is given by the generalized principal eigenvector of $[\mathbf{R}_i, \mathbf{T}_i]$. Compared to Max SNR, this criterion may lead to inadequate power being transmitted to the desired user, or equivalently, lead to increased transmitted power that results in type-II outage. Intuitively, there is no benefit putting too much emphasis on interference minimization at the cost of reduced energy to the desired user, since the noise term can not be eliminated. In the packet-switched system, our goal is to maximum the network throughputs with the maximum transmit power, so the power allocation is known in advance. In this case, Max SINR can be exploited as follows:

$$\arg\max_{\mathbf{w}_i} \frac{\mathbf{w}_i^H \mathbf{R}_i \mathbf{w}_i}{\mathbf{w}_i^H \mathbf{T}_i \mathbf{w}_i} \quad \text{with} \quad \mathbf{T}_i = \sum_{k \neq i} \mathbf{Q}_k + \frac{\sigma_i^2}{P_i} \mathbf{I}$$

(51)

which can be seen as a tradeoff between Max SNR and Max SIR scheme.

4. Joint Power Control and Maximum SINR Beamforming

The maximum SINR approach given previously is not the optimum downlink beamforming. While uplink beamforming is a decoupled problem (a chosen weight vector impact only the desired receiver), in transmit beamforming each transmit weighting affects all the receivers. So downlink beamforming should be done jointly for all users.

The joint power control and beamforming problem was first considered and solved in part in [15], [16], where uplink joint algorithm is proposed and proved to converge to the optimal solution, and feasible solution is obtained for downlink through virtual uplink construction. A complete solution to the joint optimal power control and downlink beamforming is given in [18] through normalization with the noise term:

$$\tilde{\mathbf{R}}_i = \frac{\mathbf{R}_i}{\sum_t |\alpha_{it}|^2 \sigma_i^2} \quad \text{and} \quad \tilde{\mathbf{Q}}_i = \frac{\mathbf{Q}_i}{\sum_t |\alpha_{it}|^2 \sigma_i^2}.$$  

(52)

We just briefly summarize the algorithm in the following. The reader is referred to the original papers for details.
The optimization problem is given by

$$\min_{\mathbf{w}_1, \ldots, \mathbf{w}_K} \sum_{i=1}^{K} P_i \quad \text{subject to} \quad \text{SINR}_i \geq \gamma_i \quad \text{and} \quad \|\mathbf{w}_i\| = 1$$  \hspace{1cm} (53)

with the SINR formula given by

$$\frac{P_i \mathbf{w}_i^H \bar{\mathbf{R}}_{i} \mathbf{w}_i}{\sum_{k \neq i} P_k \mathbf{w}_k^H \bar{\mathbf{Q}}_{i} \mathbf{w}_k + 1}. \hspace{1cm} (54)$$

The idea is to construct a virtual uplink problem with SINR

$$\frac{P_i \mathbf{w}_i^H \bar{\mathbf{R}}_{i} \mathbf{w}_i}{\sum_{k \neq i} P_k \mathbf{w}_k^H \bar{\mathbf{Q}}_{i} \mathbf{w}_k + \|\mathbf{w}_i\|^2}. \hspace{1cm} (55)$$

The following iterations converge to the optimal beamforming vector and power allocation from any initial values [15], [16].

1) for $1 \leq i \leq K$, $\mathbf{w}_i^n = \arg \max_{\mathbf{w}_i} \frac{\mathbf{w}_i^H \bar{\mathbf{R}}_{i} \mathbf{w}_i}{\mathbf{w}_i^H \bar{\mathbf{T}}_{i} \mathbf{w}_i}$, where $\bar{\mathbf{T}}_{i} = \sum_{k \neq i} \mathbf{p}_k^n \mathbf{Q}_k + \mathbf{I}$, with

$$\mathbf{p}_k^n = [(P_1)^n, \ldots, (P_K)^n]^T$$

collecting the power at the $n$-th iteration. This is the decentralized Max SINR scheme whose solution is the principal generalized eigenvector of $[\bar{\mathbf{R}}_{i}, \bar{\mathbf{T}}_{i}]$;

