A PWR operates under the nominal full power conditions given below.

a) Given an arbitrary axial heat flux profile of the form

\[ q'(z) = q_o Z(z) \]

show that prior to boiling (i.e. heat transfer is by single phase forced convection), the position of maximum clad temperature is independent of the magnitude of the heat flux \( q_o \).

b) Assuming nucleation begins at the point of maximum clad temperature, determine the reactor power at which nucleation begins in the highest powered channel.

c) Determine the reactor power at which fully developed nucleate boiling begins in the highest powered channel.

d) Determine the position and value of the maximum fuel centerline temperature under nominal full power conditions.

For parts b) - d), you may assume an axial heat flux profile of the form

\[ q'(z) = q_o \sin \left( \frac{\pi(z + \lambda)}{H_o} \right), \quad z \in [0, H] \]

where \( \lambda \) is the extrapolation distance and is chosen to match the axial peak to average ratio. Assume the Thom correlation is valid in the Nucleate Boiling regime.

**Problem Data**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core Thermal Output</td>
<td>3411 Mw</td>
</tr>
<tr>
<td>Fuel Height</td>
<td>144 inches</td>
</tr>
<tr>
<td>Rod Pitch</td>
<td>0.496 inches</td>
</tr>
<tr>
<td>Outer Clad Diameter</td>
<td>0.374 inches</td>
</tr>
<tr>
<td>Clad Thickness</td>
<td>0.0225 inches</td>
</tr>
<tr>
<td>Pellet Diameter</td>
<td>0.3225 inches</td>
</tr>
<tr>
<td>Gap Conductance</td>
<td>1000 Btu/hr-ft²-F</td>
</tr>
<tr>
<td>Clad Thermal Conductivity</td>
<td>9.6 Btu/hr-ft-F</td>
</tr>
<tr>
<td>System Pressure</td>
<td>2250 psia</td>
</tr>
<tr>
<td>Core Mass Flux</td>
<td>2.48 x 10⁶ lbm/hr-ft²</td>
</tr>
<tr>
<td>Core Inlet Temperature</td>
<td>552 F</td>
</tr>
<tr>
<td>Power Peaking Factor</td>
<td>2.50</td>
</tr>
<tr>
<td>Axial Peak to Average Ratio (Fz)</td>
<td>1.50</td>
</tr>
<tr>
<td>Number of Fuel Rods</td>
<td>50,952</td>
</tr>
<tr>
<td>Energy Deposited in Fuel</td>
<td>97.4 %</td>
</tr>
</tbody>
</table>

**SOLUTIONS**

a) The outer clad temperature satisfies

\[ T_{oc}(z) = T_o(z) + \frac{q'(z)}{h_c} \]

where the bulk fluid temperature is the solution of the simple energy balance.
\[
\frac{dT_{\text{en}}}{dz} = \frac{q^*(z) \pi D_u}{\dot{m} C_p}
\]

The maximum clad temperature occurs at the location \( z_{\text{max}} \) such that

\[
\frac{dT_{\text{en}}}{dz} \bigg|_{z_{\text{max}}} = 0 = \frac{dT_{\text{en}}}{dz} \bigg|_{z_{\text{max}}} + \frac{1}{h_c} \frac{dq^*}{dz} \bigg|_{z_{\text{max}}}
\]

For an arbitrary heat flux profile of the form \( q^*(z) = q_0^* Z(z) \)

\[
0 = \frac{q_0^* Z(z_{\text{max}}) \pi D_u}{\dot{m} C_p} + \frac{q_0^*}{h_c} \frac{dZ(z)}{dz} \bigg|_{z_{\text{max}}}
\]

or

\[
0 = \frac{Z(z_{\text{max}}) \pi D_u}{\dot{m} C_p} + \frac{1}{h_c} \frac{dZ(z)}{dz} \bigg|_{z_{\text{max}}}
\]

which is independent of the magnitude of the heat flux profile and depends only on the shape function \( Z(z) \).

