Recall that by definition
\[ g(\alpha) = (T^t f)(\alpha) = f(T(\alpha)) , \quad \alpha \in F^2 , \]

(a) \[ f(T(x_1, x_2)) = f(x_1, 0) = ax_1 \]
\[ \Rightarrow g(x_1, x_2) = ax_1 . \]

(b) \[ g(x_1, x_2) = f(T(x_1, x_2)) = f(-x_2, x_1) \]
\[ = a(-x_2) + b x_1 = bx_1 - ax_2 . \]

(c) \[ g(x_1, x_2) = f(T(x_1, x_2)) = f(x_1-x_2, x_1+x_2) \]
\[ = a(x_1-x_2) + b(x_1+x_2) = (a+b)x_1 + (-a+b)x_2 . \]

\[ (D^t f)(\rho) = f(D \rho) = f(p') = \int_a^b p'(x) \, dx \]
\[ = p(b) - p(a) . \]
(3) For $A \in V = F^{n \times n}$, we have

$$(T^t f)(A) = f(T(A)) = f(AB - BA)$$

$$= tr(AB - BA) = tr(AB) - tr(BA) = 0$$

$\Rightarrow T^t f = 0.$

(6) $V = F[x]_{\deg \leq n}$ has the standard basis

$\{1, x, x^2, \ldots, x^n\}$, and $\dim V = n + 1$. The dual basis of $V^*$ is $\{f_0, f_1, \ldots, f_n\}$ where

$$f_k(p) = \frac{1}{k!} p^{(k)}(0) \quad (p^{(k)} \text{ denotes the } k\text{-th derivative}).$$

Then for $p \in V$, we have

$$(T^t f_k)(p) = f_k(Dp) = f_k(p') = \frac{1}{k!} p^{(k+1)}(0)$$

$$= \begin{cases} (k+1) f_{k+1}(p), & \text{if } k \leq n-1 \\ 0, & \text{if } k = n. \end{cases}$$
Then for \( f = c_0 f_0 + c_1 f_1 + \ldots + c_n f_n \in V^* \),
we have \( D^t f = c_0 f_0 + 2c_1 f_1 + 3c_2 f_2 + \ldots + nc_n f_n \).

If \( D^t f = 0 \), then \( c_0 = c_1 = \ldots = c_{n-1} = 0 \)
\( \Rightarrow f = c_n f_n \). Therefore, \( \ker(D^t) \) is
1-dimensional with basis \( \{ f_n \} \).