1. (a) $A = LU$ where

\[
L = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
-5 & -4 & 1
\end{bmatrix}, \quad U = \begin{bmatrix}
-5 & 2 & 6 \\
0 & 1 & -2 \\
0 & 0 & 4
\end{bmatrix}.
\]

(b) $x_1 = -\frac{21}{5}$, $x_2 = -6$, $x_3 = -\frac{1}{2}$.

(c) $\det(A) = -20$, $\det(A^{-1}) = -\frac{1}{20}$, $\det(2A) = -160$, $\det(AA^T) = 400$.

2. (a) Check that $0 \in S$ and $S$ is closed under vector addition and scalar multiplication. Another way to see that $S$ is a subspace is to observe that $S = \text{span}\{(1,0,2)^T, (0,1,1)^T\}$.

(b) A basis for $S$ is $\{(1,0,2)^T, (0,1,1)^T\}$, and $\dim S = 2$.

(c) An orthonormal basis for $S$ is $\{\frac{1}{\sqrt{5}}(1,0,2)^T, \frac{1}{\sqrt{30}}(-2,5,1)^T\}$.

3. (a) The vectors are linearly dependent, since $v_2 = 3v_1 + v_3$. A basis for $V$ is $\{v_1, v_3\}$.

(b) $v \notin V$ because the system $xv_1 + yv_3 = v$ has no solution.
4. (a) \( u_1 = \frac{1}{2}(1, 1, 1)^T, u_2 = \frac{1}{2}(-1, 1, 1)^T, u_3 = \frac{1}{2}(1, -1, 1)^T. \)

(b) \( v = (v \cdot u_1)u_1 + (v \cdot u_2)u_2 + (v \cdot u_3)u_3 = 7u_1 + 3u_2 + 4u_3. \)

5. (a) \( q_1 = \frac{1}{3}(2, 1, 2)^T, q_2 = \frac{\sqrt{2}}{6}(-1, 4, -1)^T. \)

(b) \( A = QR \) where
\[
Q = \begin{bmatrix}
2/3 & -\sqrt{2}/6 \\
1/3 & 2\sqrt{2}/3 \\
2/3 & -\sqrt{2}/6
\end{bmatrix}, \quad R = \begin{bmatrix}
3 & 5/3 \\
0 & \sqrt{2}/3
\end{bmatrix}.
\]

(c) \( A^TAX = A^Tb \) has a unique solution \( X = (9, -3)^T. \)

6. (a) The eigenvalues of \( A \) are 1 and \(-1.\)

(b) The eigenspace with eigenvalue 1 is \( \text{NS}(A - I) \) and has a basis \( \{(1, 0, 0)^T, (0, 1, 1)^T\} \). The eigenspace with eigenvalue \(-1\) is \( \text{NS}(A + I) \) and has a basis \( \{(0, 1, -1)^T\} \).

(c) We have
\[
P = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 1 \\
0 & 1 & -1
\end{bmatrix}, \quad D = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -1
\end{bmatrix}.
\]

(d) \( Q = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1/\sqrt{2} & 0 \\
0 & 1/\sqrt{2} & -1/\sqrt{2}
\end{bmatrix}. \)