b) **Heat Flux**

The heat flux profile is in terms of two unknown parameters, the extrapolation distance \( \lambda \) and the amplitude \( q_0^* \). The extrapolation distance is determined by the axial peak to average ratio. The amplitude sets the magnitude of the heat flux. At nominal operating conditions

**Rod Surface Heat Fluxes**

**Average Channel**

\[
\overline{\dot{q}} = \gamma \frac{\dot{Q}}{n2\pi R_c H} = 189,400 \text{ Btu/hr-ft}^2
\]

**Hot Channel**

\[
q_{\text{max}}^* = F_z \overline{\dot{q}}^* = 473,500 \text{ Btu/hr-ft}^2
\]

**Extrapolation Distance**

The axial peak to average ratio is defined to be

\[
F_z = \frac{q^*(z_{\text{max}})}{\overline{\dot{q}}^*}
\]

where \( z_{\text{max}} \) is the position of maximum heat flux in a particular channel, and \( \overline{\dot{q}}^* \) is the axially averaged heat flux in the same channel. Note, that since for any given channel \( q^*(z_{\text{max}}) \) and \( \overline{\dot{q}}^* \) both contain the amplitude
\(q_0^a\), this parameter cancels and the axial peak to average ratio is only a function of shape. The position of maximum heat flux is that location such that

\[
\frac{d}{dz} q^a \bigg|_{z_{\text{max}}} = 0
\]

For this heat flux profile, the maximum heat flux occurs at \(\frac{H}{2}\), such that

\[
q_0^a = q^a_{\text{max}}
\]

The axially averaged heat flux is defined to be

\[
\overline{q} = \frac{1}{H} \int_0^H q^a(z)dz = \frac{1}{H} \int_0^H q_0^a \sin \left( \frac{\pi (z + \lambda)}{H_e} \right) dz
\]

\[
\overline{q} = \frac{q_0^a H_e}{\pi H} \left[ \cos \left( \frac{\pi \lambda}{H_e} \right) - \cos \left( \frac{\pi (H + \lambda)}{H_e} \right) \right]
\]

The axial peaking factor is then

\[
F_e = \frac{\pi}{H_e \left[ \cos \left( \frac{\pi \lambda}{H_e} \right) - \cos \left( \frac{\pi (H + \lambda)}{H_e} \right) \right]}
\]

For \(H_e = H + 2 \lambda\), this expression is transcendental in \(\lambda\) and must be solved iteratively. Iterating on \(\lambda\) gives \(\lambda = 0.3009\) feet.

**Coolant Enthalpy and Temperature Distributions**

The fluid temperature distribution is given by \(T_w(z) = T_w(h(z))\), where \(h(z)\) is the enthalpy distribution and is given by

\[
h(z) = h(0) + \frac{1}{m_T f} \int_0^z q^a(z') \pi D dz'
\]

which for the heat flux profile given here yields

\[
h(z) = h(0) + \frac{q_0^a H D}{m_T f} \left[ \cos \left( \frac{\pi \lambda}{H_e} \right) - \cos \left( \frac{\pi (z + \lambda)}{H_e} \right) \right]
\]

**Channel Flow Area**

\[
A_c = S^2 - \pi \frac{D^2}{4} = 9.455 \times 10^{-4} \text{ ft}^2
\]
Channel Mass Flow Rate

\[ \dot{m}_{\text{channel}} = G \times A_x = 2344.9 \ \text{lbm/hr} \]

Channel Exit Conditions

For the given problem parameters, and a channel inlet enthalpy of \( h(0) = 549.69 \ \text{Btu/lbm} \), the channel exit enthalpy in the hot channel is \( h(H) = 712.09 \ \text{Btu/lbm} \). The enthalpy of a saturated liquid at 2250 psia is 700.95 Btu/lbm such that the coolant leaves the channel as a saturated mixture. The fluid properties are evaluated at the average fluid temperature in the subcooled region of the channel

\[ T_{\text{ave}} = \frac{(T_{\text{sat}} + T_{\text{sat}})}{2} = \frac{(652.74 + 552)}{2} = 602.4 \]

giving

\[ C_p = 1.4325 \]

\[ \mu = 0.1979 \]

\[ k = 0.3052 \]

