Brent’s Algorithm

Brent’s Algorithm combines the best features of the Newton-Raphson and Bisection techniques. Similar to the Bisection technique, Brent’s algorithm assumes the root can be bracketed in an interval \([a,b]\). Due to its rapid convergence rates, Brent’s algorithm first attempts to find the root using the Newton-Raphson scheme. If the estimate of the root falls outside the interval \([a,b]\), then Bisection is used to narrow the interval and the process is repeated. Eventually, Bisection will narrow the interval sufficiently such that the Newton-Raphson scheme will converge.

Algorithm

Given a function \(F(x)\) for which the root is to be found:

a) Pick \([a,b]\)

b) If \(F(a)*F(b) > 0 \Rightarrow\) Stop, the function does not cross the axis within \([a,b]\)

c) If \(F(a)*F(b) < 0\) Then

\[
\begin{align*}
x_0 &= \frac{a+b}{2} \\
\delta &= - \frac{F(x_0)}{F'(x_0)}
\end{align*}
\]

while \(|\delta/x_0| > \varepsilon\)

\[
x = x_0 + \delta
\]

if \(x \notin [a,b]\) then \(x = \frac{a+b}{2}\)

if \(F(a)*F(x) > 0\) then \(a = x\)

if \(F(a)*F(x) < 0\) then \(b = x\)

\[
x_0 = x
\]

\[
\delta = - \frac{F(x_0)}{F'(x_0)}
\]

continue