For the BWR operating parameters given below, compute and plot:

a) The clad surface temperature assuming the Jens-Lottes Correlation
b) The clad surface temperature assuming the Thom Correlation
c) The clad surface temperature assuming the Chen Correlation

equation:

\[ q''(z) = h_{FC}[T_w(z) - T_s(z)] + h_{NB}[T_w(z) - T_{sat}] \]

where \( h_{FC} \) is an appropriate single phase forced convection heat transfer coefficient (e.g. Weisman), and \( h_{NB} \) is an appropriate nucleate boiling heat transfer coefficient (e.g. Thom or Jens-Lottes). For Parts a) and b) you can assume this superposition approach holds.

d) In the Chen correlation, compare the temperature distributions obtained with the original Dittus-Boelter correlation for \( h_{io} \) and that obtained by substituting the Weisman correlation for \( h_{io} \).

e) Of interest is the impact of heat transfer coefficient choice on the maximum clad and maximum fuel centerline temperatures. Compare the computed maximum fuel centerline temperatures computed from the approaches in parts a) – d).

**BOILING WATER REACTOR PARAMETERS**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>1040 psia</td>
</tr>
<tr>
<td>Coolant Mass Flux</td>
<td>(1.21 \times 10^6) lbm/hr-ft²</td>
</tr>
<tr>
<td>Core Inlet Enthalpy</td>
<td>527.9 Btu/lbm</td>
</tr>
<tr>
<td>Core Average Heat Flux</td>
<td>144,032 Btu/hr-ft²</td>
</tr>
<tr>
<td>Rod Pitch</td>
<td>0.640 inches</td>
</tr>
<tr>
<td>Rod Diameter</td>
<td>0.493 inches</td>
</tr>
<tr>
<td>Clad Thickness</td>
<td>0.034 inches</td>
</tr>
<tr>
<td>Pellet Diameter</td>
<td>0.416 inches</td>
</tr>
<tr>
<td>Clad Thermal Conductivity</td>
<td>9.6 Btu/hr-ft-F</td>
</tr>
<tr>
<td>Gap Conductance</td>
<td>1000 Btu/hr-ft²-F</td>
</tr>
<tr>
<td>Fuel Height</td>
<td>146 inches</td>
</tr>
<tr>
<td>Fraction of energy deposited in fuel</td>
<td>0.97</td>
</tr>
<tr>
<td>Axial Peak to average ratio</td>
<td>1.5</td>
</tr>
</tbody>
</table>

The axial heat flux may be taken to be

\[ q''(z) = q''_0 \left( \frac{\pi(H + \lambda z)}{H_s} \right) \sin \left( \frac{\pi(H + \lambda z)}{H_s} \right) \]
SOLUTION

Heat Flux

The heat flux profile is in terms of two unknown parameters, the extrapolation distance $\lambda$ and the amplitude $q'_0$. The extrapolation distance is determined by the axial peak to average ratio. The amplitude sets the magnitude of the heat flux.

**Extrapolation Distance**

The axial peak to average ratio is defined to be

$$F_e \equiv \frac{q'(z_{\text{max}})}{\bar{q}}$$

where $z_{\text{max}}$ is the position of maximum heat flux in a particular channel, and $\bar{q}$ is the axially averaged heat flux in the same channel. Note, that since for any given channel $q'(z_{\text{max}})$ and $\bar{q}$ both contain the amplitude $q'_0$, this parameter cancels and the axial peak to average ratio is only a function of shape. The position of maximum heat flux is that location such that

$$\frac{dq'^{(x)}}{dz} \bigg|_{z_{\text{max}}} = 0$$

For this heat flux profile, the maximum heat flux does not occur at $\frac{H}{2}$, nor is the function evaluated at the position of maximum heat flux equal to one, such that $q'_0 \neq q'(z_{\text{max}})$. Determination of $z_{\text{max}}$ is further complicated by the fact that the solution for $z_{\text{max}}$ contains the extrapolation distance which is as of yet unknown. We can avoid this problem by defining a new variable