The channel saturation properties are taken at 2250 psia

\[ T_{\text{sat}} = 652.74 \]

\[ h_f = 700.95 \]

\[ h_{fg} = 415.01 \]

**Convective Heat Transfer Coefficient**

From the Weisman Correlation

\[ h_c = \frac{k}{D_e} C \text{Re}^{0.8} \text{Pr}^{1/3} \]

where \( C = 0.042(S / D) - 0.024 = 0.042(0.496 / 0.374) - 0.024 = 0.0317 \).

Equivalent Diameter

\[ D_e = \frac{4A_x}{\pi D} = \frac{4(0.0386)^2}{\pi(0.0386)} = 0.0386 \ \text{ft} \]

Reynolds Number

\[ \text{Re} = \frac{GD_e}{\mu} = \frac{(2.48 \times 10^4)(0.0386)}{0.1979} = 484,100 \]

Prandtl Number

\[ \text{Pr} = \frac{C_p \mu}{k} = \frac{1.4325 \times 0.1979}{0.3052} = 0.929 \]
From which the convective heat transfer coefficient can be found to be

\[
h_e = \frac{k}{D_e} C Re^{0.8} Pr^{1/3} = \frac{0.3052}{0.0386} (0.0317)(484,100)^{0.8} (0.929)^{1/3} = 8,630 \text{ Btu/hr-ft}^2\text{-F}
\]

**Position of Maximum Clad Temperature**

Assuming heat transfer is by single phase forced convection, the position of maximum clad has been shown to satisfy

\[
0 = \frac{Z(z_{max})\pi D_e}{mC_p} + \frac{1}{h_e} \left| \frac{dZ(z)}{dz} \right|_{z_{max}}
\]

For a heat flux profile of the form

\[
q^*(z) = q_0^* \sin \left( \frac{\pi(z + \lambda)}{H_e} \right)
\]

\[
0 = \frac{\pi D_e}{mC_p} \sin \left( \frac{\pi(z_{max} + \lambda)}{H_e} \right) + \frac{1}{h_e} \frac{\pi}{H_e} \cos \left( \frac{\pi(z_{max} + \lambda)}{H_e} \right)
\]

The solution is iterative and gives \( z_{max} = 9.168 \text{ ft}. \)

**Heat Flux to Cause Nucleation at the Position of Maximum Clad Temperature**

The minimum criteria for boiling is that the clad temperature exceeds the saturation temperature. For the heat flux profile considered here, the clad temperature is given by

\[
T_{co}(z_{max}) = T_e(z_{max}) + \frac{q_0^*}{h_e} \sin \left( \frac{\pi(z_{max} + \lambda)}{H_e} \right)
\]

where the bulk coolant temperature is

\[
T_e(z_{max}) = T_e(0) + \frac{q_0^* H_e D_e}{mC_p \gamma_f} \left[ \cos \left( \frac{\pi}{H_e} \right) - \cos \left( \frac{\pi(z_{max} + \lambda)}{H_e} \right) \right]
\]

For \( T_{co}(z_{max}) = T_{sat} \), the only unknown is the magnitude of the heat flux and is equal to \( q_0^* = 351,600 \text{ Btu/hr-ft}^2 \).

The heat flux necessary to cause nucleation must then be greater than this value. The criteria for nucleation at the position of maximum clad temperature is given by

\[
q^*(z_{max}) = 15.6 \rho^{1.156} [T_{co}(z_{max}) - T_{sat}]^{2.30}
\]

For \( q^*(z_{max}) = q_0^* \sin \left( \frac{\pi(z_{max} + \lambda)}{H_e} \right) \)
\[
T_{\text{sat}} (z_{\text{sat}}) = T_{\infty} (z_{\text{sat}}) + \frac{q^*}{h_c} \sin \left( \frac{\pi (z_{\text{sat}} + \lambda)}{H_e} \right)
\]

\[
T_{\infty} (z_{\text{sat}}) = T_{\infty} (0) + \frac{q^* H D}{m C_p T_f} \left[ \cos \left( \frac{\lambda}{H_e} \right) - \cos \left( \frac{\pi (z_{\text{sat}} + \lambda)}{H_e} \right) \right]
\]

