This month our report is devoted almost exclusively to the July meeting of the Western Simulation Council at the Beckman/Berkeley Computation Center in Los Angeles. Subject was "Techniques for Accuracy Improvement of Analog Computer Solutions." Bill Kindle's "Accuracy Considerations in Using Servos" is reported in full, and an abstract of Shozo Masuno's discussion is presented.

MEETING OF WESTERN S/C OF 30 JULY ON "IMPROVING COMPUTER SOLUTIONS"

Approximately 43 representatives of 23 organizations gathered at the Beckman/Berkeley Computation center in Los Angeles on 30 July to discuss "Techniques for Accuracy Improvement of Analog Computer Solutions."

Irwin Pfeffer (Space Technology Laboratories, Los Angeles 45, Calif.), Chairman of the Western Simulation Council, presided, and Bill Kindle of EAI Computation Center (El Segundo, Calif.) was the first speaker. Your Editor was unable to attend this meeting, so we will give you Bill's notes just as he gave them to us rather than Bill's talk just as he delivered it.

Kindle on "Accuracy Considerations in Using Servos"

Servos are used to introduce non-linear operations into the mechanization of equations on the analog computer. The circuits which are constructed to perform these non-linear operations are not unique, and careful choice of the form of such circuits can do much to enhance the accuracy of computer solutions.

A general philosophy for aiming toward optimum use of servo components may be derived from the following statement:

In non-linear problems which have sensible linearizations, servos should be used to introduce the variations in the coefficients of the problem and should not be required to pass the linear portion of the signal through their electro-mechanical drive.

The further a problem deviates from having a sensible linearization** the less obvious becomes the application of this criterion. There are, however, many significant situations where it can be readily applied. Some examples follow:

**Do you mean, Bill, that you can sense them or that they make sense? Or both? —JM
**The sillier you are to try to linearize it!

In method 1 the servo must pass both the linear and non-linear variation of $C_L$ with $a$. 

Example 1—Aerodynamics

In generating an aerodynamic coefficient such as, for example, the lift coefficient of an airframe as a function of angle of attack. Consider the methods in Figures 1 and 2.
In method 2 the linear variation of \( C_L \) with \( a \) passes through the tapped-pot function generator as a voltage and can thus carry high frequencies. Only the non-linear variation of \( C_L \) with \( a \) is introduced by the servo.

This is illustrated in Figures 1a and 2a. The inherent physical properties of airframes, wherein large swings in angle of attack occur at low frequencies and high frequencies have small amplitudes, permit the servo simulation of the aerodynamics of airframes with natural frequencies higher than the natural frequency of the servo.

Figure 2a illustrates that for small values of \( a \) the value of \( C_L/a \) is independent of \( a \). Thus, for high-frequency small variations in \( a \) the fact that the servo is unable to follow \( a \) introduces no error.

Also, for symmetrical airframes, \( C_L/a \) is symmetrical about the origin of \( a \), and the function \( C_L/a \) can be generated as a function of \( |a| \), which permits twice as many points in the function.

Another advantage of this technique is that it yields more accurate steady states for small values of \( a \) than can be achieved using electronic nonlinear equipment. This added accuracy can be very important in computing induced drag, which accounts for the energy lost in drag due to lift.

**Example 2—Pneumatics**

In simulating the isothermal compression of a gas in a chamber

\[
\frac{d}{dt} PV = RT \Sigma w
\]

where \( P \) is the pressure; \( V \) is volume; \( R \) is gas constant; \( T \) is temperature, and \( \Sigma w \) is the sum of weight flows in and out of the chamber.

This equation is normally solved for the pressure with the volume variation and weight flows of gas as inputs.

Consider the following two methods:

1. \( P = \frac{1}{V} \int RT \Sigma w \, dt \)
2. \( P = \frac{1}{V} \int \frac{RT \Sigma w - P}{dt} \, dt \)

as mechanized as shown in Figs. 3 and 4.

These equations are mathematically identical, and the obvious choice is method 1, which is simpler. However, method 2 is the proper choice.

By method 2 the high-frequency dynamics of the "pneumatic spring" are introduced by \( dv/dt \), and the non-linearity of the "fluid spring" is introduced by the multiplication by \( P \) and the division by \( V \).

By method 1 the \( V \) servo is required to pass the linear spring signal in addition to introducing the nonlinear effects.

