A four loop Pressurized Water Reactor is designed with the operating characteristics given below.

### Problem Data

- **Fuel Melt Temperature**: 4980 F
- **Core Thermal Output**: 3411 Mw
- **Core mass flux**: $2.62 \times 10^6$ lbm/hr-ft$^2$
- **Fuel Height**: 144 inches
- **Rod Pitch**: 0.496 inches
- **Outer Clad Diameter**: 0.374 inches
- **Pellet Diameter**: 0.3225 inches
- **Clad Thickness**: 0.0225 inches
- **Clad Thermal Conductivity**: 9.6 Btu/hr-Ft-F
- **Gap Conductance**: 1000 Btu/hr-ft$^2$-F
- **Power Peaking Factor**: 2.32
- **Core Averaged Fluid Temperature**: 590 F
- **Number of Fuel Assemblies**: 193
- **Fuel Assembly Array**: 17 x 17
- **Water Rods per Assembly**: 25
- **Energy Deposited in Fuel**: 97.4 %

a) A number of potential accidents in pressurized water reactors can lead to over power transients where the reactor power exceeds normal operating levels. One concern in setting the high power reactor trip is that the maximum fuel temperature remain below the fuel melt temperature. For the given data, determine the reactor trip set point such that $\frac{Q_{\text{melt}}}{Q_{\text{trip}}} < 1.2$, where $Q_{\text{trip}}$ is the trip set point and $Q_{\text{melt}}$ is the power level associated with the melt temperature. What is the fuel centerline temperature at the trip set point? The volumetric heat generation rate may be assumed uniform, and you can assume fully developed nucleate boiling at the point of maximum heat flux. The fuel thermal conductivity as a function of temperature can be taken to be that of unirradiated UO$_2$.

b) What would be the difference in the set point value if single phase forced convection were assumed at the position of maximum heat flux?

c) Under nominal operating conditions, what is the maximum linear heat rate and maximum fuel centerline temperature for this reactor? You can again assume fully developed nucleate boiling at the point of maximum heat flux.

d) What is the H/D for this reactor? If the same fuel bundles were to be used, how many would be required for a three loop system operating at 2560 Mw and the same maximum linear heat rate? What would be the resulting H/D for this core?

e) If the same lattice design, i.e. S/D and 17 x 17 array, were to be used for an SMR operating at 160 Mw, what would be the number of assemblies and core height to maintain the same maximum linear heat rate assuming an H/D of 0.9? Your fuel height should be in increments of 6 inches. What would the core loading pattern look like (how would the assemblies be arranged) to achieve an approximately cylindrical core?

For all solutions, you should report integer numbers of rods and assemblies.
SOLUTIONS

a) A number of potential accidents in pressurized water reactors can lead to over power transients where the reactor power exceeds normal operating levels. One concern in setting the high power reactor trip is that the maximum fuel temperature remain below the fuel melt temperature. For the given data, determine the reactor trip set point such that \( \frac{Q_{\text{melt}}}{Q_{\text{trip}}} < 1.2 \), where \( Q_{\text{trip}} \) is the trip set point and \( Q_{\text{melt}} \) is the power level associated with the melt temperature. What is the fuel centerline temperature at the trip set point? The volumetric heat generation rate may be assumed uniform, and you can assume fully developed nucleate boiling at the point of maximum heat flux. The fuel thermal conductivity as a function of temperature can be taken to be that of unirradiated UO2.

The general solution for the temperature drop across the fuel region of a solid cylindrical fuel rod with temperature dependent thermal conductivity is

\[
\int_{T(0)}^{T(R)} k(T) dT + \int_0^R \frac{1}{r'} \int_0^{r'} q''(r) r^2 dr' dr = 0
\]

for

\[
q''(r) = q''
\]

\[
\int_0^{r'} q''(r') r^2 dr' = \int_0^{r'} q'' r^2 dr' = \frac{q' r'^2}{2}
\]

Note:

\[
q' = q'(R_o) 2\pi R_o = \int_0^R q''(r) 2\pi r^2 dr' = q'' \pi R^2
\]

\[
\int_0^R \frac{1}{r'} \int_0^{r'} q''(r') r^2 dr' = \int_0^R \frac{q'' r'}{2} dr' = \frac{q' R^2}{4}
\]

