SIMULATION COUNCIL

Newsletter

Vol. XI No. 3

John McLeod, EDITOR
Suzette McLeod, SECRETARY
8484 La Jolla Shores Drive, La Jolla, California

MAUGHAN MASON

Simulation Councils Inc.
Maughan S. Mason, Thiokol Chemical Corporation, Wesotch Division, Brigham City, Utah; Chairman, Board of Directors

Western Simulation Council
Harald Skramstad, Naval Ordnance Laboratory, Corona, California; Chairman, Steering Committee

Midwestern Simulation Council
Robert T. Harnett, ASNCC, Aeronautical Systems Division, Wright-Patterson Air Force Base, Ohio; Chairman, Steering Committee

Eastern Simulation Council
David Jordan, Republic Aviation Corporation, Mineola, New York; Chairman, Steering Committee

Southeastern Simulation Council
William C. Bennett, Lockheed Aircraft Corporation, Marietta, Georgia; Chairman, Steering Committee

Central Simulation Council
Donald C. Augustin, McDonnell Automation Center, St. Louis, Missouri; Chairman, Steering Committee

Rocky Mountain Simulation Council
Dee H. Barker, Brigham Young University, Provo, Utah; Chairman, Steering Committee

DDA Council Inc.
Bill Farrand, Automationics, Anaheim, California; Chairman, Board of Directors

Bits

Pieces—The Southwestern Simulation Council meeting at Georgia Tech on November 16, 1962:
1. Analog Simulation in the Study of Tunnel Diode Oscillators by R. D. Shults, Georgia Tech, Electrical Engineering Dept.

Analog Techniques—Ways of closing circuits when A < t < B.

Just as your editor was “researching” old files in the hope of dredging up something which might be worked into this months N/L the following letter was received from James Williams of Southeastern Simulation Council:
“Our last meeting took place November 16, 1962 with the Georgia Institute of Technology as host. The program consisted of two papers which are enclosed: THE ANALOG SIMULATION IN THE STUDY OF TUNNEL DIODE OSCILLATORS by Dave Shults and AUTOMATIC OPTIMIZATION OF SIX PARAMETERS SOME PRELIMINARY WORK ON LOGIC & CONTROL CIRCUITS by Max Howell.

“Another paper entitled THE USE OF ANALOG COMPUTERS IN SANITARY ENGINEERING was given by Dr. Karl Schnelle of Vanderbilt University; however, this paper was not presented to us for public release.

“These papers were given before lunch along with a short business meeting for the purpose of selecting officers for the coming year. Following lunch, all participated in a panel discussion on the topic of MULTIPLIERS THEIR FUNCTION AND USE WITH ANALOG COMPUTORS. Members of the panel were: S. W. Robinson, Lockheed-Georgia Company, Marietta, Georgia.

J. W. Tootle, Martin-Marietta Corporation, Orlando, Florida.
Joe Hammond, Electrical Engineering Department, Georgia Institute of Technology.
Charles Riley, NASA, George C. Marshall Space Flight Center, Huntsville, Alabama. “Thirty were in attendance and enjoyed the hospitality of the Tech Engineers and the facilities of their new Electrical Engineering Building.”

Bits

(Mrs.) Janie Owen for
James L. Williams
Secretary-Treasurer
Southeastern Simulation Council

In the columns that follow we give you the Shults paper in its entirety because, like the GESEE techniques covered in the November ’62 issue of the N/L, this is an interesting and useful example of how analog computers can be used to simulate the dynamic response of electronic circuits. In this case your Ed. finds the direct recording of a phase-plane plot plus a simultaneous recording of the waveform for various combinations of circuit parameters particularly interesting.

The paper by Max Howell left this reviewer at the post when the author, without warning or explanation, started throwing Bessel Functions around in the first paragraph. Further inspection, however, disclosed some methods for search and optimization with a rep-op computer which might be of interest, so rather than attempting to rewrite the article in language I can understand, or emasculating it in an attempt to abstract, I will pass it on as received.

ROCKY MOUNTAIN SIMULATION COUNCIL

(Preliminary Announcement)
Next meeting: White Sands Proving Grounds (via El Paso, Texas)
Date: Friday, March 15
Theme: "New Techniques and Equipment"

March 1963—Instruments & Control Systems—Page 137
you never saw anything like this before

Looks like an ordinary plot of a damped system. It's a damped system, all right, but that's no ordinary plot. First, it's a recording (actual) of a 4,000 cps signal and, second, it represents over 30,000 repetitive solutions. Unless you've seen this done on a GPS Analog Computer, you've never seen it.

