Example:

The NCSU PULSTAR is a pool type research reactor with the top of the core located approximately 20 feet below the pool surface. A number of beam tubes penetrate the biological shield approximately 1 foot below the top of the core, which if broken provide the opportunity for the core coolant (and shielding) to escape. Assuming the pool surface to be 100 square feet, the beam tubes to be 1 foot in diameter and 8 feet long, and a break diameter of 1 inch in diameter, determine the time to uncover the core. Note, this example is identical to the drain down of a fuel storage pool.

Figure 1: Schematic of the PULSTAR pool

Solution:

If the top of the core is at elevation $H_0$, application of the mass conservation equation to the pool volume above the core gives

$$\frac{dM}{dt} = -\dot{m}_{\text{break}}$$

where

$$M = \rho A_{\text{pool}} (H_1 - H_0)$$

and

$$\dot{m}_{\text{break}} = \rho v_{\text{break}} A_{\text{break}}$$

For a constant pool density

$$\frac{dM}{dt} = \rho A_{\text{pool}} \frac{d(H_1 - H_0)}{dt} = -\rho v_{\text{break}} A_{\text{break}}$$

To determine the fluid velocity at the break, we apply Bernoulli’s equation between the pool surface and the break. This assumes a pseudo steady-state solution for the momentum equation and implies the temporal behavior of the solution is governed by the mass conservation equation, a reasonable assumption.
\[
\frac{P_1}{\rho} + \frac{v_1^2}{2g_c} + \frac{g}{g_c} H_1 = \frac{P_2}{\rho} + \frac{v_2^2}{2g_c} + \sum_{i} \frac{f_i D_i}{2g_c} + \sum_{j} K_j \frac{v_j^2}{2g_c} + \frac{g}{g_c} H_2
\]  

(5)

where location 1 is the pool surface and location 2 is the break. We make the following simplifications and assumptions:

1) \(P_1 = P_2 = P_{\text{amb}}\) (Free Jet Condition)

2) \(v_1\) is negligible relative to \(v_2\) \((v_1 \approx v_2 \frac{A_2}{A_1} \text{ and } A_1 \gg A_2)\)

3) \(H = H_1 - H_2\)

4) Wall friction is negligible relative to the forms losses

which gives for Bernoulli's equation

\[
\frac{g}{g_c} H = \frac{v_2^2}{2g_c} + \sum_{j} K_j \frac{v_j^2}{2g_c}
\]

(6)

Note:

\[
\sum_{j} K_j \frac{v_j^2}{2g_c} = K_{\text{in}} \frac{A_{\text{in}}^2}{A_{\text{BT}}^2} + K_{\text{break}} \frac{v_2^2}{2g_c} = \left( K_{\text{in}} \frac{A_{\text{in}}^2}{A_{\text{BT}}^2} + K_{\text{break}} \right) \frac{v_2^2}{2g_c}
\]

(7)

where the subscript BT implies the beam tube. Bernoulli's equation then becomes

\[
\frac{g}{g_c} H = \frac{v_2^2}{2g_c} + \left( K_{\text{in}} \frac{A_{\text{in}}^2}{A_{\text{BT}}^2} + K_{\text{break}} \right) \frac{v_2^2}{2g_c}
\]

(8a)

\[
\frac{g}{g_c} H = \left( 1 + K_{\text{in}} \frac{A_{\text{in}}^2}{A_{\text{BT}}^2} \right) \frac{v_2^2}{2g_c}
\]

(8b)

We may then solve for the break velocity

\[
v_2 = v_{\text{break}} = \sqrt{\frac{2gH}{\kappa}}
\]

(9)

and substitute into the mass balance equation
\[ \rho A_{\text{pool}} \frac{d(H_1 - H_0)}{dt} = -\rho A_{\text{break}} \sqrt{\frac{2gH}{\kappa}} \] 

(10)

Note:

\[ \frac{d(H_1 - H_0)}{dt} = \frac{dH_1}{dt} = \frac{dH}{dt} \]

for \( H_0 \) and \( H_2 \) constant. The differential equation for the pool height is then

\[ \frac{dH}{dt} = \frac{A_{\text{break}}}{A_{\text{pool}}} \sqrt{\frac{2g}{\kappa}} H^{1/2} \]

(11)

\[ H^{-1/2}dH = \frac{A_{\text{break}}}{A_{\text{pool}}} \sqrt{\frac{2g}{\kappa}} dt \]

(12)

Integrate from the initial pool height to the top of the core and solve for time \( t \).

