The Simulation Council

Newsletter

John H. McLeod Jr., EDITOR
Suzette McLeod, SECRETARY
8484 La Jolla Shores Drive, La Jolla, California

Bits

This month we give you what info we have available on the Southeastern Simulation Council meeting held on October 2 at the University of Alabama, and notes on one paper and the text of another presented at the 14 December '59 meeting of the Eastern Simulation Council meeting at Leeds & Northrup—more would be too much.*

The Southeastern Simulation Council material consists of a brief description of a U of A course on the applications of electronic analog computers by W. B. Stiles, an abstract of "Some Engineering Aspects of Low-Frequency Noise Measurements" by R. S. Johnson and J. L. Hammond, and an abstract of "Loop-Gain Analysis for Implicit Function Generation" by J. H. Meeks and R. S. Johnson.

The Eastern SC offers are "Notes for Simulation of Industrial Process Controllers" by R. L. Farrenkopf and F. A. Lapinski, and "Idealized versus Realistic Simulation of Cascaded Process Lags" by Morton J. Hausner.

Pieces

SOUTHEASTERN S/C MEETING OF 2 OCT ON COMPUTER COURSE, NOISE, AND FUNCTION GENERATION

At the 2 October 1959 meeting of the Southeastern Council held at the University of Alabama, Dr. W. B. Stiles discussed some computer courses offered at the University. And as we all are (or should be) interested in what our educational institutions are teaching to the future engineers and scientists about computers and simulation, we give you here a summary of Dr. Stiles remarks:

Stiles on Analog Computer Course

The Departments of Mathematics and Commerce at the University of Alabama offer courses on numerical analysis and programming for digital computers. However, the following discussion is limited to applications of electronic analog computers, and to a description of a course on such applications which is taught in the Department of Engineering Mechanics. Such computers are used to solve electric circuit problems, electromagnetic and electrostatic problems, control problems, heat-flow problems, problems of vibration, chemical rate process problems, and many other applications in the field of applied engineering in which it is necessary to solve linear or nonlinear differential equations.

The Department of Engineering Mechanics acquired a small Donner computer for use in instruction, with the thought that it could be expanded for research applications as the need developed. In addition to the basic Donner computer, an electronic function generator was obtained. Since the original equipment was purchased an additional function generator, a function multiplier, and other miscellaneous components have been ordered. A graduate student in the Department of Industrial Engineering constructed a Heath analog computer which was later made available to the Mechanics Department for instructional purposes.

Dr. Stiles said that he hoped only a few students, maybe four or five, would take the course the first time it was offered. Instead 11 students enrolled! This made it difficult to schedule laboratory problems on the single available computer. Interest has continued to grow, and 18 students are enrolled during the current semester.

About 80% of the course time is spent on theory and solving academic problems...
to laboratory use of the computer. A number of simple problems are
solved by students working in small squads, and one term project is
assigned to each student to be solved individually. An indication of
interest exhibited by the class is the fact that near end of the term
students have asked to be allowed to use the computer evenings and weekends
in order to have sufficient time to complete the problems.
Laboratory problems consisted of the following:

1. Demonstration of coupled vibrations with four degrees of freedom
and with variable damping. General precautions to be observed were
pointed out at this time.

2. A problem with a single degree of freedom involving an elastic spring
and viscous damping together with a disturbing force which varied linearly
with time. Many of the examples used in laboratory and recitation in-
volved dynamics problems because this course and a course in differential
equations were required of all students.

Various analytic functions, such as

\[ y = A \sin (5t + \frac{1}{2}) \]
\[ y = 0.5e^{-0.15t} \]
\[ y = e^{-100} \sin 5t \]
\[ y = 5 (1 - 2^{-0.25}) \]
were set up without the use of the function generator.

In addition to these analytic functions, the students used complex impedances in the input and feedback circuits so that a single amplifier could be used to solve higher-order differential equations.

4. The use of diodes. The simulation of dry friction, dead space, step
functions, and limiting operations was studied.

5. Electronic function generator. The function generator was used to
solve the problem of large oscillations of a simple pendulum in which the
approximation \( \sin t \approx t \) is not justified.

6. Electronic function multiplier. An example in which the resisting force
varies as the square of the velocity will be given.

7. Term project. Each student prepared a term project involving some
problem with which he was familiar either from other courses or from expe-
rience on a job. It was required

that the projects be not too simple, and as a criterion it was expected
that the solution would include at least a third-order differential equa-
tion, a pair of simultaneous differential equations, or the use of the func-
tion generator. The projects involved vibration problems, control problems,
an electrical oscillator, a problem on rate of variation of concentrations in
chemical engineering, and one additional electrical engineering problem
requiring the use of a function generator to indicate the output of an
electronic circuit.