2) $\mathbf{p}_{U}^{n+1} = \bar{\mathbf{D}}^n \bar{\mathbf{F}}_{U} \mathbf{p}_{U}^n + \bar{\mathbf{u}}_{U}^n$, where we define $\bar{\mathbf{D}}^n = \text{diag}(\gamma_1/(\mathbf{w}_1^n)^H \bar{\mathbf{R}}_{1} \mathbf{w}_1^n, \ldots, \gamma_K/(\mathbf{w}_K^n)^H \bar{\mathbf{R}}_{K} \mathbf{w}_K^n)$;

$$\bar{\mathbf{F}}_{U} = \begin{cases} 0 & i = j \\ (\mathbf{w}_i^n)^H \bar{\mathbf{Q}}_{j} \mathbf{w}_j^n & i \neq j \end{cases}$$

and $(\bar{\mathbf{F}}^n)_{ij} = \begin{cases} 0 & i = j \\ (\mathbf{w}_j^n)^H \bar{\mathbf{Q}}_{i} \mathbf{w}_i^n & i \neq j \end{cases}$, i.e., $\bar{\mathbf{F}}_{U} = (\bar{\mathbf{F}}^n)^T$;

$$(\bar{\mathbf{u}}_{U})_i = \frac{\gamma_i \|\mathbf{w}_i^n\|^2}{(\mathbf{w}_i^n)^H \bar{\mathbf{R}}_{i} \mathbf{w}_i^n} = \bar{\mathbf{D}}^n \mathbf{1}_w, \text{ where } \mathbf{1}_w = [\|\mathbf{w}_1^n\|, \ldots, \|\mathbf{w}_K^n\|]^T = \mathbf{1}, \text{ and}$$

$$(\mathbf{u})_i = \frac{\gamma_i}{(\mathbf{w}_i^n)^H \bar{\mathbf{R}}_{i} \mathbf{w}_i^n} = \bar{\mathbf{D}}^n \mathbf{1} = (\bar{\mathbf{u}}_{U})_i.$$

The superscript $n$ means substituting $\mathbf{w}_i^n$ for $\mathbf{w}_i$. This is the decentralized power control solution (see (17)) when beamforming vector is fixed.

The optimal virtual uplink power vector $\mathbf{p}_U = (\mathbf{I} - \bar{\mathbf{D}} \bar{\mathbf{F}}_{U})^{-1} \bar{\mathbf{u}}_{U}$ while the downlink power vector $\mathbf{p} = (\mathbf{I} - \bar{\mathbf{D}} \bar{\mathbf{F}})^{-1} \bar{\mathbf{u}}$. Note $1^T \mathbf{p} = 1^T (\mathbf{I} - \bar{\mathbf{D}} \bar{\mathbf{F}})^{-1} \bar{\mathbf{D}} \mathbf{1} = 1^T \bar{\mathbf{D}} (\mathbf{I} - \bar{\mathbf{D}} \bar{\mathbf{F}})^{-1} \mathbf{1} = (\mathbf{p}_U)^T \mathbf{1}.$ So the optimality of $\mathbf{p}$ is guaranteed by the optimality of virtual uplink solution. It can be obtained through iteration:

1) and 2) as above, and

3) $\mathbf{p}_{U}^{n+1} = \bar{\mathbf{D}}^n \bar{\mathbf{F}}_{U} \mathbf{p}_U^n + \bar{\mathbf{u}}_U^n$. 


V. Speed-up Technique for Cellular Network Simulation

Before we present the numerical results for comparison and make conclusions, we propose a speed-up technique to ease the computational burden for network simulation. As we mentioned in Section II, the CDMA system is interference limited. For the sector of interest, the interference comes from all the surrounding cells. Since we do the power control at the base station, the out-of-cell interference is decided by the transmitted power of each cell, which could be quite different for each implementation. Furthermore, the power control procedure of each cell is intermixed with each other and the convergence may be very slow. By the assumption of the identity of all cells, our idea is illustrated as follows. We also presuppose that the other-sector interference contributes to noise term with the following form

$$\frac{1}{N} \sum_j (p) \cdot G_{ji}$$  \hspace{1cm} (56)

where \( p_j \) denotes the power vector of some interfering sector \( j \) (we assume it the same for all sectors); \( G_{ji} \) is the long term link gain between the interfering sector \( j \) and use \( i \) of the sector of interest, which includes the effect of path loss, shadowing and antenna gain pattern, but averages out the effect of small-scale fading; \( N \) is the spreading gain, and \( \frac{1}{N} \) reflects the average effect of the RAKE receiver at the mobile.