\[ \int_{H_0}^{H_2} H^{-1/2}dH = \int_{0}^{t} \frac{A_{\text{break}}}{A_{\text{pool}}} \sqrt{\frac{2g}{\kappa}} dt \]

(13)

\[ 2H^{1/2} \left| H_0 \right. \left. - H_2 \right|_{H_0}^{H_2} = -\frac{A_{\text{break}}}{A_{\text{pool}}} \sqrt{\frac{2g}{\kappa}} t \]

(14)

\[ t = 2 \left( \frac{(H_1(0) - H_2)^{1/2}}{H_0 - H_2} \right) \frac{A_{\text{pool}}}{A_{\text{break}}} \left( \frac{2g}{\kappa} \right)^{-1/2} \]

(15)

For:

\[ H_1(0) - H_2 = 20 \text{ feet} \]
\[ H_0 - H_2 = 1 \text{ foot} \]
\[ A_{\text{pool}} = 100 \text{ ft}^2 \]
\[ A_{\text{break}} = \pi D^2 / 4 = \pi \left( \frac{1}{12} \right)^2 / 4 = 0.00545 \text{ ft}^2 \]
\[ A_{\text{BT}} = \pi \left( \frac{1}{4} \right)^2 / 4 = 0.7854 \text{ ft}^2 \]
\[ K_{\text{in}} = 0.5 \text{ (Flush inlet)} \]
\[ K_{\text{break}} = 1.0 \text{ (Free discharge)} \]

the time required to drain the pool is \( t = 22,465 \text{ seconds} \equiv 6 \text{ hours} \)
Multiple Flow Path Systems

Many multiple flow path systems can be treated as flow networks, where a flow network is defined to be a system of one-dimensional, single-inlet, single-outlet flow segments connected at manifolds. In the analysis of flow networks, Bernoulli’s Equation is applied to each one-dimensional segment. The pressure drop across a manifold connecting any two one-dimensional segments is given by an appropriate loss coefficient(s) plus the elevation change. Mass and energy conservation within a manifold can be treated directly by the control volume form of the mass and energy equations.

Consider the simple flow network illustrated below. We are interested in determining the steady-state flow split (i.e. \( m_1 \) and \( m_2 \)), and/or the pressure drop across the network.

![Figure 1: Dual Path Flow Network](image)

The pressures at points A and B can be viewed as the pressures at the centroids of the manifolds, and are the same for both flow paths. Due to differences in the flow geometry, pipe dimensions, etc., the flow rates within the two branches can be different. For simplicity, we assume the cross sectional area of each flow segment is constant (but different), and that the cross sectional areas of the inlet and exit streams are the same. The one-dimensional momentum equation (Bernoulli’s equation) can be assumed to hold in each segment, such that for the inlet section

\[
\frac{P_1}{\rho} + g_z = \frac{P_A}{\rho} + \frac{f_{in} L_{in}}{D_{in}} \frac{v^2}{2g_c} + \sum_{z_j \in L_{in}} K_j \frac{v^2}{2g_c} + \frac{g}{g_c} H_A
\]  

for path 1

\[
\frac{P_A}{\rho} + g_z = \frac{P_B}{\rho} + \frac{f_{in} L_{in}}{D_{in}} \frac{v^2}{2g_c} + \sum_{z_j \in L_{in}} K_j \frac{v^2}{2g_c} + \frac{g}{g_c} H_B
\]

for path 2

\[
\frac{P_A}{\rho} + g_z = \frac{P_B}{\rho} + \frac{f_{in} L_{in}}{D_{in}} \frac{v^2}{2g_c} + \sum_{z_j \in L_{in}} K_j \frac{v^2}{2g_c} + \frac{g}{g_c} H_B
\]

and for the exit section.
\[
\frac{P_B}{\rho} + \frac{g}{g_c} H_B = \frac{P_i}{\rho} + \frac{f_{ex} L_{ex}}{D_{ex}} \frac{v^2}{2g_c} + \sum_{z_j \notin L_{ex}} K_j \frac{v^2}{2g_c} + \frac{g}{g_c} H_2
\]

(4)

If we assume the elevations are the same, and define \( \sum_{z_j \notin L_{ex}} K_j \) then

i) \( \frac{P_A}{\rho} - \frac{P_B}{\rho} = \left( \frac{f_{in} L_{in}}{D_{in}} + K_{in} \right) \frac{v^2}{2g_c} \)

ii) \( \frac{P_A}{\rho} - \frac{P_{A'}}{\rho} = \left( \frac{f_{1} L_{1}}{D_{1}} + K_1 \right) \frac{v^2_1}{2g_c} \)

iii) \( \frac{P_A}{\rho} - \frac{P_B}{\rho} = \left( \frac{f_2 L_2}{D_2} + K_2 \right) \frac{v^2_2}{2g_c} \)

iv) \( \frac{P_B}{\rho} - \frac{P_2}{\rho} = \left( \frac{f_{ex} L_{ex}}{D_{ex}} + K_{ex} \right) \frac{v^2}{2g_c} \)

In addition, mass conservation requires

v) \( \dot{m} = \dot{m}_1 + \dot{m}_2 \)

where \( \dot{m} = \rho v_\text{avg} \) in the appropriate flow segment.