$$x = \pi \frac{H + \lambda - z}{H_e}$$

such that

$$q'^{(x)} = q'_0 x \sin(x)$$

and maximizing with respect to $x$

$$0 = \frac{d}{dx} \left[ q'_0 x \sin(x) \right]_{x_{\text{max}}}$$

$$0 = \sin(x_{\text{max}}) + x_{\text{max}} \cos(x_{\text{max}})$$

which is transcendental in $x_{\text{max}}$ and must be solved iteratively. Note, that since

$$z_{\text{max}} \in [0, H] \Rightarrow x_{\text{max}} \in (\pi, 0) \text{ for } \lambda > 0$$

Iterating on $x_{\text{max}}$ yields the solution $x_{\text{max}} = 2.029$. 

The axially averaged heat flux is defined to be
\[
\overline{q'} = \frac{1}{H} \int_0^H q'(z)dz = \frac{1}{H} \int_0^H q_0 \left( \frac{H + \lambda - z}{H_e} \right) \sin \left( \frac{\lambda}{H_e} \right) dz
\]
\[
\overline{q'} = q_0' \left[ \cos \left( \frac{\lambda}{H_e} \right) \lambda - \cos \left( \frac{(H + \lambda)}{H_e} \right) (H + \lambda) - \sin \left( \frac{\lambda}{H_e} \right) \frac{H_e}{\pi} + \sin \left( \frac{(H + \lambda)}{H_e} \right) \frac{H_e}{\pi} \right]^{-1}
\]

The axial peaking factor can then be written in terms of \( x_{\text{max}} \) and \( \overline{q'} \) as
\[
F_z = \frac{x_{\text{max}} \sin(x_{\text{max}})}{1 \left[ \cos \left( \frac{\lambda}{H_e} \right) \lambda - \cos \left( \frac{(H + \lambda)}{H_e} \right) (H + \lambda) - \sin \left( \frac{\lambda}{H_e} \right) \frac{H_e}{\pi} + \sin \left( \frac{(H + \lambda)}{H_e} \right) \frac{H_e}{\pi} \right]^{-1}}
\]

For \( H_e = H + 2\lambda \), this expression is transcendental in \( \lambda \) and must be solved iteratively. Iterating on \( \lambda \) gives \( \lambda = 1.797 \) feet.

**Heat Flux Profile**

From the definition of average heat flux, the magnitude of the heat flux profile is given by
\[
q_0^* = \overline{q'H} \left[ \cos \left( \frac{\lambda}{H_e} \right) \lambda - \cos \left( \frac{(H + \lambda)}{H_e} \right) (H + \lambda) - \sin \left( \frac{\lambda}{H_e} \right) \frac{H_e}{\pi} + \sin \left( \frac{(H + \lambda)}{H_e} \right) \frac{H_e}{\pi} \right]^{-1}
\]

For the data given here
\[
q_0^* = 1.18727 \times 10^4 \text{ Btu/hr-ft}^2
\]

The fluid properties assumed for this problem are
\[
\begin{align*}
\rho_f &= 45.99 \\
\rho_g &= 2.3426 \\
\mu_f &= 0.2192 \\
\mu_g &= 0.0460 \\
k_f &= 0.3289 \\
C_{pf} &= 1.2986 \\
\sigma &= 0.00118 \\
T_{sat} &= 549.43 \\
h_f &= 548.75 \\
h_{fg} &= 642.304
\end{align*}
\]

The coolant enters the channel subcooled, such that the potential exist for single phase forced convection over some portion of the channel. The outer clad surface temperature (in the absence of boiling) is given by
where the fluid temperature can be obtained directly from the enthalpy using a state equation of the form

\[ T_e(z) = T_{e_i} + \frac{q''(z)}{h_e} \]

The enthalpy is obtained from the simple energy balance

\[ h(z) = h(0) + \frac{1}{m} \int_0^z q'(z') \pi D_0 dz' \]