Computing in compressed time and recording in slow time is but one of the many capabilities of the GPS General Purpose Analog Computer. On a scope, the entire curve is seen at all times, so it is easy to manipulate equation parameters to observe their effect on the entire solution... no waiting... no need to record trial data... record only what you want to keep. □ The GPS Analog Computer System operates in real time or in compressed time of up to 3,000 to 1. Its broadband operation (amplifier bandwidths to 1 megacycle) is unique in the analog field. □ A complete line of computer modules is available... statistical units, logic control, memory units... the gamut. Write for Technical Information or call Area Code 617, DEcatur 2-8110.
Analog Simulation in the Study of Tunnel Diode Oscillators

R. D. Shults, Georgia Institute of Technology, Atlanta

Since the advent of the Esaki tunnel diode in 1957 there has been a widespread and rapidly growing interest in its application. Since the tunnel diode is a negative resistance device, it is well suited to oscillator applications.

The diode presents a static E-I characteristic of the form shown in Figure 1 and, when biased in the region of negative dynamic resistance, may be represented by the equivalent circuit shown in Figure 2, where $R_s = $ series resistance; $L_s = $ series inductance; $C_d = $ junction capacitance, $p = $ negative resistance.

A practical form of a two-terminal negative resistance oscillator using a tunnel diode is shown in Figure 3.

Under the assumption of perfect by-passing and negligible series resistance and inductance in the diode, the incremental model of the oscillator circuit is as shown in Figure 4.

The node equation for the circuit is:

$$i_n = -i_c - ip$$

$$= -C \frac{de}{dt} - f(e) \ldots (1)$$

The output voltage is:

$$e(t) = L \frac{di_n}{dt} + Ri_s \ldots (2)$$

Substitution of equation (1) into equation (2) yields the differential equation which describes the oscillator.

$$\frac{d^2e}{dt^2} + \frac{R}{L} \frac{de}{dt} + \frac{1}{LC} e + \frac{1}{C} \frac{d}{dt} [f(e)] + \frac{R}{LC} [f(e)] = 0 \ldots (3)$$

Under the assumption of negligible series resistance in the diode, the function $f(e)$ is represented by the characteristic of the diode when the origin of coordinates is placed at the bias point, as shown in Figure 5.

Since equation (3) is nonlinear, the solution is very difficult to obtain analytically. This paper is a discussion of the solution of equation (3) by means of an analog computer.

**Function Generator**

The first problem in programming equation (3) on the analog computer is the construction of a function generator which will represent the function $f(e)$. A basic circuit which illustrates the principle of the function generator is shown in Figure 6. The voltage $E_1$ is reduced to the proper level by amplifier 1, and since point A is held at virtual ground by the high gain of operational amplifier 2, the voltage $E$ appears across the tunnel diode. The diode will then draw a current $I_d = F(E)$ which produces an output from amplifier 2 proportional to the diode characteristic $F(E)$.

Because of the inherent instability of the tunnel diode when biased in the region of negative resistance, a practical form of the function generator requires that the tunnel diode be stabilized by shunting it with a resistance. The value of the resistance used must be found by con-
REAC® 500 ANALOG COMPUTER
OFFERING MAXIMUM EXPANSION FLEXIBILITY AT MINIMUM COST

The Reliability of REAC Analog Computers has been proven through years of unmatched performance, with typical unscheduled down-time averaging less than 3%.

The new REAC 500 carries forward the Reeves tradition of built-in reliability. Put this proven performance to work for you. Let us help you with your procurement planning — it will be to your advantage.

For further information on the REAC 500, write for Data File 903

Qualified engineers who are seeking rewarding opportunities for their talents in this and related fields are invited to get in touch with us.

