1) It has been shown, that void and quality and be related through the Fundamental Void-Quality-Slip Relation

\[
\alpha = \frac{1}{1 + \frac{1-x}{x} \frac{\rho_g}{\rho_r} S}
\]

Drift Velocity, defined to be \( V_g = \alpha, v_r = \alpha_s(v_g - v_r) \), is more commonly used to correlate relative phase motion than the slip ratio \( S = \frac{v_g}{v_r} \). Show that void and quality can be related by

\[
\alpha = \frac{1}{1 + \frac{1-x}{x} \frac{\rho_g}{\rho_r} + \frac{\rho_g V_g}{Gx}}
\]

2) The relative velocity, and therefore the drift velocity correlations are generally flow regime dependent. One example of these correlations are those utilized in early versions of the TRAC code given below

**Bubbly Flow**

\[
v_r = \frac{1.41}{\alpha_s} \left[ \frac{\sigma g g_r (\rho_r - \rho_g)}{\rho_r^2} \right]^{3/4}
\]

**Slug Flow**

\[
v_r = \frac{0.345}{\alpha_s} \left[ g D_t (\rho_r - \rho_g) \right]^{1/2}
\]

**Churn-Turbulent**

\[
v_r = \frac{v}{1 - C \alpha_s \frac{\alpha_s \rho_g}{\rho}} + \frac{\alpha_s \rho_g}{\rho}
\]

Where \( C = 1.1 \) and \( \alpha_s \) is limited to a maximum value of 0.8.

**Annular Flow**

\[
v_r = \frac{v}{\left[ \frac{\rho_r (76 - 75 \alpha_s)}{\rho_r \sqrt{\alpha_g \rho_g}} \right]}^{1/2} + \frac{\alpha_s \rho_g}{\rho}
\]

The corresponding flow regime map is attached, where the dashed lines mark transition regions between flow regimes. In these regions, the relative velocity is linearly interpolated between the boundary values. An alternate approach is to use a flow regime independent void/quality relation, such as the Zuber-Findlay correlation to determine void fraction. The Zuber-Findlay correlation is given as
\[
\alpha_s = \frac{1}{\left( C_o \left[ 1 + \frac{1-x}{x} \frac{\rho_g}{\rho_t} \right] + \frac{\rho_g V_g}{Gx} \right)}
\]

where again \( V_g \) is the Drift Velocity and \( C_o \) is the Concentration Parameter. Both are correlated parameters.

One such correlation by Dix is
\[
C_o = \beta \left[ 1 + \left( \frac{1}{\beta} - 1 \right)^x \right]
\]

\[
b = \left( \frac{\rho_g}{\rho_t} \right)^{0.1}
\]

\[
\beta = \frac{x}{x + (1-x)\frac{\rho_g}{\rho_t}}
\]

\[
V_g = 2.9 \left( \frac{\sigma_{gg} (\rho_t - \rho_g)}{\rho_g^2} \right)^{0.6}
\]

An alternate correlation for high pressure steam-water flows assumes
\[
C_o = 1.13
\]

\[
V_g = 1.41 \left( \frac{\sigma_{gg} (\rho_t - \rho_g)}{\rho_t^2} \right)^{0.6}
\]

independent of flow regime.

Given void and quality, the phase velocities can be computed from

\[
v_t = \frac{G(1-x)}{\alpha_s \rho_t}
\]

\[
v_g = \frac{Gx}{\alpha_s \rho_g}
\]

A uniformly heated, boiling water reactor channel has parameters listed below.

a) For the given data, compute and plot the void distribution in the channel with the void-quality relation derived in problem 1 and the flow regime dependent relative velocities given above. Indicate on your plot the associated flow regimes computed from the TRAC flow regime map and the Hewitt and Roberts Map included in the notes.

b) Repeat part a) using the flow regime independent Zuber-Findlay correlation with the two different relations for the concentration parameter and drift velocity.

c) Compare the void fraction predictions from the three different models.
For the purposes of this assignment, you can assume the flow quality is equal to the equilibrium quality.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>Heat Flux</td>
<td>144,032 Btu/hr-ft²</td>
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<tr>
<td>Pressure</td>
<td>1040 psia</td>
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<tr>
<td>Coolant Mass Flux</td>
<td>1.21 x 10⁶ lbm/hr-ft²</td>
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<tr>
<td>Channel Inlet Enthalpy</td>
<td>527.9 Btu/lbm</td>
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<tr>
<td>Channel Height</td>
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<tr>
<td>Rod Pitch</td>
<td>0.640 inches</td>
</tr>
<tr>
<td>Rod Diameter</td>
<td>0.493 inches</td>
</tr>
</tbody>
</table>