Therefore

\[
\int_{T(0)}^{T(R)} k(T) dT + \frac{q'}{4\pi} = 0
\]

For
\[
K(T) = \frac{3978.1}{692.6 + T} + 6.02366 \times 10^{-12} (T + 460)^3
\]

\[
\int_{T(0)}^{T(R)} k(T) dT = \int_{T(0)}^{T(R)} \left[ \frac{3978.1}{692.6 + T} + 6.02366 \times 10^{-12} (T + 460)^3 \right] dT
= 3978.1 \ln \left( \frac{692.6 + T(R)}{692.6 + T(0)} \right) + \frac{6.02366 \times 10^{-12}}{4} \left[ (T(R) + 460)^4 - (T(0) + 460)^4 \right]
\]
such that
\[
3978.1 \ln \left( \frac{692.6 + T(R)}{692.6 + T(0)} \right) + \frac{6.02366 \times 10^{-12}}{4} \left[ (T(R) + 460)^4 - (T(0) + 460)^4 \right] + \frac{q'}{4\pi} = 0 \quad (a)
\]

In addition we have
\[
T(R) - T_{cw} = q'(R) R \left( \frac{1}{H_{G}R_i} + \frac{1}{k_c} \ln \left( \frac{R_o}{R_i} \right) + \frac{1}{h_c R_o} \right)
\]

or
\[
T(R) - T_{cw} = \frac{q'}{2\pi} \left( \frac{1}{H_{G}R_i} + \frac{1}{k_c} \ln \left( \frac{R_o}{R_i} \right) + \frac{1}{h_c R_o} \right) \quad (b)
\]

if heat transfer at the surface is due to single phase forced convection, and
\[
T_{cw} = T_{sat} + \frac{q'}{2\pi R_c} \left( \frac{q'}{2\pi R_c \times 10^5} \right)^{1/4} \quad (c)
\]

\[
T(R) - T_{cw} = \frac{q'}{2\pi} \left( \frac{1}{H_{G}R_i} + \frac{1}{k_c} \ln \left( \frac{R_o}{R_i} \right) \right) \quad (d)
\]

if heat transfer at the rod surface is by fully developed nucleate boiling.

For a given fuel centerline temperature, Equations (a), (c) and (d) contain the single unknown \( q' \) which may be solved for iteratively.

For the fuel centerline temperature equal to the melt temperature
\[
q' = 63,126.97 \text{ Btu/hr - ft} = 18.5 \text{ kW/ft} = q'_{melt}
\]

The number of fuel rods in the reactor is
\[
n = N_{assemblies} \times (N_{array} - N_{H2O}) = 193 \times (289 - 25) = 50,952
\]

which gives for the reactor trip set point
The fuel centerline temperature corresponding to \( q'_{\text{trip}} = \frac{q'_{\text{melt}}}{1.2} \) is 52,605.8 Btu/hr-ft is 4259.2 F

b) What would be the difference in the set point value if single phase forced convection were assumed at the position of maximum heat flux?

The convective heat transfer coefficient is given by the Weisman Correlation

\[
h_c = \frac{k}{D_x} \times C \times \left( \frac{G D_x}{\mu} \right)^{0.8} \left( \frac{C_p \mu}{k} \right)^{1/3}
\]

where for square lattices

\[
C = 0.042 \times (S / D_x) - 0.024 = 0.042 \times (0.496 / 0.374) - 0.024 = 0.0317
\]

Taking the fluid properties at the average coolant temperature

\[
C_p = 1.371
\]

\[
\mu = 0.2046
\]

\[
k = 0.3136
\]

\[
D_x = \frac{4[S^2 - \pi D_x^2 / 4]}{\pi D_x} = \frac{4[0.496^2 - \pi \times 0.374^2 / 4]}{\pi \times 0.374} = 0.4636 \text{ inches} = 0.03863 \text{ ft}
\]

The convective heat transfer coefficient is

\[
h_c = \frac{k}{D_x} \times C \times \left( \frac{G D_x}{\mu} \right)^{0.8} \left( \frac{C_p \mu}{k} \right)^{1/3} = 8,909.1
\]

For a given fuel centerline temperature, Equations (a) and (b) contain the single unknown \( q' \) which may be solved for iteratively.