Dr. Stiles expressed himself as well pleased with response to the course
during the first semester and believes the course will improve each time it is
offered.

Next speaker on the program was D. N. O'Steens (University of Alab-
am) who told about their B-58 Cockpit Simulator. (His paper hasn't been
released for publication, so we'll say no more about it.)

**Meeks on Implicit Function Generation**

Following a coffee break J. H. Meeks (Georgia Institute of Tech-
nology, Atlanta, Ga.) was scheduled for a talk on "Loop Gain Analysis
for Implicit Function Generation."* We are indebted to Ray Lawrence
(Army Ballistic Missile Agency, Huntsville, Ala.), Secretary-Treasurer
for the Southeastern SC, for forwarding the following abstract:

John Meeks described a method of steady-state gain analysis useful in
programming implicit function generators on the analog computer. The
method involves diagramming the circuit in a conical form and then,
by means of a fixed set of rules, deriving a quantity termed "track-
ing gain" which depends on the functional derivatives of the forward
and feedback loops. The algebraic sign of the tracking gain indicates the
position of stable equilibrium in terms of the input and output vari-
ables. It was noted that in the generation of inverse functions by the usual
methods, it is difficult to predict the stable point if the inverse function is
multivalued or if the feedback involves a number of nonlinear paths.

In addition to the stability information, the tracking gain also provides

*Prepared in collaboration with his colleague, R. S. Johnson.

a measure of the circuit's ability to follow varying input quantities: A
large magnitude indicates good tracking ability, whereas smaller values
lead to poorer performance. As the tracking gain is normally a function of
the input and output variables, the form of automatic gain control re-
quired is usually suggested by the nature of this function.

Several applications of the method were presented. The examples in-
cluded analyses of polar-mode resolvers, square-root circuits, and di-
vision circuits.

M. E. McKoy (Glenn Martin Co., Orlando, Florida) suggested that the
technique might also be applicable to algebraic programming.

**Johnson on Low-Frequency Noise Measurement**

Another speaker on the program was Robert S. Johnson (Georgia In-
itute of Technology, Atlanta, Ga.), whose talk on "Some Engineering
Aspects of Low-Frequency Noise Measurement"** is given here in ab-
stract form.

Bob gave a brief résumé of the application of the analog computer to the
measurement of statistical parameters of very-low-frequency noise
signals. Particular emphasis was placed on the estimation of variance,
amplitude probability distribution, and power spectral density functions.

A simple combination of comparators and integrators was shown to
suffice for the estimation of amplitude probability distributions. The cir-
cuit for the measurement of a single point on the distribution curve re-
quires, at most, only two amplifiers, so that a number of circuits may be
employed in parallel to obtain several points during one measurement peri-
od. Data for the distribution curve of a Gaussian noise generator was
presented to demonstrate the precision of measurement.

The circuit submitted for the esti-
mation of variance consisted of an
absolute-value circuit (or precision
full-wave rectifier) followed by an in-
tegrator. It was shown that if the
probability distribution of the proc-
ess is known, then the integrator out-
put can be related to the process vari-
ance by a constant "form factor" de-

erived from the distribution character-
istics.

The variance circuit preceded by
an appropriate bandpass filter was
suggested as a very useful program

*Done in collaboration with J. L. Ham-
mond, also of Georgia Tech.
for the measurement of points on the power spectral-density curve. However, in addition to the form factor mentioned, it is also necessary to compute a “filter factor,” which depends on the transmission characteristics of the filter. This factor can be conveniently determined by using the computer to find the integral of the squared impulse response.

The paper brought out several reminders concerning the accuracy of sample measurements relative to true process statistics. In addition to possible errors due to departures from assumed conditions of ergodicity and stationarity in the process under study, very large errors may be introduced by failure to take sufficiently long time samples. For example, it was shown that in the estimation of the power density at 0.1 cps using a filter having a Q of 10, it requires some 11 hours of sample measurement to obtain 10% accuracy with 95% certainty.

Bob concluded his talk with a short description of the low-frequency Gaussian noise generator built at Georgia Tech, and a few comments concerning the applications of such a device.

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Farrenkopf and Lapinski on Simulation of Process Controllers

Basic Block Diagrams of Controllers

A. 2-mode—proportional plus integral (reset) is shown in Fig. 1, where $K = \text{gain} = 100/\text{proportional band}$, $T_1 = \text{reset time constant}$.