The only unknown is the magnitude of the heat flux which can be solved for iteratively. For the parameters given here \( q^* = 356,800 \) Btu/hr-ft\(^2\). The reactor power corresponding to this maximum heat flux in the hot channel is

\[
\dot{Q} = 3411 \times \frac{356,800}{473,500} = 2570 \text{ Mw}
\]

c) Under nominal full power conditions, it has been shown that the fluid leaves the channel as a saturated liquid. We can therefore assume that under nominal conditions the channel operates under single phase forced convection, mixed boiling and fully developed nucleate boiling. We first determine the transition points under these conditions.

**Location where the clad temperature exceeds the saturation temperature**

If \( z_{\text{sat}} \) is the position at which the wall temperature reaches the saturation temperature, then \( z_{\text{sat}} \) is the solution of

\[
T_{\text{sat}} = T_{\infty} (z_{\text{sat}}) + \frac{q^* (z_{\text{sat}})}{h_c}
\]

The fluid temperature distribution is given by \( T_{\infty} (z) = T_{\infty} (h(z)) \), where \( h(z) \) is the enthalpy distribution and is given by

\[
h(z) = h(0) + \frac{q^* H D}{m C_p T_f} \left[ \cos \left( \frac{\lambda}{H_e} \right) - \cos \left( \frac{\pi (z + \lambda)}{H_e} \right) \right]
\]

The solution for \( z_{\text{sat}} \) is iterative. For the given data, the solution for \( z_{\text{sat}} \) gives \( z_{\text{sat}} = 5.064 \) feet. This implies that boiling is possible over the upper half of the channel.

**Transition from Single Phase Forced Convection to Nucleate Boiling**

**Incipient Boiling Point (nucleation point)**

The transition from single phase forced convection to mixed boiling is assumed to occur where the wall temperature predicted by the single phase forced convection correlation is equal to that predicted by an incipient boiling correlation. The wall temperature under single phase forced convection is

\[
T_{\text{ceo}} (z) = T_{\infty} (z) + \frac{q'' (z)}{h_c}
\]

and assuming the incipient boiling correlation

\[
q^* (z_e) = 15.6 P^{1.156} \left[ T_{\text{ceo}} (z_e) - T_{\text{sat}} \right]^{2.3}
\]
the solution for $z_n$ is iterative. For the given data, $z_n = 5.186$ ft.

**Fully Developed Nucleate Boiling Point**

The transition from mixed boiling to fully developed nucleate boiling is assumed to occur where the wall temperature predicted by the mixed boiling correlation is equal to that predicted by the fully developed nucleate boiling correlation. Assuming the Bergles and Rohsenow correlation is valid in the mixed boiling region, then the transition point to fully developed nucleate is the location $z_B$ which satisfies

$$q^*(z_B) = q^*_{\text{FC}}(z_B) \left[ 1 + \frac{q^*(z_B)}{q^*_{\text{FC}}(z_B)} \left( 1 - \frac{q^*_{\text{NB}}(z_n)}{q^*(z_B)} \right) \right]^{1/2}$$

where: $q^*(z)$ is the operating heat flux profile

$$q^*_{\text{FC}}(z_B) = h_c [T_w(z_B) - T_n(z_B)]$$

$$T_w(z_B) = T_{\text{sat}} + \left( \frac{q^*(z_B)}{\xi \times 10^6} \right)^{1/m} \quad (\text{Wall temperature from the fully developed nucleate boiling correlation})$$

$$q^*_{\text{NB}}(z_n) = \xi \times 10^6 [T_w(z_n) - T_{\text{sat}}]^m$$

$$T_w(z_n) = T_e(z_n) + \frac{q^*(z_n)}{h_c}$$

$$T_n(z) = T_n(h(z))$$

$$h(z) = h(0) + \frac{q^*_{\text{HD}} D}{m \rho_f \gamma} \left[ \cos \left( \frac{\pi \lambda}{H_x} \right) - \cos \left( \frac{\pi (z + \lambda)}{H_x} \right) \right]$$