If the density is known, then Equations i) through v) contain the seven unknowns \( P_i, P_2, P_A, P_B, v_1, v_2, \) and \( v \). Solution then requires two of the above variables be known.

**Solution**: Given \( v \) (or equivalently \( \dot{m} \)) compute \( \Delta P, v_1 \& v_2 \)

Subtract iii) from ii)

\[
0 = \left( \frac{f_1 L_1}{D_1} + K_1 \right) \frac{v^2_1}{2g_c} - \left( \frac{f_2 L_2}{D_2} + K_2 \right) \frac{v^2_2}{2g_c}
\]

(5)

or

\[
\left( \frac{f_1 L_1}{D_1} + K_1 \right) \frac{v^2_1}{2g_c} = \left( \frac{f_2 L_2}{D_2} + K_2 \right) \frac{v^2_2}{2g_c}
\]

(6)

This implies the flow partitions itself so as to balance the friction and forms losses in the two branches.

Adding i), iii), and iv)

\[
\frac{\Delta P}{\rho} = (K_{in} + K_{ex}) \frac{v^2}{2g_c} + K_2 \frac{v^2_2}{2g_c}
\]

(7)
provides an expression for the total pressure drop across the network, where \( \Delta P = P_1 - P_2 \) (We could have just as easily added i), ii) and iv)). Given \( \dot{m}_1 \dot{v}_1 + \dot{m}_2 = \rho v_1 A_1 + \rho v_2 A_2 \) and Equations 6) and 7), we have three equations in the unknowns \( v_1, v_2, \) and \( \Delta P \). We can therefore eliminate \( v_1 \) and \( v_2 \) and solve for the pressure drop in terms of the given velocity \( v \). As the friction factors are in general functions of velocity, this will require iteration. The remaining velocities can then be easily determined. Note, that while total pressure drop across the network can be determined, we can not determine the individual pressures without additional information.

A similar approach can be used to calculate the total mass flow rate given the pressure drop across the loop.
Example:

A simple two loop representation of a PWR primary side is given below. Due to minor geometry differences in the loops, the total effective loss coefficient for Loop 2 (Hot Leg, Steam Generator and Cold Leg) is 10% higher than that for Loop 1. For the given information, compute the mass flow rate in each loop, as well as the total core mass flow rate. You may assume frictional losses are included in the total loss coefficients and the reactor coolant pumps operate at the same constant $\Delta P$.

- Core Flow Area: 28.2 ft$^2$
- Loop Averaged Temperature: 580 F
- Vessel Loss Coefficient: 43
- Hot Leg Diameter: 29 inches
- Loop 1 Loss Coefficient (referenced to hot leg velocity): 5
- Pump $\Delta P$: 100 psi

![Figure 1: Two-Loop Representation of a PWR](image)

**SOLUTION**

Apply Bernoulli's Equation between the core exit and the core inlet.

Loop 1

\[
\frac{P_T}{\rho} + \frac{g}{g_c} H_T + \frac{\Delta P_p}{\rho} = \frac{P_B}{\rho} + K_1 \frac{v_1^2}{2g_c} + \frac{g}{g_c} H_B
\]  

(1)

\[
\frac{P_T}{\rho} - \frac{P_B}{\rho} + \frac{g}{g_c} H_T - \frac{g}{g_c} H_B - K_1 \frac{v_1^2}{2g_c} + \frac{\Delta P_p}{\rho} = 0
\]  

(2)

\[
- \frac{\Delta P_c}{\rho} + \frac{g}{g_c} \Delta H_c - K_1 \frac{v_1^2}{2g_c} + \frac{\Delta P_p}{\rho} = 0
\]  

(3)
Loop 2

Similarly for Loop 2

\[- \frac{\Delta P_c}{\rho} + \frac{g}{g_c} \Delta H_c - K_2 \frac{v_c^2}{2g_c} + \frac{\Delta P_p}{\rho} = 0 \]  

(4)

and between the core inlet and the core exit

Core

\[
\frac{P_B}{\rho} + \frac{g}{g_c} H_B = \frac{P_f}{\rho} + K_c \frac{v_c^2}{2g_c} + \frac{g}{g_c} H_f
\]  