1. all cells start transmitting with maximum power \( P_{\text{max}} \), which can be seen as the worst case. After calculating the interference as (56), we do the power control as the following. We assume that the beamforming vector has been obtained through one of the scheme in Section IV.

\[ p^{(1)} = (I - DF)^{-1} u^{(0)} = (I - DF)^{-1} DC(n + \frac{1}{N} P_{\text{max}} g) \]  \hspace{1cm} (57)

where \( C \) denote some constant term, \( n = [\sigma_1^2, \cdots, \sigma_k^2]^T \) denotes the background noise, and \( g = [\sum_j G_{ji}, \cdots, \sum_k G_{jk}]^T \) denotes the link gain of the interfering sectors.

2. If

\[ \sum(p^{(1)}) > P_{\text{max}}, \]  \hspace{1cm} (58)

declare a type-II outage and stop; otherwise go to step 3.

3. Iterate as follows until it converges:

\[ p^{(m+1)} = (I - DF)^{-1} u^{(m)} = (I - DF)^{-1} DC(n + \frac{1}{N} \sum(p^{(m)}) g). \]  \hspace{1cm} (59)

For (2), it seems as a pessimistic approach since we assume the interfering cells operate at maximum transmitted power. Alternatively, one can assume the interfering cells operate at zero
power, which can be seen as the best case. In our simulation, we find that these two approaches have indistinguishable performance behavior. Another reason for our taking this approach here is that the convergence of (3) is guaranteed, which will be shown in the following.

1. The Contraction Principle

Our theoretical background is the so-called contraction principle, or Banach fixed point theorem. By a contraction mapping, we mean a mapping onto a vector space \( X \), \( f : X \rightarrow X \) satisfying

\[
d(f(x), f(y)) \leq Kd(x, y), \quad \forall x, y \in X
\]

for some constant \( 0 \leq K < 1 \), where \( d \) is a metric for space \( X \). The contraction principle is given as follows.

Suppose \( X = (S, d) \) is a complete metric space, and \( f \) is a contraction mapping on it, then there is a unique point \( x \) in \( X \) such that \( f(x) = x \). If \( x_0 \) is any point in \( X \), and \( x_n = f^n(x_0) \), then

\[
d(x_n, x_0) \leq K^n / (1 - K) \quad \text{with} \quad \lim x_n = x.
\]

2. Convergence of the Speed-up Technique

We define a metric space \( X = \{ x \in \mathbb{R}^K : 0 \leq \sum(x) \leq P \max \} \) with Euclidean distance. It is easily seen that this is a complete metric space. All we need to show is that

\[
f(x) = (I - DF)^{-1}DC(n + \frac{1}{N}\sum(x)g)
\]

is a contraction mapping on \( X \). We have

\[
\|f(x_1) - f(x_2)\| = \frac{1}{N}\| (I - DF)^{-1}DC \| \| \sum(x_1) - \sum(x_2) \|
\]

Further,

\[
|\sum(x_1) - \sum(x_2)| \leq \sqrt{K}\|x_1 - x_2\|
\]

Note that condition \( \sum(p^{(i)}) \leq P_{\max} \) ensure that mapping \( f : X \rightarrow X \), so we only need to show

\[
\frac{\sqrt{K}}{N}\| (I - DF)^{-1}DCg \| < 1
\]

We have

\[
K < N
\]
$$
\min \sum_{i,j} t_{ij} \leq r \leq \max \sum_{i,j} t_{ij}
$$

(65)

where $t_{ij}$ is the element of matrix $(I - D\bar{F})^{-1}$, $r < 1$ is the spectral radius of matrix $(I - D\bar{F})^{-1}$. (65) holds by the property (e) of the Perron-Frobenius theorem, provided there is no type-I outage. So usually we will expect the row sums of matrix $(I - D\bar{F})^{-1}$ to be less than 1. What is important is that $g$ collects the link gains of the interfering sectors, which are far less than 1 (assume mobiles already do the home cell selection). Usually $\|g\|$ is on the order of $10^{-2}$ or less (empirical statistics). Therefore, the convergence of the proposed technique is guaranteed and usually fairly fast. Hopefully the fixed point of the algorithm will represent a steady point of the cellular network and reflect the average performance of the network.