The Thom Correlation gives

$$\xi = \frac{\exp \left( \frac{2P}{1260} \right)}{72^2}$$

$m = 2$

The solution for $z_B$ gives $z_B = 8.793$ feet. Nucleate boiling is assumed for all elevations above this point. As the magnitude of the heat flux is reduced from the nominal full power value, both $z_n$ and $z_B$ increase. The easiest way to determine the power level at which fully developed nucleate boiling begins is to decrease the magnitude of the heat flux until the point $z_B$ reaches the channel exit. The table below illustrates this.
The final value is sufficiently close and yields a reactor thermal output of

\[ \dot{Q} = 3411 \times \left( \frac{4.2235}{4.735} \right) = 3042.5 \text{ Mw} \]

d) **Fuel Centerline Temperature**

The fuel centerline temperature for a uniform radial volumetric heat generation rate is given by solution of the transcendental equation

\[
3978.1 \ln \left( \frac{692.61 + T_c(z)}{692.61 + T_w(z)} \right) + \frac{6.02366 \times 10^{-12}}{4} \left( T_c(z) + 460 \right)^4 - \left( T_w(z) + 460 \right)^4 = -q''(z)R^2/4
\]

where the volumetric heat generation rate is given in terms of the heat flux by

\[ q''(z) = \frac{2 \pi R_0 q''(z)}{\pi R^2} = \frac{2 R_0 q''(z)}{R^2} \cdot \]

The fuel pellet surface temperature is related to the clad surface temperature and the axial heat flux through

\[ T_s(z) = T_{w0}(z) + q''(z) \left( \frac{R_o}{R_s H_{Gc}} + \frac{R_w}{k_c} \ln \left( \frac{R_o}{R_w} \right) \right) \]

The peak fuel centerline temperature lies in the vicinity of the peak heat generation rate. For this problem, the peak heat generation rate occurs at \( H/2 \), such that the peak fuel centerline temperature will occur in the mixed boiling region where the wall temperature is obtained from

\[ q''(z) = q''_{FC}(z) \left( 1 + \left[ \frac{q''_{sat}(z)}{q''_{FC}(z)} \left( 1 - \frac{q''_{sat}(z)}{q''_{FC}(z)} \right) \right]^{2/3} \right) \]

where:

\[ q''(z) \text{ is the operating heat flux profile} \]

\[ q''_{FC}(z) = h_c [T_{w0}(z) - T_{w}(z)] \]

\[ q''_{sat}(z) = 6 \times 10^6 [T_{w0}(z) - T_{sat}] \]
\[ q_{wb}^*(z) = \xi \times 10^6 [T_{wb}(z) - T_{wb}]^\eta \]

\[ T_{co}(z) = T_{co}(z) + \frac{q^*(z)}{h_c} \]

\[ T_o(z) = T_o(h(z)) \]

Solution for the outer clad temperature, fuel pellet surface temperature and fuel centerline temperature in the vicinity of H/2 is given in the table below.

<table>
<thead>
<tr>
<th>z (inches)</th>
<th>(T_{co})</th>
<th>(T_c)</th>
<th>(T_o)</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>658.7</td>
<td>1295</td>
<td>3771</td>
</tr>
<tr>
<td>71</td>
<td>658.9</td>
<td>1296</td>
<td>3773</td>
</tr>
<tr>
<td>72</td>
<td>659.1</td>
<td>1296</td>
<td>3774</td>
</tr>
<tr>
<td>73</td>
<td>659.2</td>
<td>1296</td>
<td>3774</td>
</tr>
<tr>
<td>74</td>
<td>659.3</td>
<td>1296</td>
<td>3772</td>
</tr>
</tbody>
</table>

The maximum fuel centerline temperature is 3774 °F and occurs between 72 and 73 inches in the channel.