The pressure rise across a pump is generally a function of the flow rate (or mass flux) through the pump, i.e.

\[ \Delta P_p = \Delta P_p(G) \]

A simple flow network is illustrated below. Opening or closing the valve in line 2 will alter the overall loss coefficient in the line, which in turn will alter the mass flux and \( \Delta P \) across the pump in both lines 1 and 2. Give a step by step procedure (including all equations) for determining the flow through the network assuming different loss coefficients in the two pump lines. You may assume the inlet and exit pressures to the network are known. You may assume the overall loss coefficients for the pump lines and the discharge lines are known and constant (independent of \( G \)). You may also assume the flow areas in all lines are equal. The loss coefficients can be assumed to reflect any losses associated with flow through the manifold.

The momentum equation across line 1 to the manifold is

\[ P_0 + \Delta P_p(G_1) = P_m + K_1 \frac{G_1^2}{2 \rho} \]  

(1)

Similarly, the momentum equation across line 2 to the manifold is

\[ P_0 + \Delta P_p(G_2) = P_m + K_2 \frac{G_2^2}{2 \rho} \]  

(2)

The momentum equation from the manifold to the discharge point is

\[ P_m = P_{dis} + K_0 \frac{G_0^2}{2 \rho} \]  

(3)

Since the flow areas are all equal, mass conservation requires

\[ G_1 + G_2 = G_0 \]  

(4)

Subtract equations (1) and (2)

\[ \Delta P_p(G_1) - \Delta P_p(G_2) = K_1 \frac{G_1^2}{2 \rho} - K_2 \frac{G_2^2}{2 \rho} \]  

(5)
and add equations (2) and (3)

\[ P_0 + \Delta P_p (G_2) = P_{\text{dis}} + K_2 \frac{G_2^2}{2\rho} + K_0 \frac{G_0^2}{2\rho} \]  \hspace{1cm} (6)

or taking advantage of the mass conservation equation

\[ P_0 + \Delta P_p (G_2) = P_{\text{dis}} + K_2 \frac{G_2^2}{2\rho} + K_0 \frac{(G_1 + G_2)^2}{2\rho} \]  \hspace{1cm} (7)

Equations 5 and 7 are two nonlinear equations in the unknown mass fluxes \(G_1\) and \(G_2\) which can be solved iteratively. The solution may be further simplified by solving equation 7 for \(G_1\) in terms of \(G_2\)

\[ G_1 = \sqrt{\frac{2\rho(P_0 + \Delta P_p (G_2) - P_{\text{dis}}) - K_2 G_2^2}{K_0} - G_2} \]  \hspace{1cm} (8)

which in combination with equation (5) produces a single nonlinear equation in \(G_2\) which may be solved iteratively.

Given \(G_1\) and \(G_2\) from 5 and 8, \(G_0\) may be found directly from equation (4). Given \(G_0\), the mass flow rate through the network is

\[ \dot{m} = G_0 A_0 \]