![Fig. 1](image)

$$\text{ERROR} \cdot \frac{K}{ST_1 + 1} \cdot \text{OUTPUT}$$

B. 3-mode reset before rate—proportional plus integral and compensated derivative (rate), is shown in Fig. 2, where $K = \text{gain}$, $T_1 = \text{reset}$, $T_2 = \text{time constant}$, $\alpha = \text{rate constant ratio}$.

![Fig. 2](image)

$$\text{ERROR} \cdot \frac{K}{ST_1 + 1} \cdot \frac{ST_2 + 1}{ST_2 / \alpha + 1} \cdot \text{OUTPUT}$$

C. 3-mode reset before rate—proportional plus integral and compensated derivative, is shown in Fig. 3. For linear simulation this becomes the same as Fig. 2.

![Fig. 3](image)

$$\text{ERROR} + \frac{\frac{ST_1}{ST_2 / \alpha + 1}}{ST_1 + 1} \cdot \frac{1}{K} \cdot \text{HIGH GAIN} \cdot \text{OUTPUT}$$

Basic linear computer diagrams are shown in Figs. 4 (2 mode) and 5 (3 mode).

![Fig. 4](image)

These simple simulations are often inadequate. Use of the above simple simulations usually depends on the assumption that: (1) no element in the real controller saturates, (2) the controller amplifier has infinite gain and input impedance, (3) frequency characteristics are determined solely by the modes. However, most controllers are built in the way shown in Fig. 6, and often (due primarily to amplifier characteristics) the above assumptions are not justified.

Example: Simulate the 2-mode electronic controller in Fig. 7. By summing currents at the grid and Laplace transforming, the result is:

$$E_o(s) = \frac{E_i(s)}{R(C_1 + C_2) s + (1 + R/R_1)}$$

$$+ \frac{(aRC_2 s)}{R(C_1 + C_2) s + (1 + R/R_1)}$$

This equation holds for all the controller states.

Assume the amplifier to have saturation limits and to be described by $e_o = A(e_i) e_p$. Then the simulation is as shown in Fig. 8 A and B.

As the amplifier gain approaches infinity this reduces to Fig. 9.

Electric three mode controllers can be handled similarly.

Pneumatic controllers

A usual characteristic of pneumatic controllers is the isolation between their input and feedback networks. A typical 2-mode controller is illustrated in Fig. 10 and its simulation in Fig. 11.
Some people can prove man and mouse are identical

It's all according to the points of similarity you choose. Differences are what really prove the superiority of man over mouse. Computers have differences, too. In fact, it's in these differences that the CSI-designed MC-5800 obsoletes every other Analog Computer. The best proof lies in:

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- Only diode function generators utilizing resistors, potentiometers, and diodes of equal quality to those in computing networks.
- Only diode function generators with individual hi-lo gain positions for each segment.
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Computer Systems, Inc.—designers and manufacturers of the highest precision analog computers and computer accessories.
Simulation of a compensated proportional plus rate unit is shown in Fig. 12. A cascade of these two circuits (with the rate unit first) provides a three-mode controller of the rate-before-reset variety.

Background

Many process control engineers entering the simulation field possess a good background in frequency-response techniques. By manipulating process and controller characteristics on Bode diagrams, the engineer has at his command a powerful tool for both synthesis and analysis, with relatively little mathematics involved. The process is usually treated as a series of lags (time constants) which can be graphically added to find the overall response of the process. The logarithms of the gains (magnitude ratios) of the lags are graphically added to form the logarithm of the product of the gains and the phase angles are graphically added to form the total phase angle. This is a powerful tool when used correctly. This tendency to treat some processes as a series of isolated lags might possibly carry over into simulation studies causing erroneous results. This is especially true when working on a process which is in the design stage where no field data can be obtained.

Definitions

To illustrate the basic differences between isolated (non-interacting) and non-isolated (interacting) process lags, several analogies will be used. Fig. 14 represents the electrical analogy of a two-time-constant process. Fig. A is called interacting because current flows between \( V_2 \) and \( V_0 \). In other words, the second time constant loads down the first one. Fig. B is called non-interacting because an isolation amplifier is placed between \( V_2 \) and \( R_2 \), preventing any current from flowing between the first and second time constants. Fig. 15 represents the pneumatic analogy of the same situation where pressures replace voltages, restrictions replace resistors, volumes replace capacitors, and the isolation amplifier takes the form of a 1:1 pneumatic booster relay.

Unfortunately a great many industrial processes consist of series of interacting lags so that frequency-response techniques begin to lose some of their original power.