The channel mass flow rate is \( \dot{m} = GA_e \), where the cross sectional flow area is given by

\[ A_e = S^2 - \pi D^2 / 4 = 0.64^2 - \pi(0.493)^2 / 4 = 0.2187 \text{ in}^2 = 1.519 \times 10^{-3} \text{ ft}^2 \]

The channel mass flow rate is then

\[ \dot{m} = 1.21 \times 10^6 \times 1.519 \times 10^{-3} = 1838 \text{ lbm/hr} \]

**Convective Heat Transfer Coefficient**

From the Weisman Correlation

\[ h_e = \frac{k}{D_e} C \text{Re}^{0.8} \text{Pr}^{1/3} \]

where \( C = 0.042(S / D) - 0.024 = 0.042(0.64 / 0.493) - 0.024 = 0.0305 \).

If the Dittus-Boelter Correlation is to be used

\[ h_e = \frac{k}{D_e} \times 0.023 \times \text{Re}^{0.8} \text{Pr}^{0.4} \]

**Equivalent Diameter**

\[ D_e = \frac{4A_e}{\pi D} = \frac{4\left|S^2 - \pi D^2 / 4\right|}{\pi D} = 0.047 \text{ ft} \]

**Reynolds Number**

\[ \text{Re} = \frac{GD_e}{\mu} = \frac{(1.21 \times 10^6)(0.047)}{0.2192} = 259,833 \]
Prandtl Number

\[
Pr = \frac{C_p \mu}{k} = \frac{1.2986 \times 0.2192}{0.3289} = 0.865
\]

From which the convective heat transfer coefficient can be found

Weisman Correlation

\[
h_c = \frac{k}{D_e} C Re^{0.8} Pr^{1/3} = \frac{0.3289}{0.047} (0.0305)(259,833)^{0.8}(0.865)^{1/3} = 4,363 \text{ Btu/hr-ft}^2\text{-F}
\]

Dittus-Boelter Correlation

\[
h_c = \frac{k}{D_e} \times 0.023 \times Re^{0.8} Pr^{0.4} = \frac{0.3289}{0.047} (0.023)(259,833)^{0.8}(0.865)^{0.4} = 3,256 \text{ Btu/hr-ft}^2\text{-F}
\]

Location where the clad temperature exceeds the saturation temperature

The minimum criteria for boiling is that the wall temperature exceed the saturation temperature. If \( z_{sat} \) is the position at which the wall temperature reaches the saturation temperature, then \( z_{sat} \) is the solution of

\[
T_{sat} = T_w(z_{sat}) + \frac{q^*(z_{sat})}{h_c}
\]

Assuming the Weisman Correlation for the convective heat transfer coefficient, the solution for \( z_{sat} \) is iterative. For the data given here, the solution for \( z_{sat} \) gives \( z_{sat} = -0.53 \) feet. The negative sign implies that boiling is possible over the entire channel.

Wall Temperature Distributions

If the wall temperature is assumed to be given by the super position equation

\[
q''(z) = h_{FC} [T_w(z) - T_w(z)] + h_{NB} (z) [T_w(z) - T_{sat}]
\]

where the nucleate boiling heat transfer coefficient is given by

\[
h_{NB}(z) = \xi \times 10^6 (T_w(z) - T_{sat})^{m-1}
\]

this is a single nonlinear equation in the wall temperature and may be solved iteratively.

If the Chen correlation is to be used, once the wall temperature exceeds the saturation temperature, the wall temperature is the solution of

\[
q^*(z) = h_{bo} [T_{co}(z) - T_{co}(z)] + h_{2b} [T_{co}(z) - T_{sat}]
\]

where
\[ h_{lo} = \left[ 0.042 \times \frac{S}{D} - 0.024 \right] \times \left[ \frac{G(1-x)D_e}{\mu} \right]^{0.8} \left( \frac{C_p \mu}{k} \right)^{1/3} \left( \frac{k}{D_e} \right) F \]

if the Weisman Correlation is assumed for the Liquid Only portion of the heat transfer coefficient, and

\[ h_{lo} = 0.023 \times \left[ \frac{G(1-x)D_e}{\mu} \right]^{0.8} \left( \frac{C_p \mu}{k} \right)^{0.4} \left( \frac{k}{D_e} \right) F \]

if the Dittus-Boelter Correlation is assumed.