(5a)

\[
\frac{\Delta P_c}{\rho} - \frac{g}{g_c} \Delta H_c - K_c \frac{v_c^2}{2g_c} = 0
\]  

(5b)

In terms of mass flow rates, these equations have the form

Loop 1

\[- \frac{\Delta P_c}{\rho_c} + \frac{g}{g_c} \Delta H_c - \frac{K_1}{A_1^2} \frac{\dot{m}_1^2}{2\rho g_c} + \Delta P_p = 0 \]  

(6)

Loop 2

\[- \frac{\Delta P_c}{\rho_c} + \frac{g}{g_c} \Delta H_c - \frac{K_2}{A_2^2} \frac{\dot{m}_2^2}{2\rho g_c} + \Delta P_p = 0 \]  

(7)

Core

\[
\Delta P_c - \frac{\rho}{\rho_c} \Delta H_c - \frac{K_c}{A_c^2} \frac{\dot{m}_c^2}{2\rho g_c} = 0
\]  

(8)

Equations 6, 7, and 8 contain four unknowns: \(\Delta P_c\), \(\dot{m}_1\), \(\dot{m}_2\) and \(\dot{m}_c\). To complete the set of equations, we use the mass conservation equation

\[
\dot{m}_c = \dot{m}_1 + \dot{m}_2.
\]  

(9)

Subtract Equation 7 from Equation 6

\[
\frac{K_1}{A_1^2} \frac{\dot{m}_1^2}{2\rho g_c} = \frac{K_2}{A_2^2} \frac{\dot{m}_2^2}{2\rho g_c}
\]  

(10)

and solve for the mass flow rate in loop 1

\[
\dot{m}_1 = \dot{m}_2 \sqrt{\frac{K_2}{K_1}}
\]  

(11)

From Equation 9,
\[ \dot{m}_c = \dot{m}_1 + \dot{m}_2 \Rightarrow \dot{m}_c = \dot{m}_2 \left(1 + \sqrt{\frac{K_2}{K_1}}\right) \]  

(12)

Add Equations 7 and 8

\[ \Delta P_p = \frac{K_2}{A_2^2} \frac{\dot{m}_2^2}{2 \rho g_c} + \frac{K_c}{A_c^2} \frac{\dot{m}_1^2}{2 \rho g_c} = \frac{\dot{m}_2^2}{2 \rho g_c} \left(\frac{K_2}{A_2^2} + \frac{K_c}{A_c^2} \left(1 + \sqrt{\frac{K_2}{K_1}}\right)^2\right) \]  

(13)

From which the mass flow rate in Loop 2 may be solved for as

\[ \dot{m}_2 = \frac{2 \rho g_c \Delta P_p}{\sqrt{\frac{K_2}{A_2^2} + \frac{K_c}{A_c^2} \left(1 + \sqrt{\frac{K_2}{K_1}}\right)^2}} \]  

(14)

Note:
\[ A_1 = A_2 = \pi R_{itl}^2 = \pi \left(\frac{29}{24}\right)^2 = 4.587 \text{ ft}^2 \]

\[ \dot{m}_2 = \frac{2(44)(4.17 \times 10^8)(100)(144)}{\sqrt{\frac{5.5}{4.587^2} + \frac{43}{28.2^2} \left(1 + \sqrt{\frac{5.5}{5}}\right)^2}} = 32.9 \times 10^6 \text{ lbm/hr} \]

\[ \dot{m}_1 = \dot{m}_2 \sqrt{\frac{K_2}{K_1}} = 32.9 \times 10^6 \sqrt{1.1} = 34.5 \times 10^6 \text{ lbm/hr} \]

\[ \dot{m}_c = \dot{m}_1 + \dot{m}_2 = 32.9 \times 10^6 + 34.5 \times 10^6 = 67.4 \times 10^6 \text{ lbm/hr} \]

This treatment of flow in piping networks assumes the flow path can be broken into a number of independent one-dimensional flow segments, and that momentum mixing and multi-dimensional effects can be taken into account by appropriate loss coefficients applied at the manifolds. While this is a reasonable approximation for many practical engineering systems, in certain reactor components these effects are important and can not be adequately treated by a simple Bernoulli’s equation approach. The jet pumps in Boiling Water Reactor systems are examples of where the momentum mixing must be accounted for explicitly.
JET PUMP

A simple diagram of a jet pump assembly is given below. Recirculation pumps take suction from the downcomer and discharge through the jet pump nozzle into the suction section of the jet pump. Acceleration of the liquid in the nozzle creates a low pressure area at the nozzle discharge, such that water is drawn through the suction region into the jet pump throat where it mixes with the nozzle flow, finally discharging into the lower core plenum.