VI. Numerical Results

In this section, we examine the performance of various downlink transmission techniques discussed above through computer simulation. For the circuit-switched system, power control is carried out and outage is declared whether there is no feasible power allocation to satisfy all the link requirements (type-I) or the total transmitted power exceeds the maximum value (type-II). We will evaluate and compare the performance of these downlink transmission techniques through the metric of total outage. In all cases, at least \( \frac{10}{\text{outage}} \) simulations are run, and for outage > 1%, 5000 simulations are run. For the packet-switched system, we allow each base station to transmit at the maximum power and equally divide the power between the active users. We examine the CDF of the SINR of a typical mobile user for performance comparison since the SINR is directly related to the achievable rate of the user. We also examine the effect of the optimal power assignment scheme in [20]. 5000 simulations are run for packet case.

In our setting, the maximum transmitted power to the background noise ratio is set to be 30 dB, and we assume all the users see the same background noise, i.e., $\sigma_1^2 = \cdots = \sigma_K^2 = \sigma^2$, and $10\log_{10} \frac{P_{\text{max}}}{\sigma^2} = 30$. The path loss parameter $\eta = 4$, the standard deviation of log-normal shadowing is 8 dB. The small-scale fading coefficients are generated through TUx model used in W-CDMA 3G studies. The users are distributed uniformly within the sector of interest, with the antenna gain pattern given in Fig. 2. We assume each user has three multipaths, i.e., $L_1 = \cdots = L_K = L = 3$, the angle of which is Gaussian distributed around the direction of line-of-sight, with standard deviation of 10 degrees. The spreading gain is $N = 64$, and the number of users per sector $K$ varies from 2 to 50. The number of antennas $M$ in our study is 2, 4, or 8 per sector. We assume each cell has three 120-degree sectors unless noted. When studying the transmit diversity scheme in six sector case, we assume the total number of users and antennas per cell keep constant. So in six-sector scenario, the number of users and antennas per sector is reduced to one half.

1. Circuit-Switched System
Fig. 3 to Fig. 5 presents the performance of six transmission techniques combined with power control for CDMA downlink, namely, transmit diversity, transmit diversity with sectorization, Max SNR beamforming, Max SIR beamforming, beam steering, and joint power control and (Max SINR) beamforming. We assume no feedback from the mobile, while the loss due to without feedback is also examined in the following. For the sake of comparison, the number of users that can be supported in one cell with 5% outage is given in Table 1.

From above data, several conclusions can be made for CDMA downlink circuit-switched system.

- We note that Max SNR beamforming almost approaches the optimal performance (that of joint power control and Max SINR beamforming) in the outage range of interest, while with much lower complexity.

- Max SIR has totally unacceptable performance and thus is omitted in Table 1. As we said before, putting too much emphasis on minimizing the interference to other users will hurt the desired energy, so more power has to be assigned to achieve the SINR threshold, resulting in type-II outage. Another problem of Max SIR is due to insufficient degree of freedom the antenna array can offer compared to the number of users.

- Beam steering has good performance only when the antenna elements is small (M=2); the gap between beam steering and Max SNR beamforming enlarges as M increases.

- For transmit diversity, sectorization significantly improves the performance (6 to 30 users more as M goes from 2 to 8 with 5% outage); but Max SNR beamforming technique still outperforms the six sector transmit diversity scheme (6 to 12 users more as M goes from 2 to 8 with 5% outage).

Fig. 6 to Fig. 10 show the performance of the four transmission techniques as number of antennas per sector varies from 2, 4 to 8. Max SIR beamforming is not of interest due to its unacceptable performance. The performance of joint power control and beamforming is similar to that of Max SNR and omitted here.

From these figures, we see that

- For transmit diversity, the gain through exploiting more antennas diminishes. This result is also obtained in [10].

- For Max SNR beamforming the gain through exploiting more antenna elements doesn’t diminish but is restricted due to imperfect channel knowledge; on the other hand, if we assume ideal feedback, the gain through exploiting more antenna elements increases.

- For beam steering we observe an interesting phenomenon: the performance improves from \( M = 2 \) to \( M = 4 \), but deteriorates as \( M \) further increases. One possible explanation is that beam steering scheme forms a beam toward the physical position of the mobile. Due to the angle spread model we use, it certainly points to wrong directions. As more antennas used, more precise calibration of line-of-sight actually means greater angle estimation errors. This effect will counteract the benefit of antenna gains with more antennas.
Fig. 3 Performance Comparison of Various Transmission Techniques with $M = 2$ Antennas per Sector (6 Antennas per Cell) — Circuit-Switched System

Fig. 4 Performance Comparison of Various Transmission Techniques with $M = 4$ Antennas per Sector (12 Antennas per Cell) — Circuit-Switched System
Table 1 Number of Users Supported in a Cell with 5% Outage