For the given data

\[
q' = 62,128.12 \text{ Btu/hr - ft} = 18.5 \text{ kw/ft} = q'_{\text{melt}}
\]

which gives the same reactor trip set point

c) Under nominal operating conditions, what is the maximum linear heat rate and maximum fuel centerline temperature for this reactor? You can again assume fully developed nucleate boiling at the position of maximum heat flux.

Under nominal conditions, the maximum linear heat rate is given by
Assuming fully developed nucleate boiling at the rod surface, the corresponding outer clad, fuel surface and fuel centerline temperatures are given by

\[
T_{oc} = T_{os} + \xi q' \left( \frac{q'}{2\pi R_c \times 10^6} \right)^{1/\nu} = 660.7 \, \text{F}
\]

\[
T(R) - T_{os} = \frac{q'}{2\pi} \left( \frac{1}{H_c R_i} + \frac{1}{k_c} \ln \left( \frac{R_o}{R_i} \right) \right) = 1251.6 \, \text{F}
\]

\[
3978.1 \ln \left( \frac{692.6 + T(R)}{692.6 + T(0)} \right) + 6.02366 \times 10^{-12} \left\{ \left[ T(R) + 460 \right]^4 - \left[ T(0) + 460 \right]^4 \right\} + \frac{q'}{4\pi} = 0
\]

\[
T(0) = 3506.4 \, \text{F}
\]

d) **What is the H/D for this reactor?**

The area of the core is given by

\[
A_{core} = N_{assemblies} \times N_{array} \times S^2 = 95.29 \, \text{ft}^2
\]

The effective core diameter is then

\[
D_{eff} = \sqrt{\frac{4A_{core}}{\pi}} = 11.01 \, \text{ft}
\]

giving for the height to diameter ratio

\[
\frac{H}{D_{eff}} = 1.089
\]

**If the same fuel bundles were to be used, how many would be required for a three loop system operating at 2560 Mw and the same maximum linear heat rate?**

The number of fuel rods is given in terms of the number of assemblies by

\[
n = N_{assemblies} \times (N_{array} - N_{H/O})
\]

such that

\[
N_{assemblies} = \frac{\dot{Q}r'}{q' \times H(N_{array} - N_{H/O})}
\]

For a reactor thermal output of 2560 Mw, and a maximum linear heat rate of 12.61 kw/ft, the number of assemblies required is
\[ N_{\text{assemblies}} = 144.85 \rightarrow 145 \text{ since only integer numbers of assemblies are allowed.} \]

**What would be the resulting H/D for this core?**

The area of this core is then
\[ A_{\text{core}} = N_{\text{assemblies}} \times N_{\text{array}} \times S^2 = 71.59 \text{ ft}^2 \]

The effective core diameter is
\[ D_{\text{eff}} = \sqrt{\frac{4A_{\text{core}}}{\pi}} = 9.55 \text{ ft} \]

giving for the height to diameter ratio
\[ \frac{H}{D_{\text{eff}}} = 1.26 \]

e) If the same lattice design, i.e. S/D and 17 x 17 array, were to be used for an SMR operating at 160 Mw, what would be the number of assemblies and core height to maintain the same maximum linear heat rate assuming an H/D of 0.9? You’re fuel height should be in increments of 6 inches.

For a target H/D = \( \xi = 0.9 \)

The number of fuel rods is given in terms of the number of assemblies by
\[ n(N_{\text{assemblies}}) = N_{\text{assemblies}} \times (N_{\text{array}} - N_{H/D}) \]

The core area can be written in terms of the number of assemblies as
\[ A_{\text{core}}(N_{\text{assemblies}}) = N_{\text{assemblies}} \times N_{\text{array}} \times S^2 \]

and the effective core diameter as a function of the number of assemblies is
\[ D_{\text{eff}}(N_{\text{assemblies}}) = \sqrt{\frac{4A_{\text{core}}(N_{\text{assemblies}})}{\pi}} \]