Figure 1: Jet Pump Assembly and Recirculation Loop

The suction and driver section of the jet pump assembly is illustrated in more detail below.

Figure 2: Jet Pump Suction and Driver Section
Consider first the recirculation line. Write a momentum balance (Bernoulli’s Equation) between a point in the downcomer above the jet pump assembly, through the recirculation pump to the driver nozzle.

$$ \begin{align*} P_{dc} + \Delta P_{p} + \rho \frac{g}{g_{c}} H_{dc} &= P_{2} + \kappa_{dc} \frac{\rho v_{dc}^{2}}{2g_{c}} + \kappa_{1} \frac{\rho v_{drv}^{2}}{2g_{c}} + \rho \frac{g}{g_{c}} H_{drv} \tag{1} \end{align*} $$

where the total effective loss coefficients (κ) account for all friction losses, forms losses and area changes along the path. We next write a momentum balance (Bernoulli’s equation) from the same downcomer location through the suction line to the throat of the jet pump at the discharge of the driver nozzle.

$$ \begin{align*} P_{dc} + \rho \frac{g}{g_{c}} H_{dc} &= P_{2} + \kappa_{dc} \frac{\rho v_{dc}^{2}}{2g_{c}} + \kappa_{2} \frac{\rho v_{suction}^{2}}{2g_{c}} + \rho \frac{g}{g_{c}} H_{drv} \tag{2} \end{align*} $$

where κ₂ represents the total friction and forms loss coefficient for this flow path referenced to the throat velocity.

Note: A free jet condition has been used to equate the pressure at the discharge nozzle and the suction region.

### Mixing Volume

A momentum balance on the mixing volume, accounting for multiple inlets gives

$$ \frac{1}{g_{c}} \left( \rho v_{3} A_{3} \frac{g}{g_{c}} A_{drv} - \rho v_{suction}^{2} A_{suction} \right) = -(P_{3} - \bar{P}_{3}) A_{3} - \rho A_{3} \frac{g}{g_{c}} (H_{3} - H_{drv}) $$

or

$$ \begin{align*} P_{2} + \rho \frac{v_{drv}^{2}}{g_{c}} A_{drv} + \rho \frac{v_{suction}^{2}}{g_{c}} A_{suction} + \rho \frac{g}{g_{c}} H_{drv} &= P_{3} + \rho \frac{v_{3}^{2}}{g_{c}} A_{3} + \rho \frac{g}{g_{c}} H_{3} \tag{3} \end{align*} $$

To complete the flow loop, we assume a very simple momentum balance (Bernoulli’s Equation) from the exit of the mixing volume, through the core and back to our reference location in the downcomer.

$$ \begin{align*} P_{3} + \rho \frac{g}{g_{c}} H_{3} &= P_{dc} + \kappa_{c} \frac{\rho v_{dc}^{2}}{2g_{c}} + \rho \frac{g}{g_{c}} H_{dc} \tag{4} \end{align*} $$

Note: This simple momentum balance is for illustration purposes only, as it is unlikely a Bernoulli’s Equation approach would be adequate in a Boiling Water Reactor core, particularly the constant density assumption. The modifications necessary to account for the two-phase conditions in the core will be considered later.

### Subtract Equation 2 from Equation 1

$$ \Delta P_{p} = \kappa_{1} \frac{\rho v_{drv}^{2}}{2g_{c}} - \kappa_{2} \frac{\rho v_{suction}^{2}}{2g_{c}} \tag{5} $$

### Add Equations 2-4 and simplify to give

$$ \frac{\rho v_{drv}^{2}}{g_{c}} A_{drv} = \frac{\rho v_{dc}^{2}}{2g_{c}} \left( \kappa_{dc} + \kappa_{c} + 2 \frac{A_{dc}}{A_{3}} \right) + \rho \frac{v_{suction}^{2}}{2g_{c}} \left( \kappa_{2} - 2 \frac{A_{suction}}{A_{3}} \right) \tag{6} $$

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Equations 5 and 6 contain three unknowns, \( v_{dc} \), \( v_{suction} \) and \( v_{drv} \). The final equation needed for solution is the mass balance equation

\[
\rho v_{dc} A_{dc} = \rho v_3 A_3 = \rho v_{drv} A_{drv} + \rho v_{suction} A_{suction}
\]  

(7)

For a given set of system parameters, the system velocities and flow rates can then be determined. Due to the nonlinear nature of the equations this generally requires a trial and error solution.