MA 587: Homework assignment #1, due January 29.

**Goal:** Get more familiar with elementary Finite Elements

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**Problem 1** Consider the chord problem

\[-u''(x) = f(x) \quad \forall x \in (0,1),
\]

\[u(0) = u(1) = 0.\]

1. Discretize the problem using the “hat functions” seen in class. Show that

\[V_h = \text{span}\{\psi_1, \ldots, \psi_N\} = \{w; w \in C^0(0,1), w(0) = w(1) = 0, w\big|_{(x_i, x_{i+1})} \in P_1, i = 1, \ldots, N+1\}.\]

2. Compute the stiffness matrix.

3. Write a code allowing for arbitrary mesh and solve a test problem of your choice. Investigate the order of convergence as \(h = \max_{i=1, \ldots, N} h_i\) goes to 0.

4. Repeat the exercise with piecewise parabolic elements. (Bonus)

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**Problem 2** Consider the beam problem

\[-u^{(IV)}(x) = f(x) \quad \forall x \in (0,1),
\]

\[u(0) = u(1) = 0,
\]

\[u'(0) = u'(1) = 0.\]

1. Give a variational formulation of the problem

2. Propose and fully describe a Finite Element method to solve the problem based the use of continuous piecewise polynomial functions of degree 3 (give chosen basis function and expression of the stiffness matrix).