REEVES INSTRUMENT CORPORATION
A Subsidiary of Dynamics Corporation of America • Roosevelt Field, Garden City, New York
CIRCLE 133 ON READER-SERVICE CARD
Considering the equivalent circuit of the parallel combination, shown in Figure 7. The driving point impedance, \( Z(s) \), is
\[
Z(s) = \frac{R_0 \left[ S^2 + \frac{R_e S}{L_e} + \frac{G_a}{C_a} S + \left( \frac{1 - R_e G_a}{L_e C_a} \right) \right]}{S^2 + \left( \frac{R_e + R_p}{L_e} \right) S + \frac{G_a}{L_e C_a} S + \frac{1 - (R_e + R_p) G_a}{L_e C_a}}.
\]

(4)

If the network is to be non-oscillatory, then \( Z(s) \) must be short-circuit stable which requires that all its poles fall in the left half S plane. This requires that \( R_p \) fall in the range corresponding to
\[
L_e \frac{G_a}{C_a} < R_e + R_p < \frac{1}{G_a}.
\]

(5)

For a GE IN2940 tunnel diode, an \( R_p \) of 100 ohms is a suitable value so that the parallel combination of \( R_p \) and the diode is stable and at the same time the impedance of the combination is high enough to avoid excessive current drain from the operational amplifier. The resistance \( R_p \) must be mounted as close as is physically possible to the diode in order to keep the parasitic circuit parameters at a minimum. After the addition of \( R_p \), a stabilized E-I characteristic is obtained as shown in Figure 8.

An output which is proportional to the actual diode characteristic \( F(E) \) is then obtained by subtracting the characteristic of the stabilizing resistor from the overall characteristic of Figure 8.

The resulting function generator is shown in Figure 9, and its operation may be described as follows. Since point A is held at virtual ground, the voltage \( E \) appears across the parallel combination of the 100 \( \Omega \) resistor and the tunnel diode. The current drawn by the combination consists of the diode current \( F(E) \), and the current in the resistor \( \frac{E}{100} \). This total current flows through the 5000 ohm feedback resistance around amplifier 2 so that the output voltage of the amplifier is
\[
E_o = 5000 \left[ F(E) + \frac{E}{100} \right] - 5000 \left[ \frac{E}{100} \right] \]

(6)

The voltage \( E_o \) is then fed to the sign-changer of amplifier 3 so that its output is:
\[
E_o = -5000 \left[ F(E) + \frac{E}{100} \right] - 5000 \left[ \frac{E}{100} \right] \]

(7)

By inspection, the output of amplifier 4 is:
\[
E_o = 5000 \left[ - \frac{E}{100} \right] \]

(8)

The voltages \( E_o \) and \( E_4 \) are then fed to the summing amplifier 5 whose output is:
\[
E_o = - \frac{E_o + E_4}{5000} \]

(9)

Therefore, the output of the circuit is:
\[
E_o = 5000 F(E) \]

(10)

which is the desired result.

The feedback resistance around amplifiers 2 and 4 was chosen as low as 5000 ohms so that the effective gain of 2 and 4 is not so high that small disturbances will cause these amplifiers to saturate. The overall circuit may then be calibrated by replacing the tunnel diode by a precision resistor. Since the output is 5000 \( F(E) \), the diodes in use, having a typical peak current of 1 milliampere over the range of interest, produce a peak output of 5 volts. An example of the function generator output is shown in Figure 10.

The circuit of Figure 9 is actually not a function generator in the true sense of the phrase. Rather, it is a characteristic generator, since it gives the characteristic of the diode with respect to the true origin. The function generator required is one which gives an output as shown in Figure 5. The origin of the coordinates must be movable and the output must be zero at that point.

The circuit of Figure 9 is easily modified so that it is a true function generator. Consider the situation depicted by Figure 11. The origin of the function \( f(e) \) is to be fixed at the bias voltage \( E_b \). At this point the quiescent current is \( I_1 \), and the output of the circuit is then 5000 \( I_1 \) volts. The circuit of Figure 9 may be modified.
FIG. 11. TRANSLATION of origin.

fed by adding an additional DC
input to amplifier 1 so that any bias
voltage $E_1$ may be chosen for the
coordinates. The DC output voltage,
5000 $I_a$, is then eliminated by feeding
a DC voltage of the proper magnitude
and polarity to the input of the sum-
mixing amplifier 5 so as to cancel the
voltage at the output.

The true function generator is
shown in Figure 12, and its output is
proportional to the true $f(e)$ of
Figure 5. The origin of $f(e)$ may be
chosen at any point on the diode
characteristic, and the output may be
adjusted to zero at that point.

**Computer Solution**

The final machine equation is ob-
tained by integrating equation (3),
which yields

$$\frac{de}{dt} + \frac{R}{L} e + \frac{1}{LC} \int e \, dt + \frac{1}{C} f(e) + \frac{R}{LC} f(e) \, dt = 0.$$  \hspace{1cm} (11)

The unscaled block diagram of the
computer solution to equation (11)
is shown in Figure 13.

