A natural circulation boiling water reactor operates at a pressure of 1000 psia. The average densities in the non boiling and boiling heights are 47 and 38 lbm/ft$^3$ respectively. The corresponding non boiling and boiling heights are 2 and 3 feet respectively. Assuming a downcomer temperature of 520 F, a core exit quality of 8 percent and a core exit slip ratio of 2, show how you would compute the chimney height if the total friction and forms losses around the loop are 0.527 psi.

**SOLUTION**

At steady state, the buoyancy forces must equal the total pressure loss around the loop. The buoyancy forces are obtained by integrating the density distribution around the closed loop such that

\[
\Delta P = -\int \rho \frac{g}{g_c} \, dH
\]

\[
\int \rho \frac{g}{g_c} \, dH = \int \rho \frac{g}{g_c} \, dH \left( c_o \right) + \int \rho \frac{g}{g_c} \, dH \left( H_o \right) + \int \rho \frac{g}{g_c} \, dH \left( H_b \right)
\]

\[
\int \rho \frac{g}{g_c} \, dH = -\rho_{dc} \frac{g}{g_c} H_{dc} + \bar{\rho}_{H_o} \frac{g}{g_c} H_o + \bar{\rho}_{H_b} \frac{g}{g_c} H_B + \rho_{ch} \frac{g}{g_c} H_{ch}
\]

Note: \( H_{dc} = H_{ch} + H_o + H_B \)

\[
\int \rho \frac{g}{g_c} \, dH = -\rho_{dc} \frac{g}{g_c} (H_{ch} + H_o + H_B) + \bar{\rho}_{H_o} \frac{g}{g_c} H_o + \bar{\rho}_{H_b} \frac{g}{g_c} H_B + \rho_{ch} \frac{g}{g_c} H_{ch}
\]

\[
\Delta P = -\left( \bar{\rho}_{H_o} - \rho_{dc} \right) \frac{g}{g_c} H_o - \left( \bar{\rho}_{H_b} - \rho_{dc} \right) \frac{g}{g_c} H_B - \left( \rho_{ch} - \rho_{dc} \right) \frac{g}{g_c} H_{ch}
\]

\[
\Delta P = \left( \rho_{dc} - \rho_{H_o} \right) \frac{g}{g_c} H_o + \left( \rho_{dc} - \rho_{H_b} \right) \frac{g}{g_c} H_B + \left( \rho_{ch} - \rho_{dc} \right) \frac{g}{g_c} H_{ch}
\]

\[
\Delta P - \left( \rho_{dc} - \bar{\rho}_{H_o} \right) \frac{g}{g_c} H_o - \left( \rho_{dc} - \bar{\rho}_{H_b} \right) \frac{g}{g_c} H_B = (\rho_{dc} - \rho_{ch}) \frac{g}{g_c} H_{ch}
\]

\[
\frac{\Delta P - \left( \rho_{dc} - \bar{\rho}_{H_o} \right) \frac{g}{g_c} H_o - \left( \rho_{dc} - \bar{\rho}_{H_b} \right) \frac{g}{g_c} H_B}{\left( \rho_{dc} - \rho_{ch} \right) \frac{g}{g_c}} = H_{ch}
\]

The total pressure drop around the loop includes the friction, forms and acceleration losses. The friction and forms losses are given. The acceleration loss is given by
\[
\Delta P_{acc} = \frac{G^2}{g_c} \left[ \left( \frac{1-x}{x} \right)^2 + \frac{x^2}{H \rho_H} \right] - \frac{1}{\rho_{dc}} \]

For the given data:

\[ \rho_{dc} \approx \rho_f \quad \text{at} \quad 520 \quad \text{F} = 47.82 \]

\[ \bar{\rho}_{H_a} = 47 \]

\[ \bar{\rho}_{H_b} = 38 \]

\[ H_a = 2 \]

\[ H_B = 3 \]

The density in the chimney is given by \( \rho_{ch} = \alpha_f \rho_f + \alpha_g \rho_g \) where the \( \alpha_k \)'s are the phasic volume fractions at the core exit. The vapor volume fraction can be obtained from the Fundamental Void-Quality-Slip relationship

\[
\alpha_g = \frac{1}{1 + \left( \frac{1-x}{x} \right) \rho_g S} \]

Given the core exit void fraction and quality, the acceleration loss can be determined directly for any given mass flux such that

\[ \Delta P = \Delta P_{friction} + \Delta P_{forms} + \Delta P_{acceleration} \]

and the chimney height can be determined directly from

\[
\Delta P = \left( \rho_{dc} - \bar{\rho}_{H_a} \right) \frac{g}{g_c} H_a - \left( \rho_{dc} - \bar{\rho}_{H_b} \right) \frac{g}{g_c} H_B \]

\[
\left( \rho_{dc} - \rho_{ch} \right) \frac{g}{g_c} = H_{ch} \]