<table>
<thead>
<tr>
<th>Transmit Diversity</th>
<th>Six Sector</th>
<th>Beam Steering</th>
<th>Max SNR</th>
<th>Max SNR with feedback</th>
<th>Joint</th>
<th>Joint with feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>36</td>
<td>39</td>
<td>42</td>
<td>42</td>
<td>42</td>
<td>42</td>
</tr>
<tr>
<td>36</td>
<td>57</td>
<td>51</td>
<td>66</td>
<td>72</td>
<td>66</td>
<td>72</td>
</tr>
<tr>
<td>39</td>
<td>69</td>
<td>12</td>
<td>81</td>
<td>117</td>
<td>84</td>
<td>117</td>
</tr>
</tbody>
</table>

Finally, Fig. 11 to 13 compares the Max SNR beamforming with and without feedback with \( M = 2, 4 \) and 8. Again those of Max SIR scheme are not of interest and those of joint power control and beamforming scheme are similar and omitted. We note that the gap between that of no feedback and that with feedback increases as the number of antenna increases, but for small number of antennas (\( M = 2, 4 \)), the loss due to approximation of channel parameters is insignificant. That means that for small number of antennas, Max SNR beamforming is the best choice even without feedback information.
Fig. 6 Performance of Transmit Diversity with 2, 4 and 8 Antennas per Sector

Fig. 7 Performance of Transmit Diversity with Sectorization with 2, 4 and 8 Antennas per Sector
Fig. 8 Performance of Max SNR Beamforming without Feedback with 2, 4 and 8 Antennas per Sector

Fig. 9 Performance of Max SNR Beamforming with Feedback with 2, 4 and 8 Antennas per Sector
Fig. 10 Performance of Beam Steering with 2, 4 and 8 Antennas per Sector

Fig. 11 Performance of Max SNR Beamforming with 2 Antennas per Sector: With and Without Feedback Channel Information
Fig. 12 Performance of Max SNR Beamforming with 4 Antennas per Sector: With and Without Feedback Channel Information

Fig. 13 Performance of Max SNR Beamforming with 8 Antennas per Sector: With and Without Feedback Channel Information
2. **Packet-Switched System**

As a counterpart to the circuit-switched case, Fig. 14 to Fig. 16 presents the performance of six transmission techniques for packet-switched system. Because we don’t have power control in this case, we study the optimal power assignment combined with Max SNR instead. We do the equal power assignment unless noted. For the sake of comparison, the mean (50% CDF) and peak (90% CDF) SINR value of a typical user are given in Table 2 and 3, respectively.

Note that for $M=8$ and $K=4$ case, the simultaneously transmitted users are doubled. One should consider this in an effort of translating SINR to achievable rates and network throughput. (f) means with feedback channel parameter information.

From above data, several conclusions can be made for CDMA downlink packet-switched system.

- **Optimal power allocation scheme** has no benefit in packet-switched system. We see from Fig. 14 – 16 that, the Max SNR with optimal power allocation, compared with Max SNR with equal power allocation, favors low-rate (low SINR) users but harms high-rate (high SINR) users. As we discussed in Section III, the optimal power allocation is like a socialism scheme which seeks for absolute fairness. It can not achieve best throughput and, without jointly considered with link and network schedule, can not guarantee fairness either. The similar phenomenon can be observed for Max SINR scheme and is omitted here.

- **Contrary to circuit case**, Max SINR beamforming has the best performance in peak rate; it is also good at mean rate with small number of users, while comparable with others with more users communicated at the same time.

- **Max SNR beamforming** is almost the best in mean rate; it is also good at peak rate performance.

- **Beam steering** is almost as good as max SNR in peak rate performance, while a little worse (1 dB) at mean rate performance.

- **For transmit diversity**, sectorization significantly improves the performance (4-7 dB).

- **Max SNR beamforming technique outperforms** the six sector transmit diversity scheme for $M=8$ (1dB in mean and 2-3 dB in peak); for $M=4$, six sector transmit diversity is better.