\[
F = \begin{cases} 
1.0 & \frac{1}{\chi_n} \leq 0.10 \\
2.35 \left( \frac{1}{\chi_n} + 0.213 \right)^{0.736} & \frac{1}{\chi_n} > 0.10 
\end{cases}
\]

\[
\frac{1}{\chi_n} = \left( \frac{x}{1-x} \right)^{0.9} \left( \frac{\rho_f}{\rho_g} \right)^{0.5} \left( \frac{\mu_g}{\mu_f} \right)^{0.1}
\]

\[ h_{2f} = 0.00122 \left[ \frac{k_f^{0.79} c_f^{0.45} P_f^{0.49} g_c^{0.25} (h_{fg} J T_{co} - T_{sat})^{0.75}}{\sigma^{0.5} \mu_f^{0.29} k_{fg}^{0.24} \rho_f^{0.24} T_{sat}^{0.75}} \right]^{0.99} S
\]

\[ S = 0.9622 - 0.5822 \times \tan^{-1} \left( \frac{Re_{2f}}{6.18 \times 10^4} \right)
\]

\[ Re_{2f} = \left( \frac{G(1-x)D_e}{\mu} \right)^{3.25}
\]

The single phase liquid component of the Chen correlation is equivalent to that for single phase forced convection prior to the fluid reaching the saturation point. If the nucleate boiling coefficient (\( h_{nb} \)) is set to zero prior to the wall temperature exceeding the saturation temperature, then the Chen correlation can be used over the entire channel. At any axial location, the heat flux, fluid temperature and enthalpy (quality) can be determined. The Chen Correlation is then in terms of the single unknown wall temperature \( T_{co} \). The dependence on wall temperature is nonlinear and \( T_{co} \) must be solved iteratively at each spatial location. The fluid temperature profile and the wall temperatures computed from the different correlations are indicated below. The maximum wall temperatures are 561.4 F if the Jens-Lottes correlation is assumed for the nucleate boiling correlation in the superposition approach, 562.1 F if the Thom correlation is assumed and 574.8 F according to the Chen correlation.
A comparison between the wall temperatures computed using the Weisman and Dittus-Boelter Correlations for the Liquid Only component is illustrated below. The maximum wall temperature using Dittus-Boelter for the Liquid Only heat transfer coefficient is 577.8F.
Fuel Pellet Surface Temperature

The fuel pellet surface temperature is related to the clad surface temperature and the axial heat flux through

\[ T_c(z) = T_{cs}(z) + q''(z) \left[ \frac{R_o}{R_l H_G} + \frac{R_o}{k_e} \ln \left( \frac{R_o}{R_i} \right) \right] \]

Fuel Centerline Temperature

The fuel centerline temperature is given by solution of the transcendental equation

\[ 3978.1 \ln \left( \frac{692.61 + T_c(z)}{692.61 + T_0(z)} \right) + \frac{6.02366 \times 10^{-12}}{4} \left\{ \left( T_c(z) + 460 \right)^4 - \left( T_e(z) + 460 \right)^4 \right\} = \frac{g''''(z) R^2}{4} \]

The following table summarizes the peak clad temperatures and peak fuel centerline temperatures for the different heat transfer coefficient choices.

<table>
<thead>
<tr>
<th>Heat Transfer Correlation</th>
<th>Peak Clad Temperature (F)</th>
<th>Peak Centerline Temperature (F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Superposition: Jens Lottes</td>
<td>561.4</td>
<td>2020</td>
</tr>
<tr>
<td>Superposition: Thom</td>
<td>562.1</td>
<td>2021</td>
</tr>
<tr>
<td>Chen: Weisman</td>
<td>574.8</td>
<td>2041</td>
</tr>
<tr>
<td>Chen: Dittus-Boelter</td>
<td>577.8</td>
<td>2046</td>
</tr>
</tbody>
</table>