The outputs of amplifiers A and
B of Figure 13 were used to drive
an X-Y Recorder, which traced out
the phase-plane trajectory from the
initial condition to the limit cycle.
The output of amplifier B, when fed
to one channel of a Sanborn Recor-
der, traces out the time-varying wave-
form simultaneously. An example of the
computer solution recorded in
this manner is shown in Figure 14.

The computer may be used to great
advantage in determining the in-
fluence of the circuit parameters
($R$, $L$, $C$, and bias) on the over-all
behavior of the oscillator. Since the
computer solution is in graphical
form, a circuit designer could rapid-
ly catalog the results for many differ-
ent combinations of circuit param-
eters. In this way, he would be
prepared to evaluate the potential the
oscillator may have in the solution of
specific problems in waveform
generation.

**FIG. 12. FUNCTION GENERATOR** with bias and zero adjust.

**FIG. 13. COMPUTER solution to equation 11.**

**FIG. 14. X-Y RECORDER** plots phase-plane plot and waveform.
Offner adaptability in ink rectilinear recordings

Only OFFNER offers the advantages of ink rectilinear recording with the flexibility of fast input coupler exchange. Servo loop at the stylus point forces locked-in accuracy. The OFFNER input couplers change the function of the amplifier system and provide all necessary bridge balancing, calibration and "computing" facilities…thus obviating the need for expensive special-purpose amplifiers.

Specifications:

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Channels</td>
<td>1-8 standard; to 24 special</td>
</tr>
<tr>
<td>Sensitivity</td>
<td>With preamp 1µm/mm to 5v/mm</td>
</tr>
<tr>
<td></td>
<td>Without preamp 1mv/mm to 5v/mm</td>
</tr>
<tr>
<td></td>
<td>DC to 150 cps</td>
</tr>
<tr>
<td>Frequency Response</td>
<td>0-120 cps, 0.05 msec max. delay error</td>
</tr>
<tr>
<td>Phase Error</td>
<td>0.15% (full scale) for DC; or AC within maximum amplitude envelope</td>
</tr>
<tr>
<td>Linearity</td>
<td>(shorted input) With preamp 1µv/hr at max. gain</td>
</tr>
<tr>
<td></td>
<td>Without preamp &lt;0.02µv/hr</td>
</tr>
<tr>
<td>Drift</td>
<td></td>
</tr>
<tr>
<td>Recording Amplitude</td>
<td>Full chart channel width from DC-30 cps with progressive reduction to 5mm at 150 cps</td>
</tr>
</tbody>
</table>

Input Impedance: Without preamp 2 megohms
Warm-up Time: Instantaneous
Nominal Cost/Channel: With preamp $1,250
Without preamp $850

Booths 3822-3824, IEEE Show, New York

Beckman INSTRUMENTS, INC.
OFFNER DIVISION
Schiller Park, Illinois

International Subsidiaries: Geneva, Switzerland; Munich, Germany; Glenrothes, Scotland
Automatic Parameter Optimization as Applied to Transducer Design

MAX HOWELL, Martin-Orlando

It was desired to design a transducer array consisting of a circular piston source with three concentric rings in the same plane and having the same center. The rings were to be adjusted in diameter and source strength to increase the directivity of the piston by narrowing the main beam and reducing the level of the minor lobes. The directivity of a piston is given by:

\[ D_i(u) = 2J_i(u)/u \]

where \( u = (\pi d) \sin \theta/\lambda, \) \( d \) is the diameter of the piston, \( \lambda \) is the wavelength of the sound, and \( \theta \) is the angular direction with respect to the main beam axis. The directivity of a ring is given by \( J_r(u) \). If \( y_1 \) \((<1)\) is the fraction of the piston source strength used for ring source strength and \( x(>1) \) is the ratio of the ring diameter to piston, the resultant directivity of the piston plus the rings can be expressed by:

\[ D(u) = 2J_i(u)/u + y_1 J_r(x_u) + y_2 J_r(x_u) \]

This equation can be normalized so that the program can minimize the amplitude of the oscillations beyond the 1st axis crossing and also minimize the value of \( u \) at the first axis crossing. For the normalized form of the directivity we have:

\[ D(u) = \frac{N(u)}{1 + y_1 + y_2 + y_3} \]

The range of values of \( u \) to be considered are from 0 to 16.