When the circuit of method 1 is closed with other circuits of a problem, it will yield spurious oscillations due to the lag introduced by the servo.

Of particular interest in the fact that experience has shown that electronic multipliers used in the circuit of method 1 will generate spurious oscillations for problems where servos used in the circuit of method 2 will not. Thus, the use of electronic components in place of servos does not relieve the analog computer user of his concern for utilization of his equipment for best dynamic accuracy.

**Example 3—Hydraulics**

In liquid fluid dynamics, flows often depend on the difference of squares of pressures, and forces may depend on pressure differences. Thus, the occasion arises where it is very important to have the pressure difference and the difference in the squares go to zero simultaneously to avoid spurious offsets.

This can be achieved by expanding the difference in the squares:

\[ P_1^2 - P_2^2 = (P_1 + P_2)(P_1 - P_2) \]

Consider the methods of Figs. 5 and 6:

When \( P_1 = P_2 \), method 1 depends on the accuracy of the squaring devices to achieve a null in \( P_1^2 - P_2^2 \).

By method 2, a null in \( P_1 - P_2 \) assures a null in \( P_1^2 - P_2^2 \) regardless of errors introduced by the servos.

It is interesting to note that some of the advantages of this technique are sacrificed if the servo multipliers are replaced by electronic multipliers; this does not take advantage of the inherent property of servo multipliers to give an absolute zero output for zero input to the pot.

**Example 4—High Q Resonance**

In computing (algebraically) the amplitude versus frequency of a high Q circuit, an equation arises of the form:

\[ \epsilon_0/\epsilon_1 = 1/(1 + D^2) \]

where \( D_M < D < D_N \) and \( D_M > 1 \).

Good accuracy is required around \( D = 0 \).
Consider the methods of Figs. 7 and 8:

Note 1: The “one” in \(1 + D^2\) is provided by the normal amplifier feedback in both circuits.

Note 2: The \(G's\) denote gains necessary to allow \(D > 1\).

Note 3: In circuit 2 the equality, \(D^2 = D \cdot |D|\) is used to save amplifiers.

If the servo error for zero input is \(\epsilon\), then method 1 gives:
\[
e_o/\epsilon_1 = 1/(1 + \epsilon^2)
\]
whereas method 2 gives
\[
e_o/\epsilon_2 = 1/[1 + (D + \epsilon)^2]
\]
\[
= 1/(1 + \epsilon^2 + 2\epsilon D + \epsilon^2)
\]

Thus the error for method 1 approaches \(\epsilon^2\) as \(D\) approaches zero, whereas the error for method 2 approaches \(\epsilon^2\). Since \(\epsilon\) is of the order of 0.001, method 2 is about 1000 times better than method 1 around \(D = 0\).

Although this technique is not restricted to servos, the stability of the circuit of method 2 is somewhat easier to achieve using servos than using electronic multipliers.

Example 5—Resolvers

In many applications, the lag of a servo resolver imposes severe limitations in analog simulation. The following discussion suggests a way of appreciably extending the frequency response characteristics of rectangular and polar resolvers by adding “tweeter” channels to the already existing “woofer” channels.

For a rectangular resolver,
\[
x_1 = x_o \cos \theta + y_o \sin \theta
\]
\[
y_1 = -x_o \sin \theta + y_o \cos \theta
\]

When the resolver lags such that the shaft position is \(\theta_o\) instead of \(\theta\), and the error is defined as \(\Delta\theta\), i.e.,
\[
\theta = \theta_0 + \Delta\theta
\]

Then for small \(\Delta\theta\),
\[
\sin \theta \approx \sin \theta_0 + \Delta\theta \cos \theta_0
\]
\[
\cos \theta \approx \cos \theta_0 - \Delta\theta \sin \theta_0
\]

And the equations for the rotation of axes become
\[
x_1 = (x_o + y_o \Delta\theta) \cos \theta_0
\]
\[
+ (y_o - x_o \Delta\theta) \sin \theta_0
\]
\[
y_1 = -(x_o + y_o \Delta\theta) \sin \theta_0
\]
\[
+ (y_o - x_o \Delta\theta) \cos \theta_0
\]

By picking off \(\Delta\theta\) from the difference between the servo input voltage and the follow-up pot voltage, and using an electronic multiplier, the correction terms for the “tweeter” channel are introduced as shown in Fig. 9.

