3) It has been shown, that the loss coefficient for a sudden expansion referenced to the velocity prior to the expansion can be expressed as

\[ K_{\text{exp}} = \left( 1 - \frac{A_1}{A_2} \right)^2 \]

Repeat the derivation for the expansion coefficient referenced to the velocity after the expansion (i.e. \( A_2 \)).

SOLUTION

Consider the conservative form of the steady state momentum equation for a horizontal channel

\[ \frac{1}{g_c} \frac{1}{A_x} \frac{\partial}{\partial z} \left( \rho \nu A_x \right) = - \frac{\partial P}{\partial z} - \frac{\tau_w}{A_x} \]

We assume that between the separation point and the reattachment point, the flow streams do not contact the wall, such that the frictional terms are zero.

The change in fluid pressure is only due to deceleration of the fluid

\[ \frac{1}{g_c} \frac{1}{A_x} \frac{\partial}{\partial z} (\rho \nu A_x) = - \frac{\partial P}{\partial z} \]

or equivalently

\[ \frac{1}{g_c} \frac{\partial}{\partial z} (\rho \nu v) = - A_x \frac{\partial P}{\partial z} \]

Integrate from the separation point (1) to the reattachment point (2).

\[ \frac{\dot{m}}{g_c} \int_{1}^{2} \frac{\partial}{\partial z} (v) dz = - A_2 \int_{1}^{2} \frac{\partial P}{\partial z} dz \]
The pressure at (1) and (2) acts uniformly over the area $A_2$ (this is an application of the free jet condition at the expansion)

$$\frac{\dot{m}}{g_c}(v_2 - v_1) = -A_2(P_2 - P_1)$$

Divide by $A_2$ and rearrange ($\dot{m}/A_2 = \rho v_2$)

$$P_1 - P_2 = \frac{\rho v_2}{g_c}(v_2 - v_1)$$

The forms loss coefficient is defined such that the pressure change given above is equal to that given by Bernoulli's Equation applied between the same points

$$P_1 + \frac{\rho v_1^2}{2} = P_2 + \frac{\rho v_2^2}{2} + K_{\text{exp}}\frac{\rho v_2^2}{2}$$

Therefore

$$P_1 - P_2 = \frac{\rho v_2^2}{2g_c} - \frac{\rho v_1^2}{2g_c} + K_{\text{exp}}\frac{\rho v_2^2}{2g_c} = \frac{\rho v_2}{g_c}(v_2 - v_1)$$

mass conservation gives

$$v_2 A_2 = v_1 A_1$$

or

$$v_1 = v_2 \frac{A_1}{A_2}$$

Eliminating $v_1$

$$\frac{\rho v_2^2}{2g_c} \frac{A_2^2}{A_1^2} + \frac{\rho v_2^2}{2g_c} \frac{A_2}{A_1} = K_{\text{exp}}\frac{\rho v_2^2}{2g_c}$$

which can be solved for $K_{\text{exp}}$.

$$\frac{A_2^2}{A_1^2} + 1 - \frac{2A_2}{A_1} = K_{\text{exp}}$$

$$K_{\text{exp}} = \frac{A_2^2}{A_1^2} - \frac{2A_2}{A_1} + 1$$

$$K_{\text{exp}} = \left(1 - \frac{A_2}{A_1}\right)^2$$