In order to get the desired directivity it was decided to construct a model having a fast crossing of the \( u \) (independent variable) axis and with little overshoot. Since a highly damped second-order system with the same initial condition is of this nature, it was used as a model. Now by solving for the normalized directivity \( N(u) \) as a function of \( u \) and comparing with the second order model \( M(u) \) we have a \( u \) history of the error \( \epsilon(u) \). If we integrate the absolute value of the function \( \epsilon(u) \), we have a criterion to compare the desirability of solutions \( N(u) \) for any given values of the parameters \((x_1, x_2, x_3, y_1, y_2, y_3)\). If we start with a fixed set of the values \((x_1, x_2, x_3, y_1, y_2, y_3)\) and adjust one of the parameters, say \( x_1 \) by a fixed amount \( \xi \), we have the new set of parameters \((x_1 + \xi, x_2, x_3, y_1, y_2, y_3)\). Now if we compare the functions \( f[\epsilon(u_1, x_1 + \xi, x_2, x_3, y_1, y_2, y_3)du] \) and \( f[\epsilon(u_1, x_1, x_2, x_3, y_1, y_2, y_3)du] \) we can determine if the adjustment \( \xi \) has improved \( N(u) \). If \( N(u) \) has been improved we again make the adjustment \( \xi \) and continue to do so until \( N(u) \) is not improved. When \( N(u) \) is not improved, we make the adjustment \( -\xi \) and continue to do so until \( N(u) \) is not improved. Then we make the adjustment \( \xi \), and switch to adjustments in another parameter, say \( y_1 \). We go through the same process and when finished we adjust another parameter. After all parameters have been adjusted, we repeat the cycle starting again with \( x_1 \). This is continued until we no longer have any improvements in \( \epsilon(u) \). The resultant values of \((x_1, x_2, x_3, y_1, y_2, y_3)\) are now recorded, and they establish a minimum of the function \( \epsilon(u) \). Other minimas of \( \epsilon(u) \) can be found by starting with different initial conditions of the six parameters.

By implementing this procedure at 1 unit of \( u \) equals 5 milliseconds and a repetitive rate of one solution over 80 milliseconds, time for convergence was in the order of one minute.

The equations and mechanization for the ring contribution are as follows (Fig. 15):

\[ x_1 u \left(\frac{d^2 J_i(x_1)}{du^2} \right) + \frac{d J_i(x_1)}{dx(u)} + x_1 u J_i(x_1) = 0 \]

For the piston we have:

\[ u^2 \left(\frac{d^2 J_i(u)}{du^2} \right) + u \frac{d J_i(u)}{du} \]

\[ (u^2 - 1) J_i(u) = 0 \]

\[ \frac{d J_i(0)}{du} = \frac{1}{2} \]
However, since we are interested in the function \(2J_1(u)/u\) we make the substitution \(J_1(u) = u Y_1(u)\), as shown in Fig. 16:

\[
\frac{d^2Y_1(u)}{du^2} + 3\frac{dY_1(u)}{du} + uY_1(u) = 0
\]

\[
\lim_{u \to 0} \frac{dY_1(u)}{du} = 0; \quad \lim_{u \to 0} Y_1(u) = \frac{1}{2}
\]

The mechanization of the normalized directivity with scale factors, etc., is shown in Figure 17.

Having now established the mechanization of the directivity as a function of the six parameters, we must determine a means to adjust the parameters so that the directivity \(N(u)\) will converge on the model \(M(u)\). First we will establish the error criterion \(\int |e| du\), which for a given run \(N\) will be called \(E_N\) (Fig. 18).

The next step is to have a device to remember the previous error, \(E_{N-1}\), so it can be compared with \(E_N\) to tell whether or not we are converging. In this case, a "track and hold" integrator followed by a "hold and track" integrator will suffice. This is followed with a comparator so that small differences can be measured and also to give a digital type intelligence (Fig. 19).

The third step is a circuit to tell the parameter if it is moving in the wrong direction to turn around and go the other way. It requires a remembrance of the preceding direction and a means of reversing this whenever \(E_{N-1} - E_N\) goes negative. This was mechanized in Fig. 20.

In Fig. 20 we compute the changes in the parameters. Next we need a circuit to switch from one parameter's optimization to another. It was decided the best time to do this is when a better result is computed after two worse results. The reason for this criterion is that if we initially started in the wrong direction the computer would be told to turn around and proceed until the next worse result is obtained; then, take a step back and change parameters. Before this could be done the value of \(50(E_{N-1} - E_N)/|E_{N-1} - E_N|\) had to be remembered through the next hold cycle. Incidentally, all of the adjustments in the parameters were done during the hold (reset) portion and the error was computed in the track (operate) portion. This was mechanized in Fig. 21.