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For a polar resolver,
\[ R = x \cos \theta + y \sin \theta \]
\[ x \sin \theta - y \cos \theta = 0 \]

With lag \( \Delta \theta \), these equations become
\[ R = x (\cos \theta_0 - \Delta \theta \sin \theta_0) + y (\sin \theta_0 + \Delta \theta \cos \theta_0) \]
\[ x (\sin \theta_0 + \Delta \theta \cos \theta_0) - y (\cos \theta_0 - \Delta \theta \sin \theta_0) = 0 \]
Rearranging:
\[ R = (x \cos \theta_0 + y \cos \theta_0) [1 + (\Delta \theta)^2] \]
\[ \Delta \theta = \frac{y \cos \theta_0 - x \sin \theta_0}{x \cos \theta_0 + y \sin \theta_0} \]

Using an electronic multiplier again, the “tweeter” channel is added as shown in Fig. 10.

As much as the “tweeter” channels for both the rectangular and polar resolvers should introduce corrections which are small compared to the signals through the “woofer” channels, the electronic multipliers can be relatively low in accuracy.

In many applications of resolvers where frequency response is a serious concern, the amplitude of an oscillation tends to decrease with increasing frequency. This helps to preserve the validity of the small-angle assumption, and in fact there would be many problems where the high-frequency oscillations would not even be excited so long as the resolver does not introduce spurious lags.

\[ \hat{R} = \hat{X} \cos \sigma + \hat{Y} \sin \sigma \]

Example 6—General Multiplication

High-frequency paths can be provided for both the variables in a product using servos as follows:
\[ x = x_0 + \Delta x \]
\[ y = y_0 + \Delta y \]
\[ xy = (x_0 + \Delta x) (y_0 + \Delta y) \]
For \( x_0 \gg \Delta x \)
\[ xy = xy_0 + x_0 \Delta y \]

The circuit is then as shown in Fig. 11.

Matsuno on Improving Terminal Readout Accuracy

The next speaker listed on the program was Bruno Ulrich (Hughes Systems Development Laboratories, Culver City, Calif.), but the man who actually spoke was his colleague Shocho Matsuno, who was kind enough to furnish us with a brief, very brief, résumé of his talk as follows:

“My discussion consisted of a few specific techniques used in improving terminal readout accuracy of homing missile simulations. In particular, focus was directed to the improvement of terminal kinematics (geometry) representation of the missile system in order to achieve accuracy comparable to the accuracy of the over-all simulation.

“Two inherent problem areas which directly affect accuracy of terminal readout were:

“a. The presence of terminal singularity (overload problem).

“b. Large variation of variables (scaling problem).

“Compensations and techniques discussed were:

“1. Elimination of polar resolver in the geometry to improve terminal computer stability by utilizing the relationships:

\[ \hat{R} = \hat{X} \cos \sigma + \hat{Y} \sin \sigma \]

where
\[ R = \sqrt{x^2 + y^2} \]
\[ \sigma = \tan^{-1} (Y/X) \]

“2. Continuous extrapolation technique to obtain miss-distance, avoiding the undesirable effects of the singularity.

“3. An extension of the extrapolation method to read out variables with high terminal rate of change (a variable \( y \) with high rate of change is redefined as \( \dot{y} = \dot{y} + \Delta y \). Now, a small term \( \Delta y \) is defined by minimizing its terminal rate of change. Then, \( \Delta y \) can be scaled up properly and will be less sensitive to readout timing).

“4. The use of time variable scale factor \( K(t) \) for reading quantities with large variation of its value, viz. missile-to-target range. Two methods of scaling were given:

“a. Continuously varying scale factor using the relationship:

\[ \frac{d}{dt} (KX) = \dot{K}X + K \dot{X} \]

“b. Step change in scale factor:

\[ KX \rightarrow K'X \text{ where } K' >> K \]

Shocho tells us that he will be glad to furnish further information and details on the subject.

* * *

Third speaker on the program was Lucien W. Neustadt of Space Technology Laboratory, but as he did not send us any information on his talk, and nobody else reported on the subject, we don’t know that he spoke (though he signed the register), and if he spoke, what he said. If Lucien will give us the word we will be happy to pass it on. Otherwise thus endeth our report on the July 1959 Western Simulation Council Meeting.