Fig. 17 to Fig. 19 compares the Max SNR beamforming with and without feedback, while Fig. 20 to 22 shows the Max SINR case. We find that Feedback doesn’t help much for beamforming technique in packet-switched system.
Fig. 14 Performance Comparison of Various Transmission Techniques with $M = 4$ Antennas and 2 Active users — Packet-Switched System

Fig. 15 Performance Comparison of Various Transmission Techniques with $M = 8$ Antennas and 2 Active users — Packet-Switched System
Fig. 16 Performance Comparison of Various Transmission Techniques with $M = 8$ Antennas and 4 Active users — Packet-Switched System

Table 2 Mean SINR (50% CDF) of a typical mobile user (dB)

<table>
<thead>
<tr>
<th>Transmit Diversity</th>
<th>Six Sector</th>
<th>Beam Steering</th>
<th>Max SNR</th>
<th>Max SNR(f)</th>
<th>Max SINR</th>
<th>Max SINR(f)</th>
<th>Opt PA &amp; Max SNR</th>
<th>Opt PA &amp; Max SNR(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M=4$ &amp; $K=2$</td>
<td>8</td>
<td>11.8</td>
<td>10.8</td>
<td>11</td>
<td>11.1</td>
<td>10.6</td>
<td>10.8</td>
<td>10.7</td>
</tr>
<tr>
<td>$M=8$ &amp; $K=2$</td>
<td>8</td>
<td>12</td>
<td>11.4</td>
<td>12.4</td>
<td>12.7</td>
<td>13.2</td>
<td>13.7</td>
<td>11.9</td>
</tr>
<tr>
<td>$M=8$ &amp; $K=4$</td>
<td>4.2</td>
<td>7.2</td>
<td>7.6</td>
<td>8.5</td>
<td>8.8</td>
<td>6.4</td>
<td>7.2</td>
<td>7.7</td>
</tr>
</tbody>
</table>
Table 3 Peak SINR (90% CDF) of a typical mobile user (dB)

<table>
<thead>
<tr>
<th>Transmit Diversity</th>
<th>Six Sector</th>
<th>Beam Steering</th>
<th>Max SNR</th>
<th>Max SNR(f)</th>
<th>Max SINR</th>
<th>Max SINR(f)</th>
<th>Opt PA &amp; Max SNR</th>
<th>Opt PA &amp; Max SNR(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>11</td>
<td>17.5</td>
<td>15.5</td>
<td>15.7</td>
<td>21.6</td>
<td>21.6</td>
<td>13.8</td>
<td>13.8</td>
</tr>
<tr>
<td></td>
<td>10.8</td>
<td>17.2</td>
<td>19.3</td>
<td>19.5</td>
<td>24.3</td>
<td>24.6</td>
<td>16.8</td>
<td>17.1</td>
</tr>
<tr>
<td></td>
<td>6.8</td>
<td>10</td>
<td>12.7</td>
<td>13.3</td>
<td>18.8</td>
<td>19.3</td>
<td>10.3</td>
<td>10.6</td>
</tr>
</tbody>
</table>

Fig. 17 Performance of Max SNR Beamforming with 4 Antennas and 2 active users: With and Without Feedback Channel Information
Fig. 18 Performance of Max SNR Beamforming with 8 Antennas and 2 active users: With and Without Feedback Channel Information

Fig. 19 Performance of Max SNR Beamforming with 8 Antennas and 4 active users: With and Without Feedback Channel Information
Fig. 20 Performance of Max SINR Beamforming with 4 Antennas and 2 active users: With and Without Feedback Channel Information

Fig. 21 Performance of Max SINR Beamforming with 8 Antennas and 2 active users: With and Without Feedback Channel Information
VII. Conclusions and Future Work

We see that Traffic type impacts algorithm choice. For circuit-switched downlink CDMA system, Max SNR beamforming scheme is the best choice (accommodate 12 to 42 more users than transmit diversity). For packet-switched system, Max SINR Beamforming has the best performance in peak rate (10-14 dB more than transmit diversity); Max SNR beamforming is almost the best in mean rate (3-4 dB more than transmit diversity), but beam steering and transmit diversity with sectorization are also good choices. We also see that sectorization greatly improves the system performance, both for the circuit and for the packet case. For transmit diversity, the gain through exploiting more antennas diminishes, while it is not the case for beamforming technique, especially with feedback channel parameters.

The following ideas are what we think deserved further study.

- Incorporate parameter estimation error in covariance matrix calculation;
- Combine link and network schedule with transmission techniques in packet-switched system;
- Max SNR techniques for widely separated antennas;
- Antenna array processing combined with sectorization less than 60°, where physical implementation of half wavelength spacing and directional gain pattern should be considered.
VIII. Bibliography