An alternate method was to use digital logic for this application. This was less complex, but for this purpose was more susceptible to extraneous

---

**FIG. 17. MECHANIZATION of directivity, with scale factors.**

**FIG. 18. DEVELOPING error criterion.**

**FIG. 19. MEMORY circuit.**
The symbol for the above integrator with the disabled i.c. network.

**FIG. 20. REVERSAL circuit.**

**FIG. 21. MECHANIZATION for error computation.**

**FIG. 22. DIGITAL circuit for same logic.**

Better after two worse

TO PARAMETER

STEPPING RELAY/mechanization.

**FIG. 23. STEPPING RELAY**

Changes in parameter

**FIG. 24. REAL-TIME integrator was satisfactory accumulator.**

ous disturbances and therefore gave incorrect signals to the parameter changes. The circuit for this was as shown in Fig. 22.

This method would have been more desirable had the logic components been synchronous (clock driven). In this case there would be less chance for external interruption.

Now it is necessary to count these “better after two worse” signals from one to six, and then reset the counter to one and start over. Each state of the counter will determine which parameter is receiving plus or minus voltages from the “change directions if worse” circuit. The parameter that is receiving these voltages will accumulate them; otherwise, it will hold its present value. This was accomplished by means of a six position stepping relay. The advantage of the relay is that when a parameter was not being optimized there was absolutely no bias feeding into its accumulator. A ring counter in conjunction with six diode gates was also tried for this purpose, but proved undesirable because of the gate offsets feeding into the accumulators. The stepping relay was acceptable for 80 millisecond track and 40 millisecond hold periods. However, if a shorter rep-op cycle had been used, high speed switching would have been necessary and parameter drift may not have been such a problem.

The stepping relay mechanization is shown in Fig. 23.

The final link in the optimization is the accumulator. Two methods were tried in this case, the accounting circuit and a real time integrator. It was thought that the accounting circuit would be desirable because it would store the changes in the parameters discretely. However, the accounting circuit will tend to converge or diverge by itself without any inputs. This is because if the loop gain is greater than unity, each time the information is recycled it will be slightly increased. Furthermore, if the loop gain is less than unity when the information is recycled it will be slightly decreased. Therefore, the real time integrator approach was tried, and proved satisfactory. The circuits for these methods are shown in Fig. 24.

Now by putting all of these components together, we are able to optimize the given system. The complete logic and control diagram is shown in Fig. 25.
Analog Techniques

We are indebted this month to Moughan Mason (Thiocil, Brigham City) who sent us the following gem from an internal memo by Bill McClosky.

Advantages and disadvantages are often quite subjective. Now there are some people who have great aversion to wiring multiple relay contacts and there are others who just happen to find that they have more integrators than relays at their disposal. Imagine the following problem: activate a relay when $t > A$ and then deactivate it when $t > B$. Now any fool knows that relays don’t work that way and said fool sets up something like in Fig. 26.

However, Gerald McBeanbag who likes to do things cleverly, whether it’s really called for or not, decided that there was a different way he could do this trivial task, and launched out on the following tack (he chose a tack to prevent the unimaginative clods from sitting on his idea): Fig. 27.

Of course he will speak of saving patchboard wiring, relays—especially when more than one is required for the many tasks at $t_A$—reliability of multiple series contacts, and mention that he only turned a relay amplifier into an integrator anyway. And if you believe him, go ahead.

Further, there will always be people who are either short of relays, or who like to hopelessly complicate matters anyhow (or who have brothers-in-law making analog computer components) who will note that when the control variable is not $t$ (or even when it is for any of the above reasons), a quarter-square (or other low-grade) multiplier can produce $x^2$. Even a two-line function generator can be used (Fig. 28).

![Diagram](https://example.com/diagram.png)

And then he will try to find all nature of problems* where he can use this new found toy in Fig. 29. After all, DFG’s do have 20 segments and it would be a shame to waste any. It’s people like this who are real trouble makers and make me sorry I thought of this in the first place.

*For non-believers this can actually come up on some types of trajectory optimizations—and I only thought of this 4 years too late, too.

March 1963—Instruments & Control Systems—Page 149