Analysis

The mathematical differences between the two types covered above will now be covered using the familiar water tank analogy. Fig. 16 shows a series of water tanks connected in non-interacting fashion. The second tank does not, in any way, affect the behavior of the head or flow out of the first tank. In other words, the second tank does not load the first one.

The process equations are:

\[
\begin{align*}
C_1h_1S & = Q_1 - Q_2 \\
Q_2 &= h_1/R_1 \\
C_2h_2S &= Q_2 - Q_3 \\
Q_3 &= h_2/R_2
\end{align*}
\]  

By combining (1) and (2) to find \( h_1 \), and combining (3) and (4) to find \( h_2 \), we find that

\[
h_2 = \frac{R_2}{Q_1} \left( \frac{R_1C_1S + 1}{R_2C_2S + 1} \right)
\]

In this example the transfer function indicates two non-interacting lags.
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Fig. 17 shows the same tanks tied together in interacting fashion. The level in the second tank affects the flow out of the first tank.

\[ \text{The process equations are:} \]
\[ C_1 h_1 S = Q_1 - Q_2 \]
\[ Q_2 = (h_1 - h_2)/R_1 \]
\[ C_2 h_2 S = Q_2 - Q_3 \]
\[ Q_3 = h_2/R_2 \]

By combining we find that

\[ h_2 = \frac{R_2}{Q_1 (R_2 C_2 S + 1) (R_2 C_1 S + 1) + R_3 C_1 S} \]

A comparison of transfer functions shows an additional term in the denominator which may be attributed to interaction. For use in frequency response, the transfer function of the interacting system may be factored to form an equivalent non-interacting process.

For example, suppose \( R_1 = R_2 = 0.1 \text{ min/ft}^2 \) and \( C_1 = C_2 = 5 \text{ ft}^2 \), so that \( R_1 C_1 = R_2 C_2 = R_3 C_1 = 0.5 \text{ min} \). The transfer function is now:

\[ h_2 = \frac{0.1}{(0.5S+1)(0.5S+1)+0.5S} \]

\[ h_3 = \frac{0.1}{0.25S^2+1.5S+1} \frac{1}{0.1} \]

Note that the interacting process was changed from two equal lags to a non-interacting process consisting of a larger and a smaller lag.

In general, when a series of interacting lags is converted into an equivalent series of non-interacting lags the new values of time constants will be spread further apart in the frequency spectrum.

To illustrate this phenomenon, consider a process composed of three equal time constants where:

\[ R_1 = R_2 = R_3 = 2 \text{ minutes/ft}^2 \text{ and } C_1 = C_2 = C_3 = 1 \text{ ft}^2 \]

For the non-interacting case, the transfer function is:

\[ 1/(2S+1)(2S+1)(2S+1) \]

For the interacting case, the transfer function is:

\[ 1/8S^3+20S^2+12S+1 \]

When this expression is factored the resulting transfer function is:

\[ 1/(10S+1)(1.33S+1)(0.60S+1) \]

The time constants are seen to be spread far apart. When the original time-constants are unequal to begin with, the equivalent non-interacting time constants will be spread further apart.

The interacting process is an "easier" one to control. The frequency at which the process reaches 150° phase shift is used to determine the permissible proportional controller gain. The interacting system reaches a higher frequency before shifting 150°. At the same time it has attenuated much more than the non-interacting process. The interacting process will tolerate much higher controller gain before instability occurs. This can be seen from the Bode plot.

As the number of time constants gets larger in an interacting system, frequency-response techniques become unwieldy and analog simulation becomes more justified.

Applications

As a more realistic example of an interacting process, we will study the case of the often used batch heat up of a jacketed vessel shown in Fig. 18.

The vessel is assumed to contain a perfectly mixed batch with negligible heat loss through the insulation. The
Time Delay Unit Simulates Fluid Transport

Used to Study Reactors and Chemical Processes

A prominent feature of nuclear power plants and many chemical processes is the flow of fluids from one energy exchange point to another. In the case of reactors there is the flow of coolant from core to heat exchanger and back; in chemical processes, the movement of chemicals from one reaction chamber to another. When simulating such systems it becomes necessary to introduce rather long time delays corresponding to the time the fluid is in transport but otherwise unchanged.

The delays required are likely to fall in the range of a tenth of a second to ten seconds. Because fluid turbulence removes sudden changes in the temperature or chemical composition of fluids, the signal to be delayed usually consists of low frequency components — on the order of one cps or less.

The EASE 1152 transport delay shown below was developed to cope with these conditions.

