A tank of area $A_1$ initially contains water at an elevation of $H_1(0)$. Tank one is connected to a second tank through a line of area $A_0$. The second tank is initially empty. At time $t = 0$ liquid is allowed to flow from tank 1 to tank 2. Develop an equation for the time necessary for the tank levels to become equal. You may assume the line between the two tanks is initially filled with water and the bottom of the tanks are at the same elevation. You may also assume all flow losses can be characterized by a known constant loss coefficient $K_0$, referenced to the velocity in the connecting line. Both tanks are open to atmosphere.

**SOLUTION**

**Mass Balance on Tank 1**

\[
\frac{dM_1}{dt} = -\rho v_0 A_0
\]

For $M_1 = \rho A_1 H_1$

\[
\frac{dH_1}{dt} = -v_0 \frac{A_0}{A_1}
\]

A similar mass balance on tank 2 gives

**Mass Balance on Tank 2**

\[
\frac{dM_2}{dt} = \rho v_0 A_0
\]

For $M_2 = \rho A_2 H_2$

\[
\frac{dH_2}{dt} = v_0 \frac{A_0}{A_2}
\]

Subtracting gives

\[
\frac{d(H_1 - H_2)}{dt} = -v_0 \left( \frac{A_0}{A_1} + \frac{A_0}{A_2} \right)
\]
Defining $H = H_1 - H_2$ gives

$$\frac{dH}{dt} = -v_0 \left\{ \frac{A_0}{A_1} + \frac{A_0}{A_2} \right\}$$

An equation is then required which gives the velocity in the connecting line in terms of the elevation difference in the tanks.

**Momentum Balance**

Apply Bernoulli’s Equation between the liquid surface in Tank 1 and the liquid surface in Tank 2

$$P_1 + \rho g H_1 = P_2 + \rho g H_2 + K \frac{\rho v_0^2}{2}$$

For $P_1 = P_2$

$$\rho g (H_1 - H_2) = \rho g H = K \frac{\rho v_0^2}{2}$$

$$v_0^2 = \frac{2gH}{K}$$

$$v_0 = \sqrt{\frac{2gH^{1/2}}{K}}$$

Substitute into the mass balance

$$\frac{dH}{dt} = -\sqrt{\frac{2g}{K} \left\{ \frac{A_0}{A_1} + \frac{A_0}{A_2} \right\} H^{1/2}}$$

and integrate

$$H^{-1/2} dH = -\xi dt$$

$$\int_{H_1(0)}^{0} H^{-1/2} dH = -\int_{0}^{t} \xi dt$$

$$2H^{1/2}\bigg|_{H_1(0)}^{0} = -\xi t$$

$$2\sqrt{H_1(0)} = \xi t$$

$$t = \frac{2\sqrt{H_1(0)}}{\xi}$$