Xenon and Samarium are fission products with significant neutron absorption cross sections (2 x 10^6 and 58,500 barns respectively). The time varying concentrations of these fission products can significantly affect reactor operations. Xenon is radioactive and is produced both by fission and decay of radioactive Iodine. Xe reaches equilibrium in around 40 – 50 hours, and as such can affect reactor behavior in the short term. Samarium is stable and is produced by the decay of radioactive Promethium. Samarium reaches equilibrium in about 20 days, so its short term behavior is less dramatic than that of Xenon, but still must be accounted for in reactivity calculations.

Assuming a neutron flux of \( \phi_0 = 4.0 \times 10^{13} \) n/cm²-sec and a macroscopic fission cross section of 0.282 cm⁻¹, compute and plot the concentrations of Iodine, Xenon, Promethium and Samarium for the following scenarios:

a) Startup, Shut down and Restart from cold clean conditions

\[
\begin{align*}
\phi(t) & = \begin{cases} 
\phi_0 & 0 \leq t < 48 \text{ hrs} \\
0 & 48 \leq t < 96 \text{ hrs} \\
\phi_0 & 96 \leq t < 144 \text{ hrs} 
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\phi(t) & = \begin{cases} 
\phi_0 & 0 \leq t < 480 \text{ hrs} \\
0 & 480 \leq t < 960 \text{ hrs} \\
\phi_0 & 960 \leq t < 1444 \text{ hrs} 
\end{cases}
\end{align*}
\]

b) Load Follow from Equilibrium conditions

\[
\phi(t) = \phi_0 \left(1 + 0.25 \sin \left( \frac{\pi t}{12} \right) \right)
\]

for \( t \in [0, 96] \) hours.

Since reactivity is directly related to the number density of the fission product poison, comment on the magnitude of the reactivity changes associated with these scenarios.
The equations governing the time dependent Xenon and Samarium concentrations are

**Xenon**

Iodine Concentration

\[
\frac{dI}{dt} = \gamma_I \sum \phi(t) - \lambda_I I
\]

Xenon Concentration

\[
\frac{dX}{dt} = \gamma_X \sum \phi(t) + \lambda_X I - \lambda_X X - \sigma_X^X \phi(t) X
\]

**Samarium**

Promethium Concentration

\[
\frac{dP}{dt} = \gamma_P \sum \phi(t) - \lambda_P P
\]

Samarium Concentration

\[
\frac{dS}{dt} = \lambda_P P - \sigma_s^S \phi(t) S
\]

From Duderstadt and Hamilton, the effective yields and decay constants are

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Fission Yield</th>
<th>Decay Constant (hrs(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iodine</td>
<td>(\gamma_I = 6.386 \times 10^{-2})</td>
<td>(\lambda_I = 0.1035)</td>
</tr>
<tr>
<td>Xenon</td>
<td>(\gamma_X = 0.228 \times 10^{-2})</td>
<td>(\lambda_X = 0.0753)</td>
</tr>
<tr>
<td>Promethium</td>
<td>(\gamma_P = 1.13 \times 10^{-2})</td>
<td>(\lambda_P = 0.0128)</td>
</tr>
</tbody>
</table>

a) Startup, Shut down and Restart from cold clean conditions

**Xenon**

\[
\phi(t) = \begin{cases} 
\phi_o & 0 \leq t < 48 \text{ hrs} \\
0 & 48 \leq t < 96 \text{ hrs} \\
\phi_o & 96 \leq t < 144 \text{ hrs}
\end{cases}
\]

For cold, clean conditions, the initial Iodine and Xenon concentrations are zero. For the power profile given, the Iodine and Xenon concentrations as a function of time are given below.
Similarly, for cold, clean conditions, the initial Promethium and Samarium concentrations are zero. For the power profile given, the Promethium and Samarium concentrations as a function of time are given below.

\[ \phi(t) = \begin{cases} \phi_i & 0 \leq t < 480 \text{ hrs} \\ 0 & 480 \leq t < 960 \text{ hrs} \\ \phi_i & 960 \leq t < 1444 \text{ hrs} \end{cases} \]
b) Load Follow from Equilibrium conditions

\[ \phi(t) = \phi_e \left( 1 + 0.25 \sin \left( \frac{\pi t}{12} \right) \right) \]

for \( t \in [0, 96] \) hours.

**Xenon**

The equilibrium concentrations of Iodine and Xenon are given by

\[ I_e = \frac{\gamma_i \Sigma_f \phi_e}{\lambda_i} \]

\[ X_e = \frac{(\gamma_i + \gamma_x) \Sigma_f \phi_e}{\lambda_x + \sigma_x \phi_e} \]

For the power profile given above, the Iodine and Xenon concentrations as a function of time are illustrated below.
Samarium

The equilibrium concentrations of Promethium and Samarium are given by

\[ P_{\infty} = \frac{\gamma_r \Sigma_r \phi_0}{\lambda_p} \]

\[ S_{\infty} = \frac{\gamma_r \Sigma_f}{\sigma_s} \]

For the power profile given above, the Promethium and Samarium concentrations as a function of time are illustrated below.
For the shutdown and restart scenarios, the reactivity associated with the buildup of Xenon and Samarium is nearly twice their equilibrium values at their peak. Being radioactive, Xenon decays away in a relatively short time. Samarium being stable remains at its elevated level until restart. For the load follow scenario, Xenon varies approximately 30% over a 24 hour cycle. The variation in the Samarium concentration is much less at about 2% over a 24 hour cycle.