Panel Controls Select Delay Quickly and Precisely

Four Model 1152 delay units are shown above mounted in 8 3/4" of rack space. In operation each delay unit must be supplemented by three operational amplifiers. Each unit has a 10-position switch (top) for selecting the integral number of seconds and a 10-turn dial (bottom) for selecting the fractional increment. Signal delay is continuously adjustable from about .010 secs to 10.000 secs. The shape and amplitude of the signal remain essentially unchanged. (Amplitude error is less than ±.01% at dc and ±.3% at highest specified input frequency.) In this respect the unit is far more accurate than magnetic storage delays which are subject to considerable dc drift.

The maximum usable delay depends upon input frequency. In order to operate with specified time accuracy, the delay (t) in seconds should be limited to

$$t = \frac{1.2}{f}$$

where f is the frequency of the highest frequency component of interest. Thus, if the highest component is 1 cps, 1.2 seconds is the maximum delay. The slower signals commonly encountered may be delayed for several seconds as diagramed in Figure 1. Moreover, maximum delay time may be doubled, tripled or quadrupled by cascading delay units. The effect is to add delay times while preserving percentage accuracy.

Note that it makes no difference what time scale is used in the problem. For example, if solution time is doubled, maximum delay time also doubles because frequency in equation (1) is reduced to one-half.

Design Optimizes Accuracy, Reduces Need for Amplifiers

The basis of the time delay is a circuit which induces phase shift that varies linearly with input frequency. Thus, for example, if frequency doubles, phase shift doubles and time delay remains exactly the same.

In the circuit used, phase shifts up to 450° occur as a linear function of frequency. This range is approximately 40% larger than that which can be achieved using the Padé approximation. Figure 2 shows phase error of only 1.1% at 450°.

The total possible error in delay time is equal to the phase error plus a slight error introduced by the finite resolution of the delay control. Resolution error amounts to only 0.25% of value for delays in the range of 0.4 to 10 seconds.

Figure 1. Graphic representation of a delayed signal.

Figure 2. Phase error plotted against phase shift. 450° phase shift corresponds to maximum usable delay (t = 1.2),

Figure 3. The delay circuit used. Note that only three operational amplifiers are required. This simple circuit possesses the complex transfer function required for accurate delays: 4th order in the numerator and 5th order in the denominator.

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equations will be derived working from point-to-point, starting at the heat input. An equivalent electrical network helps to visualize the problem (Fig. 19).

\[ T_H = \text{temperature of heating medium, } \circ\text{F}; T_W = \text{temperature of inner wall, } \circ\text{F}; T_B = \text{batch temperature, } \circ\text{F}; T_A = \text{ambient temperature, } \circ\text{F}; R_{HW} = \text{equivalent thermal resistance between heating medium and inner wall, } \circ\text{F}/\text{BTU/min}; R_{WB} = \text{equivalent thermal resistance between inner wall and batch, } \circ\text{F}/\text{BTU/min}; C_H = \text{thermal capacity of heating medium, BTU/}^\circ\text{F}; C_W = \text{thermal capacity of inner wall, BTU/}^\circ\text{F}; C_B = \text{thermal capacity of the batch, BTU/}^\circ\text{F}; k = \text{conduction constant between batch and ambient, BTU/min}^\circ\text{F}.\]

\[ Q_{in} = T_H C_H S + (T_H - T_W) / R_{HW} \]
\[ (T_H - T_W) / R_{HW} = T_W C_W S + (T_W - T_B) / R_{WB} \]
\[ (T_W - T_B) / R_{WB} = T_B C_B S + (T_B - T_A) / k \]

By taking the equations one at a time, converting them into convenient form and drawing the individual computer diagram the complete system simulation is accomplished. All the interactions are included in the simulation and any of the intermediate temperatures may be examined. The effects of vessel design changes are easily studied by changing the proper potentiometers. (Fig. 20).

Another possibility for simulation that has been used is a passive network such as the circuit in Fig. 19. A current output is needed to form the analog of heat into the system. A fine example of this direct simulation technique is a distillation column study by Rijnsdorp and Maarseveld\(^2\), where most of the process was simulated using passive elements, thus saving a tremendous amount of amplifiers.

**Conclusions**

A reiteration of the important points is in order. The transition from frequency-response techniques to analog simulation requires a change in thinking with regards to processes in the design stage. A close look at the process by means of equations or equivalent circuits will help take into account the interactions essential to a realistic simulation.

**References**


* * *

In the Leeds & Northrup Eastern Simulation Council meeting there was a discussion of method of steepest ascent and a reference to an article by Box and Wilson. Hans Meissinger has done a lot of work in this area:


Reference 3 in particular is believed to represent some original work of unusual interest.