1. (15%) Let 
\[ A = \begin{pmatrix} -5 & 2 & 6 \\ 0 & 1 & -2 \\ 25 & -14 & -18 \end{pmatrix}, \quad b = \begin{pmatrix} 6 \\ -5 \\ -12 \end{pmatrix} \]
(a) Find the LU decomposition of A.
(b) Solve the linear system AX = b.
(c) Evaluate: \( \det(A) \), \( \det(A^{-1}) \), \( \det(2A) \), \( \det(AA^T) \).

2. (15%) Let \( S = \{ (x_1, x_2, x_3)^T \in \mathbb{R}^3 | x_3 = 2x_1 + x_2 \} \).
(a) Show that S is a subspace.
(b) Find a basis and determine the dimension of S.
(c) Find an orthonormal basis for S.

3. (15%) Consider the vectors 
\[ v_1 = (1, 2, 1)^T, v_2 = (3, -1, 2)^T, v_3 = (0, -7, -1)^T. \]
(a) Is \( \{v_1, v_2, v_3\} \) linearly independent? If not, find a basis for the subspace \( V = \text{span}\{v_1, v_2, v_3\} \).
(b) Is \( v = (1, 2, 3) \in V? \) Justify your answer.

4. Let \( v_1 = (1, 1, 1, 1)^T; v_2 = (-1, 4, 4, -1)^T; v_3 = (4, -2, 2, 0)^T \).
(a) (15%) Apply Gram-Schmidt Process to \( \{v_1, v_2, v_3\} \) and obtain an orthonormal basis 
\( \{u_1, u_2, u_3\} \) for \( \text{span}\{v_1, v_2, v_3\} \).
(b) (5%) Write \( v = (4, 3, 7, 0)^T \) as a linear combination of \( \{u_1, u_2, u_3\} \).

5. (15%) Let 
\[ A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \\ 2 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 12 \\ 6 \\ 18 \end{pmatrix}. \]
(a) Find an orthonormal basis for the column space \( C(A) \).
(b) Find the QR- decomposition of A.
(c) Find the least square solution to \( AX = b \).

6. (20%) Let 
\[ A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \]
(a) Find the eigenvalues of A.
(b) Find a basis for each eigenspace of A.
(c) Find an invertible matrix \( P \) and a diagonal matrix \( D \) such that \( A = PDP^{-1} \).
(d) Find an orthogonal matrix \( Q \) such that \( A = QDQ^T \).