Information
(Without Theory)

Thanks largely to Dave Miller (Colorado Research Corp., Broomfield, Colorado), there has been considerable Simulation Council-type activity in the Denver area. As a result, the SCI Board of Directors is serious-
ly considering a charter presented by this group as "The Rocky Mountain Simulation Council," pending adjustment of the territory that they will represent.

One of the objectives of the Simulation Council was, has been, and I hope is, to stimulate discussion. Somehow we have gotten into a rather formal rut where few people talk back. The correspondence touched off by my comments on McGregor's talk (Newsletter of July 1959) is very gratifying. The first response, that by Martin Dost, Junior Engineer in the Mechanical Analysis Laboratory, IBM, Poughkeepsie, N. Y., was as follows:

"Your coverage of W. K. McGregor's talk on the "Determination of Forcing Functions from the Response of a Known Physical System" in the July '59 Newsletter was of considerable interest to me, being to some extent familiar with analog computing and feedback control theory. Would you please let me know where I can obtain a copy of McGregor's paper.

"From abstract and block diagram of the postulated system it is not obvious to me how the error signal is obtained. If the block in the feedback path (signified as "Feedback System") is another summing point that produces the difference of e_{18} and e_{18} = e_{18} + c_{19}, I can imagine to obtain e_{18} + e_{19}. But is not e_{18} + e_{19} the output of the open loop system with input e_{18} (similar to the measuring system, whose input is e_{18} and whose output is e_{18} = e_{18} + c_{18}). The closed loop system sketched, if well compensated, will yield a output e_{18} = e_{18}; where, therefore, does the error signal e_{18} come from if the loop is closed?"

To this your Editor replied, with copy to Mac:

"Thank you for your letter of August 24. I have a very good reason for being so slow in answering. Every time I have intended to answer and reread McGregor's abstract, then my explanation, then your question, I got so confused that I put your letter aside and tackled something I could understand.

"If I read your letter correctly, we are in perfect agreement until the last phrase of the last sentence in which you ask where the error signal e_{18} comes from if the loop is closed. The answer is that it comes from the block labeled Simulated System, just as it always did. Closing the feedback loop does nothing to the transfer characteristics of this block, but only alters the input so that instead of e_{18}, it now sees e_{18} the input. So for the closed loop the output e_{18} is now equal to e_{18} + e_{19} instead of e_{18} + c_{19}. Because we have postulated a feedback system that will make this equal to e_{18}, which is equal to e_{18} + c_{19}, then we have: e_{18} + e_{19} = e_{18} = e_{18} + c_{18}, and as the error in both systems is the same: e_{19} = e_{18} and e_{18} = e_{18}.

I hope that this explanation either clarifies the operation of the system or gets you as confused as I am."

The copy to McGregor elicited the full whammy—"The Extrusion of Transient Data from Instrument Systems by a Simulation Method"—to Dost as well as to your confused editor, who at this point was ready to give up. Mac's covering letter read as follows:

"Enclosed is a more complete treatment of the transient data correction method reported at the January meeting of the Southeastern Simulation Council. I really don't know if this is the same thing John McLeod said that I said or not. I think he has to swap simulated system and feedback system blocks to simplify the problem so that I can understand it. I enclosed also a "corrected?" diagram similar to Mr. McLeod's.

"I appreciate your interest and invite your comments always."

But back came bouncing Dost's reply:

"Thank you for your letter of September 22, concerning your note on Mr. McGregor's talk on the transient data correction method.

"After I received Mr. McGregor's original paper, I could understand the concept involved. I believe that the block diagram on p. 1050 should have looked like Fig. 12.

"The function of the control system, mechanized, e.g., by an analog computer, is to generate the approximate inverse transference of the measuring system and thereby to extend the frequency response of the measuring system greatly or yielding an overall transfer function of KG/(1 + KG) = 1 because the gain K is assumed to be very large."

Now this sort of thing leaves ye Ed bewildered but fulfills the purpose of the Simulation Councils. Any chit-chat from youse guys?

**Thot**

(not concerting)

Either I have no readers (hope Milt doesn't find out), or I am making no mistakes (impossible), or my mistakes are not very important, interesting, or amusing; or you guys have no spirit. How about another two-, three-, or multi-cornered discussion (or even a good rhubarb) like the McGregor-McLeod-Dost one?