1. The possible rational forms of $3 \times 3$ nilpotent matrices are:

\[
\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}
\]

and these have minimal polynomials $x, x^2, x^3$, respectively. Then $N_1$ and $N_2$ are similar $\iff$ they have the same rational form $\iff$ they have the same min. polyn.

2. $A$ and $B$ have the same eigenvalues. For every $i$, the generalized eigenspace $W_i = \ker (A-c_i I)^{d_i}$ has $\dim = d_i \leq 3$.

The operator $(A-c_i I)_{|W_i}$ is nilpotent and by Exercise 1, its rational form is uniquely determined by its min. polyn. $x^{r_i}$. 
This determines the Jordan form of \( A \) uniquely in terms of the min. polyn. 
\[ p(x) = (x-c_1)^{r_1} \cdots (x-c_k)^{r_k}. \]

(3) If the Jordan blocks with eigenvalue 2 have sizes \( l_1 \geq l_2 \geq \cdots \geq l_s \geq 1 \), then 
\[ l_1 = 2, \; l_1 + \cdots + l_s = 3 \Rightarrow s = 2, \; l_1 = 2, l_2 = 1. \]
If the Jordan blocks with eigenvalue \(-7\) have sizes \( m_1 \geq \cdots \geq m_t \geq 1 \), then 
\[ m_1 = 1, \; m_1 + \cdots + m_t = 2 \Rightarrow t = 2, \; m_1 = m_2 = 1. \]
Hence the Jordan form of \( A \) is:
\[
\begin{bmatrix}
2 & 2 \\
1 & 2 \\
& & -7 \\
& & & -7
\end{bmatrix}
\]
4. \( f(x) = (x+2)^4 (x-1)^2 \Rightarrow \) the sizes of Jordan blocks with eigenvalue \(-2\) add up to 4, and the sizes of Jordan blocks with eigenvalue \(1\) add up to 2. The possibilities are:

\[
4 = 4 = 3+1 = 2+2 = 2+1+1 = 1+1+1+1, \\
2 = 2 = 1+1.
\]

We have \(5 \times 2 = 10\) possibilities.

5. \( D = \frac{d}{dx} \) satisfies \( D^4(x^k) = 0 \ \forall \ 0 \leq k \leq 3 \) and \( D^3(x^3) = 6 \neq 0 \Rightarrow \) the min. polyn. of \( D \) is \( p(x) = x^4 \Rightarrow \) all eigenvalues are 0 and there is a single Jordan block of size 4:

\[
\begin{bmatrix}
0 \\
1 & 0 \\
1 & 1 & 0 \\
& & & 1 & 0
\end{bmatrix}.
\]