It has been shown, that void and quality and be related through the Fundamental Void-Quality-Slip Relation

\[ \alpha = \frac{1}{1 + \frac{1 - x \rho_s}{x \rho_t} S} \]

Drift Velocity, defined to be \( V_g = \alpha \nu_r = \alpha (v_g - v_l) \), is more commonly used to correlate relative phase motion than the slip ratio \( S = v_g / v_l \). Show that void and quality can be related by

\[ \alpha = \frac{1}{1 + \frac{1 - x \rho_s}{x \rho_t} + \frac{\rho_s V_g}{Gx}} \]

**SOLUTION**

The Slip Ratio is defined as

\[ S = \frac{v_g}{v_l} \]

In terms of relative velocity

\[ S - 1 = \frac{v_g}{v_l} - \frac{v_l}{v_l} = \frac{v_g}{v_l} - \frac{v_l}{v_l} \]

or

\[ S = 1 + \frac{v_g}{v_l} \]

The liquid phase velocity can be written in terms of the total mass flux as

\[ v_l = \frac{G(1-x)}{\alpha \rho_t} \]

giving

\[ S = 1 + \frac{\alpha \rho_s v_l}{G(1-x)} = 1 + \frac{\rho_s V_g}{G(1-x)} \]

In addition, from the Fundamental Void-Quality-Slip Relationship

\[ S = \frac{x \rho_s}{1-x \rho_s} \frac{\rho_t}{\rho_t} = \frac{x(1-\alpha) \rho_t}{1-x \alpha \rho_s} \]
such that

$$\frac{x(1-\alpha) \rho_I}{1-x} \rho_s = 1 + \frac{\rho_{V_g}}{G(1-x)}$$

which can be solved for the void fraction

$$\alpha = \frac{1}{1 + \frac{1-x \rho_s}{x \rho_I} + \frac{\rho_{V_g}}{Gx}}$$