The core height as a function of the number of assemblies is given in terms of the effective core diameter and the target height to diameter ratio
\[ H(N_{\text{assemblies}}) = \xi \times D_{\text{eff}}(N_{\text{assemblies}}) \]

For a given core thermal output and maximum linear heat rate, the number of assemblies is the iterative solution of
\[ q'_{\text{max}} = \frac{\dot{Q}_{\text{r}}}{n(N_{\text{assemblies}})H(N_{\text{assemblies}})} \]
Four solutions are considered:

**Solution 1**

For a maximum linear heat rate of \( q'_{\text{max}} = 12.61 \text{ Kw/ft} \) and a target height to diameter ratio of \( \xi = 0.9 \), solution of

\[
q'_{\text{max}} = \frac{\dot{Q}/F_q}{n(N_{\text{assemblies}})H(N_{\text{assemblies}})}
\]

yields \( N_{\text{assemblies}} = 28.5 \Rightarrow 29 \) and number of fuel rods \( n_{\text{rods}} = 7,656 \).

The corresponding core height for \( \xi = 0.9 \) is \( H = 3.84 \Rightarrow H_{\text{act}} = 4 \text{ ft} \). The actual maximum linear heat rate and height to diameter ratio based on these dimensions is

\[
\left( q'_{\text{max}} \right)_{\text{act}} = 11.80 \text{ Kw/ft}
\]

\[
\left( \frac{H}{D_{\text{eff}}} \right)_{\text{act}} = 0.937
\]

A measure of the fuel volume is the number of rods times the height. For these dimensions

\[
n_{\text{rods}} \times H_{\text{act}} = 30,624.
\]

**Solution 2**

Assuming \( N_{\text{assemblies}} = 29 \) and \( n_{\text{rods}} = 7,656 \), the fuel height corresponding to a maximum linear heat rate of

\[
q'_{\text{max}} = 12.61 \text{ Kw/ft}
\]

is

\[
H = \frac{\dot{Q}/F_q}{n_{\text{rods}}q'_{\text{max}}} = H = 3.75 \Rightarrow H_{\text{act}} = 4 \text{ ft}
\]

which gives the same values as Solution 1 for the actual maximum linear heat rate, height to diameter ratio and fuel volume.

**Solution 3**

Assuming \( N_{\text{assemblies}} = 29 \), \( n_{\text{rods}} = 7,656 \) and \( H = 3.5 \) the corresponding maximum linear heat rate is

\[
q'_{\text{max}} = 13.49 \text{ Kw/ft}
\]

which violates the 12.61 limit, and is therefore not a valid solution.

**Solution 4**
For this solution, the number of assemblies is rounded down, i.e. \( N_{\text{assemblies}} = 28.5 \Rightarrow 28 \) and number of fuel rods \( n_{\text{rods}} = 7,392. \)

The corresponding core height for \( \zeta = 0.9 \) is \( H = 3.78 \Rightarrow H_{\text{act}} = 4 \) ft. The actual maximum linear heat rate based on these dimensions is

\[
(q'_{\text{max}})_{\text{act}} = 12.23 \text{ Kw/ft}
\]

which satisfies the \( q'_{\text{max}} = 12.61 \text{ Kw/ft} \) limit and as such is a viable solution. The height to diameter ratio is

\[
\left( \frac{H}{D_{\text{eff}}}_{\text{act}} \right) = 0.953
\]

The corresponding fuel volume metric is 29,568 ft.

Solutions 1 and 4 are both viable, in that they both produce designs that do not exceed the maximum linear heat rate limit of \( q'_{\text{max}} = 12.61 \text{ Kw/ft} \). Solution 4 has values closer to the targets, and a lower fuel volume. Solution 1 has a lower max linear heat rate, and therefore lower maximum fuel temperatures, but at the cost of an increased fuel volume. This solution would only be preferable if the price penalty for the increased amount of fuel were offset by the reduction in corrosion as a result of lower temperatures.

**Loading Pattern**

It is virtually impossible to create a loading pattern from 28 or 29 assemblies that resembles a cylinder. Pictured below is an example with 29 assemblies.

The above diagram is much closer to a parallelepiped than a cylinder. The minimum number of assemblies that realistically approaches a cylinder is 37 as illustrated below.