Critical heat flux in a uniformly heated channel has been correlated according to a critical boiling length correlation of the form

\[ x_{\text{crit}} = \frac{a L_{\text{crit}}}{b + L_{\text{crit}}} \]

where the correlation is based on a best fit of the available data. As a consequence, the critical boiling length correlation has inherent uncertainty which can be expressed in terms of a relative uncertainty as

\[ x_{\text{crit}} = \frac{a L_{\text{crit}}}{b + L_{\text{crit}}} (1 \pm \sigma) . \]

Assuming the MCPR limit associated with this correlation is given by

\[ \text{MCPR} \approx \frac{(q_{\text{crit}})_0}{(q_{\text{crit}})_i} \]

where \((q_{\text{crit}})_0\) is the predicted critical power assuming no uncertainty, and \((q_{\text{crit}})_i\) is the critical power predicted at the limit of the uncertainty band (i.e. one \(\sigma\)), derive the MCPR limit

\[ \text{MCPR} = \frac{a \bar{h}_f - h_f - h_m}{a \bar{h}_f + h_f - h_m} \]

where \(\bar{a} = a \times (1 - \sigma)\).

**SOLUTION**

The Critical Power for a uniformly heated channel is defined to be

\[ q_{\text{crit}} = q_{\text{crit}}^* \pi D H \]

where the critical heat flux satisfies the two energy balances

1) \[ h_{\text{sat}} - h_f = \frac{1}{m} \int_{\bar{h}_s}^{H} q_{\text{crit}}^*(z) \pi D dz \]

2) \[ h_f - h_m = \frac{1}{m} \int_{0}^{H_0} q_{\text{crit}}^*(z) \pi D dz \]

Equation 1) can be written in terms of the critical quality and the critical boiling length correlation by

3) \[ \frac{h_{\text{sat}} - h_f}{h_{\text{sat}}} = x_{\text{crit}} = \frac{a L_{\text{crit}}}{b + L_{\text{crit}}} = \frac{a(H - H_0)}{b + (H - H_0)} = \frac{1}{m \bar{h}_f} \int_{\bar{h}_s}^{H} q_{\text{crit}}^*(z) \pi D dz \]

For a uniform heat flux, Equations 2) and 3) reduce to

4) \[ \frac{a(H - H_0)}{b + (H - H_0)} = \frac{1}{m \bar{h}_f} q_{\text{crit}}^* \pi D (H - H_0) \]
5) \[ h_f - h_{in} = \frac{q^* \pi D H_0}{\dot{m}} \]

Equation 5) may be solved directly for the non boiling height

6) \[ H_0 = \frac{\dot{m}(h_f - h_{in})}{q^* \pi D} \]

The critical heat flux can then be obtained in terms of the critical boiling length correlation from Equations 4) and 6) as follows

\[
\frac{a(H - H_0)}{b + (H - H_0)} = \frac{1}{\dot{m} h_{fg}} q^* \pi D(H - H_0)
\]

\[
\frac{a}{b + (H - H_0)} = \frac{1}{\dot{m} h_{fg}} q^* \pi D
\]

\[
\frac{1}{b + (H - H_0)} = \frac{1}{a \dot{m} h_{fg}} q^* \pi D
\]

\[
b + (H - H_0) = \frac{a \dot{m} h_{fg}}{q^* \pi D}
\]

\[
b + H = \frac{a \dot{m} h_{fg}}{q^* \pi D} + H_0
\]

Substituting \( H_0 \) from Equation 6)

\[
b + H = \frac{a \dot{m} h_{fg}}{q^* \pi D} + \frac{\dot{m}(h_f - h_{in})}{q^* \pi D} = \frac{\dot{m}(ah_{fg} + h_f - h_{in})}{q^* \pi D}
\]

\[
q^* \pi D = \frac{\dot{m}(ah_{fg} + h_f - h_{in})}{b + H}
\]

7) \[ q_{cr} = q^* \pi D H = \frac{\dot{m}(ah_{fg} + h_f - h_{in})}{1 + b / H} \]

Equation 7 is valid for any value of \( a \) and \( b \). If \( a \) and \( b \) correspond to the zero uncertainty values, then

8) \[ (q_{cr})_0 = \frac{\dot{m}(ah_{fg} + h_f - h_{in})}{1 + b / H} \]

The critical power any non zero uncertainty is

9) \[ (\tilde{q}_{cr}) = \frac{\dot{m}(a[1 + \sigma]h_{fg} + h_f - h_{in})}{1 + b / H} \]
It should be obvious from Equation 9, that the minimum value of the critical power occurs at the lower limit of the uncertainty band, i.e.

\[ q_{\text{crit}} = \frac{\dot{m}(a[1-\sigma]h_{\text{in}} + h_f - h_{\text{in}})}{1 + b / H} = \frac{\dot{m}(\tilde{a}h_{\text{in}} + h_f - h_{\text{in}})}{1 + b / H} \]

Dividing Equation 8 by Equation 10 gives

\[ \text{MCPR} = \frac{(q_{\text{crit}})_0}{(q_{\text{crit}})_1} = \frac{\frac{a h_{\text{in}} + h_f - h_{\text{in}}}{\tilde{a} h_{\text{in}} + h_f - h_{\text{in}}}} {a h_{\text{in}} + h_f - h_{\text{in}}} \]