Critical Boiling Length Approach

Consider the boiling channel illustrated below. If dryout occurs at a location $z_{crit}$ in the channel, then there will be a corresponding quality $x_{crit}$ at this location. It has been observed that there is a strong correlation between the quality at the dryout point and the critical boiling length, where the critical boiling length is defined to be the dryout location minus the nonboiling height, $L_{crit} = z_{crit} - H_o$. This is true independent of the heat flux profile.

Critical boiling length correlations are commonly of the form

$$x_{crit} = \frac{a(G, P)L_{crit}}{L_{crit} + b(G, P)}$$

To apply the critical boiling length correlation, we make use of the following energy balances

a) Nonboiling Height

$$\dot{m}(h_f - h_{in}) = \int_{0}^{H_o} \dot{q}^*_{crit}(z) \pi Ddz$$  \hspace{1cm} (1)

b) Critical Position

$$\dot{m}(h_{crit} - h_f) = \int_{H_o}^{z_{crit}} \dot{q}^*_{crit}(z) \pi Ddz$$  \hspace{1cm} (2)

where:

$$h_{crit} = h_f + x_{crit}h_{fg}$$

$$\therefore \dot{m}x_{crit}h_{fg} = \int_{H_o}^{z_{crit}} \dot{q}^*_{crit}(z) \pi Ddz$$  \hspace{1cm} (3)

or
\[ x_{\text{crit}} = \frac{1}{\dot{m} h_{fg}} \int_{H_o}^{z_{\text{crit}}} q_{\text{crit}}^*(z) \pi Ddz \] (4)

We can replace the critical quality with the critical boiling length correlation such that

\[ \frac{a L_{\text{crit}}}{L_{\text{crit}} + b} = \frac{a(z_{\text{crit}} - H_o)}{z_{\text{crit}} - H_o + b} = \frac{1}{\dot{m} h_{fg}} \int_{H_o}^{z_{\text{crit}}} q_{\text{crit}}^*(z) \pi Ddz \] (5)

where

\[ \dot{m}(h_f - h_{in}) = \int_{0}^{H_o} q_{\text{crit}}^*(z) \pi Ddz \] (6)

If we write the heat flux profile as \( q_{\text{crit}}^*(z) = q_{0,\text{crit}}^* Z(z) \), where \( Z(z) \) is some arbitrary shape function, then given a critical location, we can solve equations 5 and 6 for the two unknowns \( H_o \) and \( q_{0,\text{crit}}^* \).

Example:

Consider a uniformly heated channel such that \( q_{\text{crit}}^*(z) = q_{\text{crit}}^* \). Equations 5 and 6 simplify to

\[ \frac{a(z_{\text{crit}} - H_o)}{z_{\text{crit}} - H_o + b} = \frac{1}{\dot{m} h_{fg}} q_{\text{crit}}^* \pi D(z_{\text{crit}} - H_o) \] (7)

and

\[ \dot{m}(h_f - h_{in}) = q_{\text{crit}}^* \pi DH_o \] (8)

Equation 8 may be used to eliminate nonboiling height from Equation 7 giving for the critical heat flux

\[ q_{\text{crit}}^* = \frac{\dot{m}(ah_{fg} + h_f - h_{in})}{\pi D(z_{\text{crit}} + b)} \] (9)

which may be solved directly for any given critical location. From Equation 9, the critical heat flux decreases monotonically with increasing critical location with the minimum critical heat flux occurring for \( z_{\text{crit}} = H \). Under these conditions, Equation 9 becomes

\[ q_{\text{crit}}^* \pi DH = \frac{\dot{m}(ah_{fg} + h_f - h_{in})}{(1 + b/H)} = q_{\text{crit}} \] (10)

where \( q_{\text{crit}} \) is called the critical power. We can then define the Critical Power Ratio (CPR)

\[ \text{CPR} = \frac{q_{\text{crit}}}{q} \] (11)

where \( q \) is the operating power of the channel. For boiling channels, critical power is a convenient parameter for expressing thermal margin. For example, if a channel operates at a CPR of 1.2, this implies we can increase the power in that channel by 20% before achieving dryout. While this example only considers uniformly heated channels, the same conclusions hold independent of heat flux profile.